CBSE Class–10 Mathematics

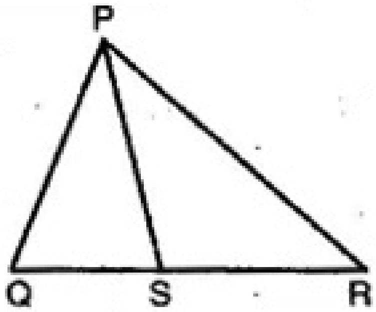
NCERT solution

Chapter - 6

Triangles - Exercise 6.6 (Optional)*

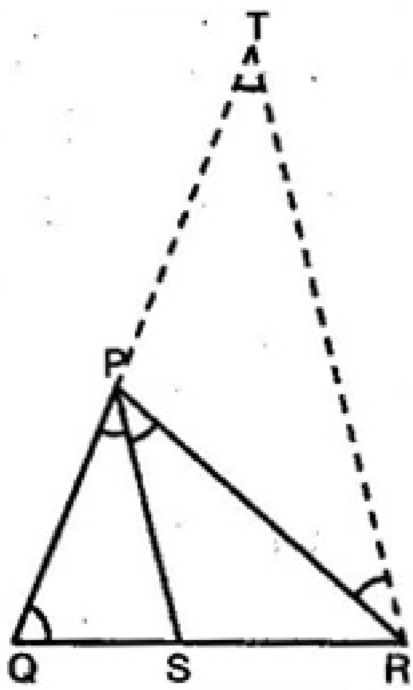
1. In the given figure, PS is the bisector of $\angle$ QPR of $\triangle$ PQR. Prove that

$$\frac{QS}{SR}=\frac{PQ}{PR}.$$



Ans. Given: PQR is a triangle and PS is the internal bisector of $\angle$ QPR

meeting QR at S.



$\therefore$ $\angle$ QPS = $\angle$ SPR

To prove: $\frac{QS}{SR}=\frac{PQ}{PR}$

Construction: Draw RT $\parallel$ SP to cut QP produced at T.

Proof: Since PS $\parallel$ TR and PR cuts them, hence,

$\angle$SPR = $\angle$PRT ....(i) [Alternate $\angle$s]

And $\angle$QPS = $\angle$PTR ....(ii)[Corresponding $\angle$s]

But $\angle$QPS = $\angle$SPR [Given]

$\therefore$ $\angle$PRT = $\angle$PTR[From eq. (i) & (ii)]

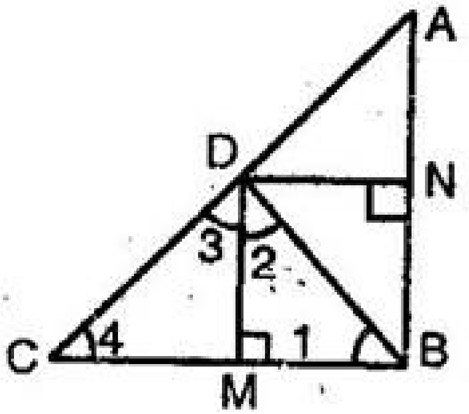
$\Rightarrow$ PT = PR..........(iii)

[Sides opposite to equal angles are equal]

Now, in $\triangle$QRT,

RT $\parallel$ SP[By construction]

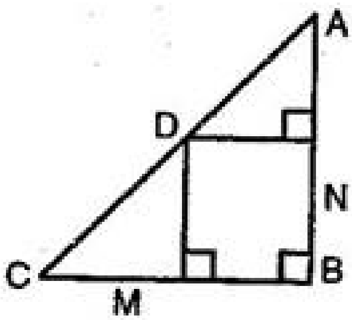
$\therefore \frac{QS}{SR}=\frac{PQ}{PT}$ [Thales theorem]



$\Rightarrow \frac{QS}{SR}=\frac{PQ}{PR}$ [From eq. (iii)]

2. In the given figure, D is a point on hypotenuse AC of $\triangle$ABC, BD $\perp$ AC, DM $\perp$ BC and

DN $\perp$ AB. Prove that:



(i) DM^2 = DN.MC

(ii) DN^2 = DM.AN

Ans. Since AB $\perp$ BC and DM $\perp$ BC

$\Rightarrow$ AB $||$ DM

Similarly, BC $\perp$ AB and DN $\perp$ AB

$\Rightarrow$ CB $||$ DN

$\therefore$ quadrilateral BMDN is a rectangle.

