CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.3

1. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\cos ec 31^\circ - \sec 59^\circ$

Ans. (i) $\frac{\sin 18^\circ}{\cos 72^\circ}$ = $\frac{\sin\left(90^\circ - 72^\circ\right)}{\cos 72^\circ}$

= $\frac{\cos 72^\circ}{\cos 72^\circ}$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

= 1

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$ = $\frac{\tan\left(90^\circ - 64^\circ\right)}{\cot 64^\circ}$

= $\frac{\cot 64^\circ}{\cot 64^\circ}$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

= 1

(iii) $\cos 48^\circ - \sin 42^\circ$

= $\cos\left(90^\circ - 42^\circ\right) - \sin 42^\circ$

= $\sin 42^\circ - \sin 42^\circ$ [Since $\cos(90^\circ - \theta) = \sin\theta$]

= 0

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

= $\operatorname{cosec}\left(90^\circ - 59^\circ\right) - \sec 59^\circ$

= $\sec 59^\circ - \sec 59^\circ$ [Since $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$]

=0

2. Show that:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Ans. (i) L.H.S. $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

= $\tan\left(90^\circ - 42^\circ\right)\tan\left(90^\circ - 67^\circ\right)\tan 42^\circ \tan 67^\circ$

= $\cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

= $\dfrac{1}{\tan 42^\circ} \cdot \dfrac{1}{\tan 67^\circ} \cdot \tan 42^\circ \cdot \tan 67^\circ$ = 1 = R.H.S.

(ii) R.H.S. $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

= $\cos\left(90^\circ - 52^\circ\right) \cdot \cos\left(90^\circ - 38^\circ\right) - \sin 38^\circ \cdot \sin 52^\circ$

= $\sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$ = 0 = R.H.S.

3. If $\tan 2A = \cot\left(A - 18^\circ\right)$, where 2A is an acute angle, find the value of A.

Ans. Given: $\tan 2A = \cot\left(A - 18^\circ\right)$

$\Rightarrow \cot\left(90^\circ - 2A\right) = \cot\left(A - 18^\circ\right)$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow -2A - A = -18^\circ - 90^\circ$

$\Rightarrow -3A = -108^\circ$

$\Rightarrow$ A = 36°

4. If $\tan A = \cot B$, **prove that** $A + B = 90^\circ$.

Ans. Given: $\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow 90^\circ = A + B$

$\Rightarrow$ A + B = 90°

5. If $\sec 4A = \cos ec(A - 20^\circ)$, **where 4A is an acute angle, find the value of A.**

Ans. Given: $\sec 4A = \cos ec(A - 20^\circ)$

$\Rightarrow \cos ec(90^\circ - 4A) = \cos ec(A - 20^\circ)$ [Since $\sec(90^\circ - \theta) = \cos ec\theta$]

$\Rightarrow 90^\circ - 4A = A - 20^\circ$

$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow -5A = -110^\circ$

$\Rightarrow$ A = 22°

6. If A, B and C are interior angles of a Δ**ABC, then show that** $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Ans. Given: A, B and C are interior angles of a $\triangle$ABC.

$\therefore$ A + B + C = $180°$ [Triangle sum property]

Dividing both sides by 2, we get

$$\Rightarrow \frac{A+B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{A}{2} + \frac{B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67^\circ + \cos 75^\circ$

= $\sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$ [Since $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$]

= $\cos 23^\circ + \sin 15^\circ$