

CBSE Class-10 Mathematics
NCERT solution
Chapter - 5
Arithmetic Progressions - Exercise 5.1

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes one fourth of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Ans. (i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = $15 + 8 = \text{Rs } 23$

Taxi fare after 3 km = $23 + 8 = \text{Rs } 31$

Taxi fare after 4 km = $31 + 8 = \text{Rs } 39$

Therefore, the sequence is 15, 23, 31, 39...

It is an arithmetic progression because difference between any two consecutive terms is equal which is 8. ($23 - 15 = 8$, $31 - 23 = 8$, $39 - 31 = 8$, ...)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump = $V - \frac{V}{4} = \frac{4V - V}{4} = \frac{3V}{4}$

Amount of air left when vacuum pump again pumps out air

$$= \frac{3}{4}V - \left(\frac{1}{4} \times \frac{3}{4}V \right) = \frac{3}{4}V - \frac{3}{16}V = \frac{12V - 3V}{16} = \frac{9}{16}V$$

So, the sequence we get is like $V, \frac{3}{4}V, \frac{9}{16}V \dots$

Checking for difference between consecutive terms ...

$$\frac{3}{4}V - V = -\frac{V}{4}, \frac{9}{16}V - \frac{3}{4}V = \frac{9V - 12V}{16} = \frac{-3V}{16}$$

Difference between consecutive terms is not equal.

Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150

Cost of digging 2 meters of well = $150 + 50 = \text{Rs } 200$

Cost of digging 3 meters of well = $200 + 50 = \text{Rs } 250$

Therefore, we get a sequence of the form 150, 200, 250...

It is an arithmetic progression because difference between any two consecutive terms is equal. ($200 - 150 = 250 - 200 = 50 \dots$)

Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after 1st year = $10000 \left(1 + \frac{8}{100} \right) \dots (1)$

Amount in bank after two years = $10000 \left(1 + \frac{8}{100} \right)^2 \dots (2)$

Amount in bank after three years = $10000 \left(1 + \frac{8}{100} \right)^3 \dots (3)$

$$\text{Amount in bank after four years} = 10000 \left(1 + \frac{8}{100} \right)^4 \dots (4)$$

It is not an arithmetic progression because $(2) - (1) \neq (3) - (2)$

(Difference between consecutive terms is not equal)

Therefore, it is not an Arithmetic Progression.

2. Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Ans. (i) First term = $a = 10, d = 10$

Second term = $a + d = 10 + 10 = 20$

Third term = second term + $d = 20 + 10 = 30$

Fourth term = third term + $d = 30 + 10 = 40$

Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = $a = -2, d = 0$

Second term = $a + d = -2 + 0 = -2$

Third term = second term + $d = -2 + 0 = -2$

Fourth term = third term + $d = -2 + 0 = -2$

Therefore, first four terms are: $-2, -2, -2, -2$

(iii) First term = $a = 4$, $d = -3$

Second term = $a + d = 4 - 3 = 1$

Third term = second term + $d = 1 - 3 = -2$

Fourth term = third term + $d = -2 - 3 = -5$

Therefore, first four terms are: $4, 1, -2, -5$

(iv) First term = $a = -1$, $d = \frac{1}{2}$

Second term = $a + d = -1 + \frac{1}{2} = -\frac{1}{2}$

Third term = second term + $d = -\frac{1}{2} + \frac{1}{2} = 0$

Fourth term = third term + $d = 0 + \frac{1}{2} = \frac{1}{2}$

Therefore, first four terms are: $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(v) First term = $a = -1.25$, $d = -0.25$

Second term = $a + d = -1.25 - 0.25 = -1.50$

Third term = second term + $d = -1.50 - 0.25 = -1.75$

Fourth term = third term + d

$= -1.75 - 0.25 = -2.00$

Therefore, first four terms are: $-1.25, -1.50, -1.75, -2.00$

3. For the following APs, write the first term and the common difference.

(i) 3, 1, -1, -3 ...

(ii) -5, -1, 3, 7...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

(iv) 0.6, 1.7, 2.8, 3.9 ...

Ans. (i) 3, 1, -1, -3...

First term = $a = 3$,

Common difference (d) = Second term – first term = Third term – second term and so on

Therefore, Common difference (d) = $1 - 3 = -2$

(ii) -5, -1, 3, 7...

First term = $a = -5$

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = $-1 - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

First term = $a = \frac{1}{3}$

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9...