$\therefore$ BM = ND

(i) In ΔBMD, $\angle 1 + \angle BMD + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + 90^\circ + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

Similarly, in ΔDMC, $\angle 3 + \angle 4 = 90^\circ$

Since BD $\perp$ AC,

$\therefore \angle 2 + \angle 3 = 90^\circ$

Now, $\angle 1 + \angle 2 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3$

Also, $\angle 3 + \angle 4 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 4 = \angle 2$

Thus, in ΔBMD and ΔDMC,

$\angle 1 = \angle 3$ and $\angle 4 = \angle 2$

$\therefore \Delta BMD \sim \Delta DMC$

$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ [BM = ND]

$\Rightarrow DM^2 = DN.MC$

(ii) Processing as in (i), we can prove that

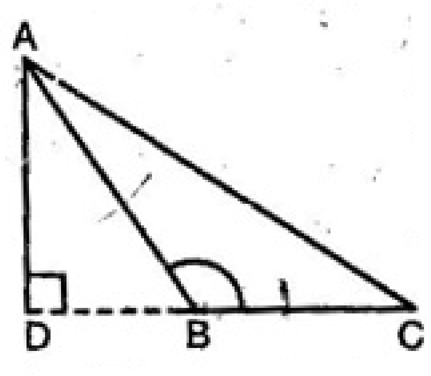
$\Delta BND \sim \Delta DNA$

$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA}$

$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN}$ [BN = DM]

$\Rightarrow DN^2 = DM.AN$

3. **In the given figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and AD $\perp$ CB produced. Prove that:**



$AC^2 = AB^2 + BC^2 + 2BC$ **.BD**

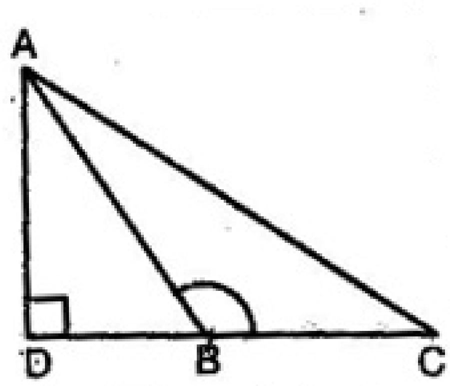
Ans. Given: ABC is a triangle in which $\angle$ABC > 90° and AD $\perp$ CB produced.

To prove: $AC^2 = AB^2 + BC^2 + 2BC$ .BD

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ ....(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,



$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB$ .BC

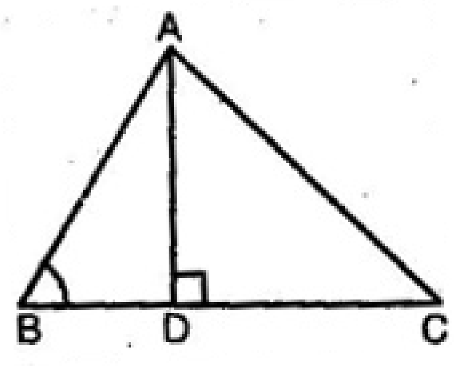
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 + 2DB$ .BC

$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB$ .BC

[Using eq. (i)]

4. **In the given figure, ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced. Prove that:**

$$AC^2 = AB^2 + BC^2 - 2BC.BD$$



Ans. Given: ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced.

