CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

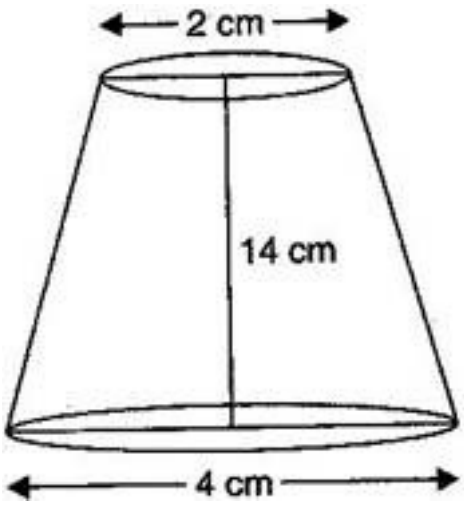
Surface Areas and Volumes -Exercise 13.4

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Ans. Here, $r_1 = \frac{4}{2}$ = 2 cm,

$r_2 = \frac{2}{2}$ = 1 cm and h = 14 cm



$\therefore$ Capacity of the glass = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 14\left(2 \times 2 + 1 \times 1 + 2 \times 1\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 7$

= $\frac{308}{3} = 102\frac{2}{3} cm^3$

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans. Let r_1 cm and r_2 cm be the radii of the ends $(r_1 > r_2)$ of the frustum of the cone.

Then, l = 4 cm

$2\pi r_1$ = 18 cm

$\Rightarrow \pi r_1$ = 9 cm

$2\pi r_2$ = 6 cm

$\Rightarrow \pi r_2$ = 3 cm

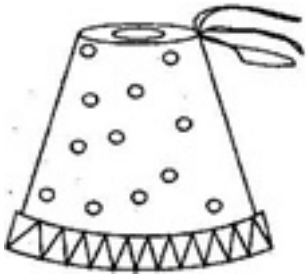
Now, CSA of the frustum = $\pi(r_1 + r_2)l$

= $(\pi r_1 + \pi r_2)l$

= $(9+3)\times 4 = 48\ cm^2$

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Ans.



Here, r_1 = 10 cm,

r_2 = 4 cm and l = 15 cm

$\therefore$ Surface area = $\pi(r_1+r_2)l+\pi r_2^2$

= $\frac{22}{7}(10+4)\times 15+\frac{22}{7}(4)^2$

= $660+\frac{352}{7}$ = $\frac{4972}{7}=710\frac{2}{7}cm^2$

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the total cost of milk which can completely fill the container at the rate of Rs. 20 per liter. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm^2. $(\text{Take } \pi = 3.14)$

Ans. Here, r_1 = 20 cm,

r_2 = 8 cm and h = 16 cm

$\therefore$ Volume of container = $\frac{1}{3}\pi h\left(r_1^2+r_2^2+r_1r_2\right)$

= $\frac{1}{3}\times 3.14\times 16\left\{(20)^2+(8)^2+20\times 8\right\}$

= $\frac{1}{3}\times 3.14\times 16(400+64+160)$

= $\frac{1}{3}\times 3.14\times 16\times 624$

= $10449.92\ cm^3$ = 10.44992 liters

$\therefore$ Cost of the milk = 10.44992×20

= Rs. 208.9984 = Rs. 209

Now, surface area = $\pi(r_1+r_2)l$ + πr_2^2

$= \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2} + \pi r_2^2$

$= 3.14(20+8)\sqrt{(16)^2 + (20-8)^2} + 3.14(8)^2$

$= 3.14 \times 28\sqrt{256+144} + 3.14 \times 64$

= 1758.4 + 200.96

$= 1959.36\ cm^2$

∴ Area of the metal sheet used = $1959.36\ cm^2$

∴ Cost of metal sheet = $1959.36 \times \frac{8}{100}$

= 156.7488 = Rs. 156.75

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans. $\tan 30^\circ = \frac{r_2}{10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{10}$

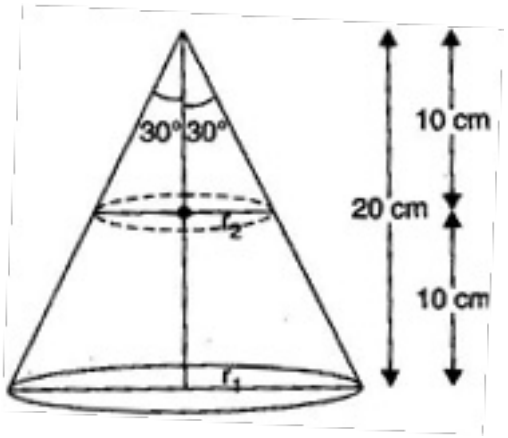
$\Rightarrow r_2 = \frac{10}{\sqrt{3}}$ cm

$\tan 30^\circ = \frac{r_1}{20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_1}{20}$

$\Rightarrow r_1 = \frac{20}{\sqrt{3}}$ cm

h = 10 cm



$\therefore$ Volume = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3} \times \frac{22}{7} \times 10\left\{\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)\left(\frac{10}{\sqrt{3}}\right)\right\}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \frac{700}{3} = \frac{22000}{9} cm^3$$

Diameter of the wire = $\frac{1}{16}$ cm

$\therefore$ Radius of the wire = $\frac{1}{32}$ cm

Let the length of the wire be l cm.

Then, Volume of the wire = $\pi r^2 l = \frac{22}{7}\left(\frac{1}{32}\right)^2 l = \frac{11l}{3584}\ cm^3$

According to the question,

$$\frac{11l}{3584} = \frac{22000}{9}$$

$$\Rightarrow l = \frac{22000 \times 3584}{11 \times 9}$$

$$\Rightarrow l = \frac{2000 \times 3584}{9}$$

$\Rightarrow l =$ 796444.44 cm = 7964.4 m