CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans. For sphere, Radius (r) = 4.2 cm

Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.2)^3 \ cm^3$

For cylinder, Radius (R) = 6 cm

Let the height of the cylinder be H cm.

Then, Volume = $\pi R^2 H = \pi(6)^2 H \ cm^3$

According to question, Volume of sphere = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi(4.2)^3 = \pi(6)^2 H$

$\Rightarrow H = \frac{4(4.2)^3}{3(6)^2}$

$\Rightarrow$ H = 2.744 cm

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans. Let the volume of resulting sphere be r cm.

According to question,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$$\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$$

$\Rightarrow r^3$ = 216 + 512 + 1000

$\Rightarrow r^3$ = 1728

$\Rightarrow r$ = 12 cm

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. Diameter of well = 7 m

$\therefore$ Radius of well (r) = $\frac{7}{2}$ m

And Depth of earth dug (h) = 20 m

Length of platform (l) = 22 m, Breadth of platform (b) = 14 m

Let height of the platform be h' m

According to question,

Volume of earth dug = Volume of platform

$$\Rightarrow \pi r^2 h = l \times b \times h'$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h'$$

$$\Rightarrow h' = \frac{22 \times 7 \times 7 \times 20}{28 \times 22 \times 14}$$

$\Rightarrow h'$ = 2.5 m

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans. Diameter of well = 3 m

$\therefore$ Radius of well $(r) = \frac{3}{2}$ m and Depth of earth dug (h) = 14 m

Width of the embankment = 4 m

$\therefore$ Radius of the well with embankment $r' = \frac{3}{2}+4=\frac{11}{2}$ m

Let the height of the embankment be h' m

According to the question,

Volume of embankment = Volume of the earth dug

$$\Rightarrow \pi\left[(r')^2-r^2\right]h' = \pi r^2 h$$

$$\Rightarrow \left[\left(\frac{11}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]h' = \left(\frac{3}{2}\right)^2\times 14$$

$$\Rightarrow \left[\frac{121}{4}-\frac{9}{4}\right]h'=\frac{9}{4}\times 14$$

$$\Rightarrow \frac{112}{4}\times h'=\frac{9}{4}\times 14$$

$$\Rightarrow h'=\frac{9\times 14\times 4}{4\times 112}$$

$\Rightarrow h'$ = 1.125 m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15

cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. For right circular cylinder, Diameter = 12 cm

$\therefore$ Radius $(r) = \frac{12}{2}$ = 6 cm and height (h) = 15 cm

For cone & Hemisphere , Diameter = 6 cm

$\therefore$ Radius $(r_1) = \frac{6}{2}$ = 3 cm and height (h_1) = 12 cm

Let n cones be filled with ice cream.

Then, According to question,

Volume of n (cones +Hemisphere) = Volume of right circular cylinder

$\Rightarrow n \times \left(\frac{1}{3}\pi(r_1)^2(h) + \frac{2}{3}\pi(r_1)^3\right) = \pi r^2 h$

$\Rightarrow n\left(\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3\right) = \frac{22}{7} \times (6)^2 \times 15$

$\Rightarrow n = \frac{22\times36\times15\times3\times7}{(7\times22\times9\times12+7\times44\times27)} = \frac{249480}{24948}$

$\Rightarrow n$ = 10

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5\ \text{cm} \times 10\ \text{cm} \times 3.5\ \text{cm}$?

Ans. For silver coin, Diameter = 1.75 cm

$\therefore$ Radius $(r) = \frac{1.75}{2} = \frac{7}{8}$ cm and Thickness (h) = 2 mm = $\frac{1}{5}$ cm

For cuboid, Length (l) = 5.5 cm, Breadth (b) = 10 cm and Height (h') = 3.5 cm

Let n coins be melted.

Then, According to question,

Volume of n coins = Volume of cuboid

$\Rightarrow n \times \pi r^2 h = l \times b \times h'$

$\Rightarrow n \times \pi \left(\frac{7}{8}\right)^2 \times \left(\frac{1}{5}\right) = 5.5 \times 10 \times 3.5$

$\Rightarrow n \times \frac{22}{7} \times \frac{49}{64} \times \frac{1}{5} = 5.5 \times 10 \times 3.5$

$\Rightarrow n = \frac{5.5 \times 10 \times 3.5 \times 7 \times 64 \times 5}{22 \times 49}$

$\Rightarrow n$ = 400

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Ans. For cylindrical bucket, Radius of the base (r) = 18 cm and height (h) = 32 cm

$\therefore$ Volume = $\pi r^2 h$ = $\pi (18)^2 \times 32$

= $10368\pi \ cm^3$

For conical heap, Height (h') = 24 cm

Let the radius be r_1 cm.

Then, Volume = $\frac{1}{3} \pi r_1^2 h'$

= $\frac{1}{3} \times \pi \times r_1^2 \times 24$ = $8\pi r_1^2 \ cm^3$

According to question, Volume of bucket = Volume of conical heap

$\Rightarrow 10368\pi = 8\pi r_1^2$

$\Rightarrow r_1^2 = \frac{10368\pi}{8\pi}$ = 1296

$\Rightarrow r_1$ = 36 cm

Now, Slant height (l) = $\sqrt{(r_1)^2+(h')^2}$

= $\sqrt{(36)^2+(24)^2}$ = $\sqrt{1296+576}$

= $\sqrt{1872}$ = $12\sqrt{13}$ cm

8. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Ans. For canal, Width = 6 m and Depth = 1.5 m = $\frac{3}{2}$ m

Speed of flow of water = 10 km/h

= $10\times1000\ m/h$ = 10000 m/h

= $\frac{10000}{60}$ m/min = $\frac{500}{3}$ m/min

∴ Speed of flow of water in 30 minutes

= $\frac{500\times30}{3}$ m/min = 5000 m/min

∴ Volume of water that flows in 30 minutes

= $6\times\frac{3}{2}\times5000$ = $45000\ m^3$

∴ The area it will irrigate = $\frac{45000}{\left(\frac{8}{100}\right)} = \frac{4500000}{8}$

= $562500\ m^2$

= $\frac{562500}{10000}$ hectares = 56.25 hectares

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Ans. For cylindrical tank, Diameter = 10 m

∴ Radius $(r) = \frac{10}{2}$ = 5 m and Depth (h) = 2 m

∴ Volume = $\pi r^2 h$ = $\pi(5)^2 \times 2$ = $50\pi m^3$

Rate of flow of water (h') = 3 km/h = 3000 m/h = $\frac{3000}{60}$ m/min = 50 m/min

For pipe, Internal diameter = 20 cm, therefore radius (r_1) = 10 cm = 0.1 m

∴ Volume of water that flows per minute = $\pi(r_1)^2 h'$

= $\pi(0.1)^2 \times 50$ = $\frac{\pi}{2} m^3$

∴ Required time = $\frac{50\pi}{\pi/2}$ = 100 minutes