

CBSE Class-10 Mathematics

NCERT solution

Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(ii) $2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans. (i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Comparing equation $x - 3y - 3 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 9y - 2 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -3, a_2 = 3, b_2 = -9, c_2 = -2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

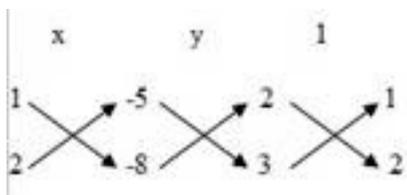
(ii) $2x + y = 5$

$3x + 2y = 8$

Comparing equation $2x + y = 5$ with $a_1x + b_1y + c_1 = 0$ and $3x + 2y = 8$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 2, b_1 = 1, c_1 = -5, a_2 = 3, b_2 = 2, c_2 = -8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1) - (2)(-5)} = \frac{y}{(-5)(3) - (-8)(2)} = \frac{1}{(2)(2) - (3)(1)}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow x = 2$ and $y = 1$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

Comparing equation $3x - 5y = 20$ with $a_1x + b_1y + c_1 = 0$ and $6x - 10y = 40$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = -5, c_1 = -20, a_2 = 6, b_2 = -10, c_2 = -40$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinitely many solutions.

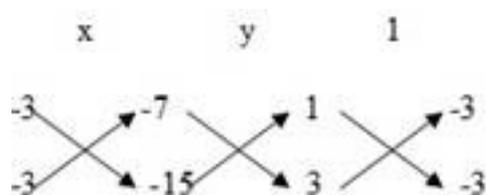
(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Comparing equation $x - 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 3y - 15 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -7, a_2 = 3, b_2 = -3, c_2 = -15$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15) - (-3)(-7)} = \frac{y}{(-7)(3) - (-15)(1)} = \frac{1}{(-3)(1) - (-3)(3)}$$

$$\Rightarrow \frac{x}{45 - 21} = \frac{y}{-21 + 15} = \frac{1}{-3 + 9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\Rightarrow x = 4 \text{ and } y = -1$$

2. (i) For which values of a and b does the following pair of linear equations have an

infinite number of solutions?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

Ans. (i) Comparing equation $2x + 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $(a - b)x + (a + b)y - 3a - b + 2 = 0$ with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2, b_1 = 3$ and $c_1 = -7, a_2 = (a - b), b_2 = (a + b)$ and $c_2 = 2 - b - 3a$

Linear equations have infinite many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-b-3a}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} \text{ and } \frac{3}{a+b} = \frac{-7}{2-b-3a}$$

$$\Rightarrow 2a + 2b = 3a - 3b \text{ and } 6 - 3b - 9a = -7a - 7b$$

$$\Rightarrow a = 5b \dots (1) \text{ and } -2a = -4b - 6 \dots (2)$$

Putting (1) in (2), we get

$$-2(5b) = -4b - 6$$

$$\Rightarrow -10b + 4b = -6$$

$$\Rightarrow -6b = -6 \Rightarrow b = 1$$

Putting value of b in (1), we get

$$a = 5b = 5(1) = 5$$

Therefore, $a = 5$ and $b = 1$

(ii) Comparing $(3x + y - 1 = 0)$ with $a_1x + b_1y + c_1 = 0$ and $(2k - 1)x + (k - 1)y - 2k - 1 = 0)$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = 1$ and $c_1 = -1, a_2 = (2k - 1), b_2 = (k - 1)$ and $c_2 = -2k - 1$

Linear equations have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{-1}{-2k - 1}$$

$$\Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1}$$

$$\Rightarrow 3(k - 1) = 2k - 1$$

$$\Rightarrow 3k - 3 = 2k - 1$$

$$\Rightarrow k = 2$$

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Ans. Substitution Method

$$8x + 5y = 9 \dots (1)$$

$$3x + 2y = 4 \dots (2)$$

From equation (1),

$$5y = 9 - 8x \Rightarrow y = \frac{9 - 8x}{5}$$

Putting this in equation (2), we get

$$3x + 2 \left(\frac{9 - 8x}{5} \right) = 4$$

$$\Rightarrow 3x + \frac{18 - 16x}{5} = 4$$

$$\Rightarrow 3x - \frac{16}{5}x = \frac{4}{1} - \frac{18}{5}$$

$$\Rightarrow 15x - 16x = 20 - 18$$

$$\Rightarrow x = -2$$

Putting value of x in (1), we get

$$8(-2) + 5y = 9$$

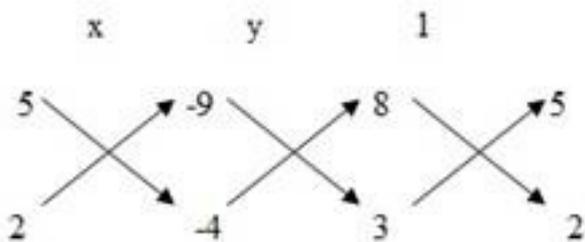
$$\Rightarrow 5y = 9 + 16 = 25 \Rightarrow y = 5$$

Therefore, $x = -2$ and $y = 5$

Cross multiplication method

$$8x + 5y = 9 \dots (1)$$

$$3x + 2y = 4 \dots (2)$$



$$\frac{x}{5(-4) - 2(-9)} = \frac{y}{(-9)3 - (-4)8} = \frac{1}{8 \times 2 - 5 \times 3}$$

$$\Rightarrow \frac{x}{-20 + 18} = \frac{y}{-27 + 32} = \frac{1}{16 - 15}$$

$$\Rightarrow \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\Rightarrow x = -2 \text{ and } y = 5$$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the

rectangle.