To prove: $AC^2 = AB^2 + BC^2 - 2BC.BD$

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ....(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

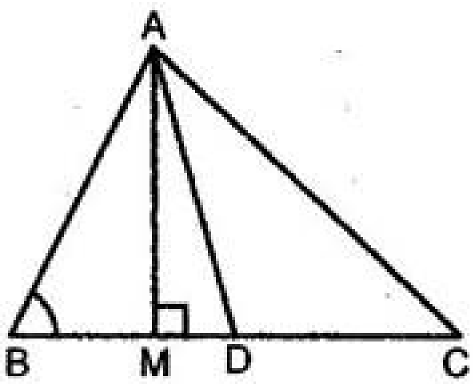
$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AD^2 + BC^2 + BD^2 - 2BC.BD$

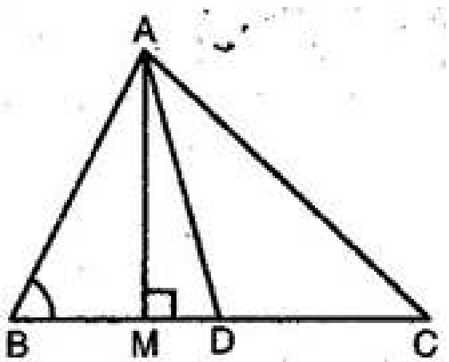
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 - 2DB.BC$

$\Rightarrow AC^2 = AB^2 + BC^2 - 2DB.BC$



[Using eq. (i)]

5. In the given figure, AD is a median of a triangle ABC and AM $\perp$ BC. Prove that:



(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC$

Ans. Since $\angle$ AMD = 90°, therefore $\angle$ ADM < 90° and $\angle$ ADC > 90°

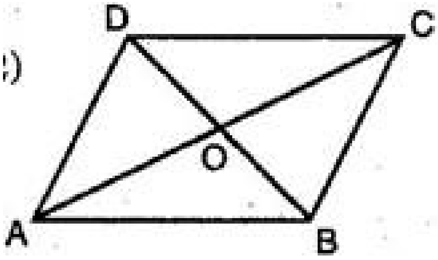
Thus, $\angle$ ADC is the acute angle and $\angle$ ADC is an obtuse angle.

(i) In $\triangle$ADC, $\angle$ ADC is an obtuse angle.

$\therefore AC^2 = AD^2 + DC^2 + 2DC.DM$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\frac{BC}{2}.DM$$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM$$



$$\Rightarrow AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 \text{..........(i)}$$

(ii) In $\triangle$ABD, $\angle$ADM is an acute angle.

$$AB^2 = AD^2 + BD^2 - 2BD.DM$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2.\frac{BC}{2}.DM$$

$$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 \text{..........(ii)}$$

(iii) From eq. (i) and eq. (ii),

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

6. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Ans. If AD is a median of $\triangle$ABC, then

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2 \text{ [See Q.5 (iii)]}$$

Since the diagonals of a parallelogram bisect each other, therefore, BO and DO are medians of triangles ABC and ADC respectively.

$\therefore AB^2 + BC^2 = 2BO^2 + \frac{1}{2} AC^2$ ....(i)

And $AD^2 + CD^2 = 2DO^2 + \frac{1}{2} AC^2$ ....(ii)

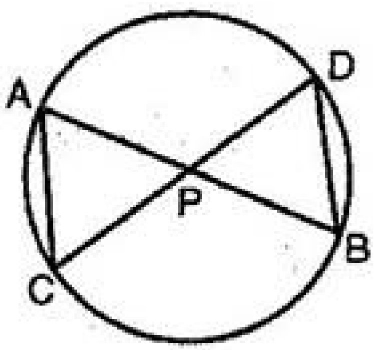
Adding eq. (i) and (ii),

$AB^2 + BC^2 + AD^2 + CD^2 = 2(BO^2 + DO^2) + AC^2$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = 2\left(\frac{1}{4}BD^2 + \frac{1}{4}BD^2\right) + AC^2 \left[DO = \frac{1}{2}BD\right]$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$

7. In the given figure, two chords AB and CD intersect each other at the point P. Prove that:



(i) $\triangle$APC $\sim$ $\triangle$DPB

(ii) AP.PB = CP.DP

Ans. (i) In the triangles APC and DPB,

$\angle$APC = $\angle$DPB [Vertically opposite angles]

$\angle$CAP = $\angle$BDP [Angles in same segment of a circle are equal]