First term = $a = 0.6$

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = $1.7 - 0.6 = 1.1$

4. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

(iii) -1.2, -3.2, -5.2, -7.2...

(iv) -10, -6, -2, 2...

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(vi) 0.2, 0.22, 0.222, 0.2222...

(vii) 0, -4, -8, -12...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

(ix) 1, 3, 9, 27...

(x) $a, 2a, 3a, 4a, \dots$

(xi) a, a^2, a^3, a^4, \dots

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

(xiv) $1^2, 3^2, 5^2, 7^2 \dots$

(xv) $1^2, 5^2, 7^2, 73 \dots$

Ans. (i) 2, 4, 8, 16...

It is not an AP because difference between consecutive terms is not equal.

$$\text{As } 4 - 2 \neq 8 - 4$$

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow \frac{5}{2} - 2 = 3 - \frac{5}{2} = \frac{1}{2}$$

Common difference (d) = $\frac{1}{2}$

Fifth term = $\frac{7}{2} + \frac{1}{2} = 4$ Sixth term = $4 + \frac{1}{2} = \frac{9}{2}$

Seventh term = $\frac{9}{2} + \frac{1}{2} = 5$

Therefore, next three terms are 4, $\frac{9}{2}$ and 5.

(iii) $-1.2, -3.2, -5.2, -7.2 \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -3.2 - (-1.2)$$

$$= -5.2 - (-3.2)$$

$$= -7.2 - (-5.2) = -2$$

Common difference (d) = -2

Fifth term = $-7.2 - 2 = -9.2$ Sixth term = $-9.2 - 2 = -11.2$

Seventh term = $-11.2 - 2 = -13.2$

Therefore, next three terms are -9.2, -11.2 and -13.2

(iv) -10, -6, -2, 2...

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -6 - (-10) = -2 - (-6)$$

$$= 2 - (-2) = 4$$

Common difference (d) = 4

Fifth term = $2 + 4 = 6$ Sixth term = $6 + 4 = 10$

Seventh term = $10 + 4 = 14$

Therefore, next three terms are 6, 10 and 14

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 3 + \sqrt{2} - 3$$

$$= \sqrt{2}, 3 + 2\sqrt{2} - (3 + \sqrt{2})$$

$$= 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

Common difference (d) = $\sqrt{2}$

Fifth term = $3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

Sixth term = $3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

Seventh term = $3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

Therefore, next three terms are $(3 + 4\sqrt{2}), (3 + 5\sqrt{2}), (3 + 6\sqrt{2})$

(vi) 0.2, 0.22, 0.222, 0.2222...

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow 0.22 - 0.2 \neq 0.222 - 0.22$$

(vii) 0, -4, -8, -12...

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -4 - 0 = -8 - (-4)$$

$$= -12 - (-8) = -4$$

Common difference (d) = -4

Fifth term = $-12 - 4 = -16$ Sixth term = $-16 - 4 = -20$

Seventh term = $-20 - 4 = -24$

Therefore, next three terms are -16, -20 and -24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$$

Common difference (d) = 0

Fifth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$ Sixth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Seventh term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2}, -\frac{1}{2}$ and $-\frac{1}{2}$

(ix) 1, 3, 9, 27...

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow 3 - 1 \neq 9 - 3$$

(x) $a, 2a, 3a, 4a...$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 2a - a = 3a - 2a = 4a - 3a = a$$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are $5a, 6a$ and $7a$

(xi) $a, a^2, a^3, a^4...$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow a^2 - a \neq a^3 - a^2$$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}...$

$$\Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

$$\text{Seventh term} = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \dots$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow \sqrt{6} - \sqrt{3} \neq \sqrt{9} - \sqrt{6}$$

(xiv) $1^2, 3^2, 5^2, 7^2 \dots$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow 3^2 - 1^2 \neq 5^2 - 3^2$$

(xv) $1^2, 5^2, 7^2, 73 \dots$

$$\Rightarrow 1, 25, 49, 73 \dots$$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 5^2 - 1^2$$

$$= 7^2 - 5^2 = 73 - 7^2 = 24$$

Common difference (d) = 24

$$\text{Fifth term} = 73 + 24 = 97 \quad \text{Sixth term} = 97 + 24 = 121$$

$$\text{Seventh term} = 121 + 24 = 145$$

Therefore, next three terms are 97, 121 and 145