Ans.(i) Let fixed monthly charge = Rs x and let charge of food for one day = Rs y

According to given conditions,

$$x + 20y = 1000 \dots (1),$$

$$\text{and } x + 26y = 1180 \dots (2)$$

Subtracting equation (1) from equation (2), we get

$$6y = 180$$

$$\Rightarrow y = 30$$

Putting value of y in (1), we get

$$x + 20(30) = 1000$$

$$\Rightarrow x = 1000 - 600 = 400$$

Therefore, fixed monthly charges = Rs 400 and, charges of food for one day = Rs 30

(ii) Let numerator = x and let denominator = y

According to given conditions,

$$\frac{x-1}{y} = \frac{1}{3} \dots(1) \quad \frac{x}{y+8} = \frac{1}{4} \dots(2)$$

$$\Rightarrow 3x - 3 = y \dots (1) \quad 4x = y + 8 \dots (1)$$

$$\Rightarrow 3x - y = 3 \dots (1) \quad 4x - y = 8 \dots (2)$$

Subtracting equation (1) from (2), we get

$$4x - y - (3x - y) = 8 - 3$$

$$\Rightarrow x = 5$$

Putting value of x in (1), we get

$$3(5) - y = 3$$

$$\Rightarrow 15 - y = 3$$

$$\Rightarrow y = 12$$

Therefore, numerator = 5 and, denominator = 12

$$\text{It means fraction} = \frac{x}{y} = \frac{5}{12}$$

(iii) Let number of correct answers = x and let number of wrong answers = y

According to given conditions,

$$3x - y = 40 \dots (1)$$

$$\text{And, } 4x - 2y = 50 \dots (2)$$

From equation (1), $y = 3x - 40$

Putting this in (2), we get

$$4x - 2(3x - 40) = 50$$

$$\Rightarrow 4x - 6x + 80 = 50$$

$$\Rightarrow -2x = -30$$

$$\Rightarrow x = 15$$

Putting value of x in (1), we get

$$3(15) - y = 40$$

$$\Rightarrow 45 - y = 40$$

$$\Rightarrow y = 45 - 40 = 5$$

Therefore, number of correct answers = $x = 15$ and number of wrong answers = $y = 5$

Total questions = $x + y = 15 + 5 = 20$

(iv) Let speed of car which starts from part A = x km/hr

Let speed of car which starts from part B = y km/hr

According to given conditions,

$$\frac{100}{x-y} = 5 \text{ (Assuming } x > y)$$

$$\Rightarrow 5x - 5y = 100$$

$$\Rightarrow x - y = 20 \dots (1)$$

$$\text{And, } \frac{100}{x+y} = 1$$

$$\Rightarrow x + y = 100 \dots (2)$$

Adding (1) and (2), we get

$$2x = 120$$

$$\Rightarrow x = 60 \text{ km/hr}$$

Putting value of x in (1), we get

$$60 - y = 20$$

$$\Rightarrow y = 60 - 20 = 40 \text{ km/hr}$$

Therefore, speed of car starting from point A = 60 km/hr

And, Speed of car starting from point B = 40 km/hr

(v) Let length of rectangle = x units and Let breadth of rectangle = y units

Area = xy square *units*. According to given conditions,

$$xy - 9 = (x - 5)(y + 3)$$

$$\Rightarrow xy - 9 = xy + 3x - 5y - 15$$

$$\Rightarrow 3x - 5y = 6 \dots (1)$$

And, $xy + 67 = (x + 3)(y + 2)$

$$\Rightarrow xy + 67 = xy + 2x + 3y + 6$$

$$\Rightarrow 2x + 3y = 61 \dots (2)$$

From equation (1),

$$3x = 6 + 5y$$

$$\Rightarrow x = \frac{6 + 5y}{3}$$

Putting this in (2), we get

$$2 \left(\frac{6 + 5y}{3} \right) + 3y = 61$$

$$\Rightarrow 12 + 10y + 9y = 183$$

$$\Rightarrow 19y = 171$$

$$\Rightarrow y = 9 \text{ units}$$

Putting value of y in (2), we get

$$2x + 3(9) = 61$$

$$\Rightarrow 2x = 61 - 27 = 34$$

$$\Rightarrow x = 17 \text{ units}$$

Therefore, length = 17 units and, breadth = 9 units