$\therefore$ By AA-criterion of similarity,

$\triangle$APC $\sim$ $\triangle$DPB

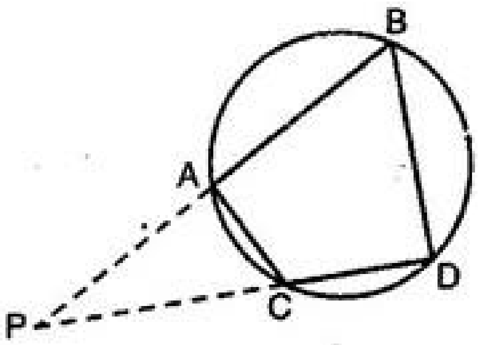
(ii) Since $\triangle$APC $\sim$ $\triangle$DPB

$\therefore \frac{AP}{DP}=\frac{CP}{PB} \Rightarrow$ AP x PB = CP x DP

8. In the give figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) $\triangle$PAC $\sim$ $\triangle$PDB

(ii) PA.PB = PC.PD



Ans. (i) In the triangles PAC and PDB,

$\angle$APC = $\angle$DPB [Common]

$\angle$CAP = $\angle$BDP [$\because$ $\angle$BAC = $180^\circ - \angle$PAC and $\angle$PDB = $\angle$CDB]

= $180^\circ - \angle$BAC = $180^\circ - (180^\circ - \angle PAC)$ = $\angle$PAC]

$\therefore$ By AA-criterion of similarity,

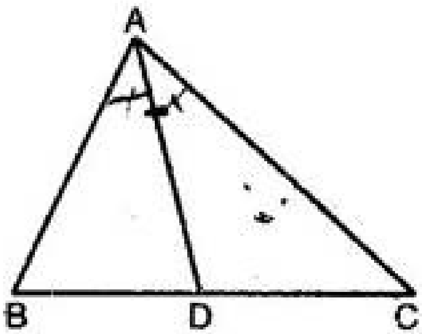
$\triangle$APC $\sim$ $\triangle$DPB

(ii) Since $\triangle$APC $\sim$ $\triangle$DPB

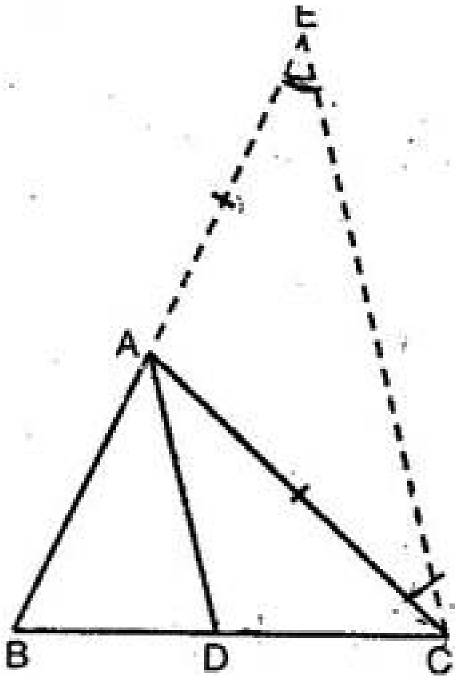
$\therefore \frac{AP}{DP}=\frac{CP}{PB}$

$\Rightarrow$ PA.PB = PC.PD

9. In the given figure, D is appointing on side BC of $\triangle$ABC such that $\frac{BD}{CD}=\frac{AB}{AC}$. Prove that AD is the bisector of $\angle$ BAC.



Ans. Given: ABC is a triangle and D is a point on BC such that $\frac{BD}{CD}=\frac{AB}{AC}$



To prove: AD is the internal bisector of $\angle$ BAC.

Construction: Produce BA to E such that AE = AC. Join CE.

Proof: In $\triangle$AEC, since AE = AC

$\therefore$ $\angle$AEC = $\angle$ACE ....(i)

[Angles opposite to equal side of a triangle are equal]

Now, $\frac{BD}{CD} = \frac{AB}{AC}$ [Given]

$\Rightarrow \frac{BD}{CD} = \frac{AB}{AE}$ [$\because$ AE = AC, by construction]

$\therefore$ By converse of Basic Proportionality Theorem,

DA $\parallel$ CE

Now, since CA is a transversal,

$\therefore$ $\angle$BAD = $\angle$AEC ....(ii) [Corresponding $\angle$s]

And $\angle$DAC = $\angle$ACE ....(iii) [Alternate $\angle$s]

Also $\angle$AEC = $\angle$ACE [From eq. (i)]

Hence, $\angle$BAD = $\angle$DAC [From eq. (ii) and (iii)]

Thus, AD bisects $\angle$BAC internally.

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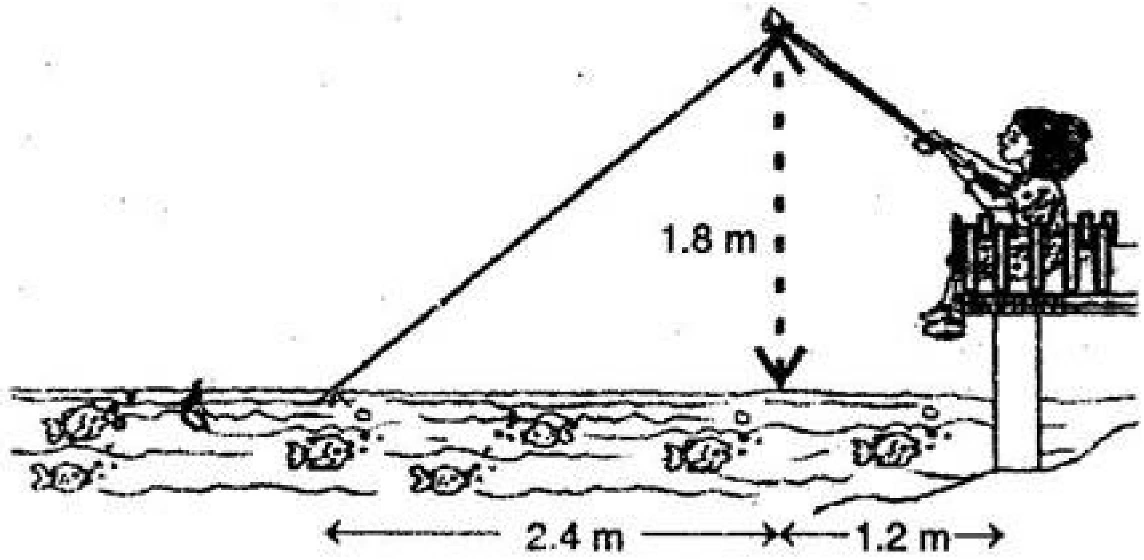
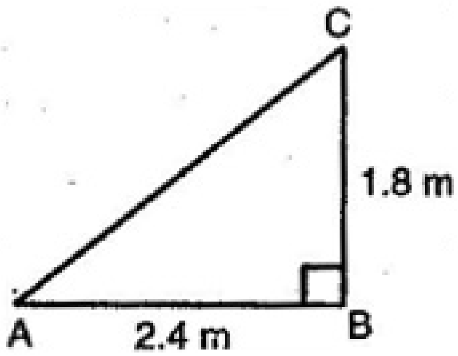
10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig.)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Ans. I. To find The length of AC.



By Pythagoras theorem,

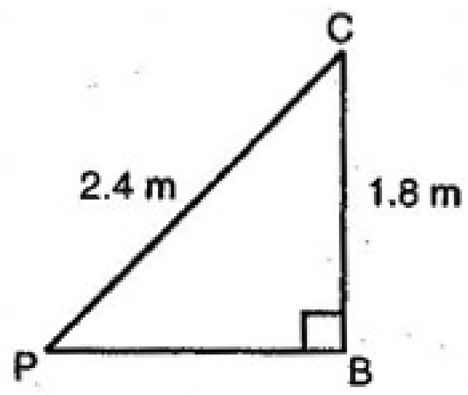
$AC^2 = (2.4)^2 + (1.8)^2$

$\Rightarrow AC^2 = 5.76 + 3.24 = 9.00$

$\Rightarrow$ AC = 3 m

$\therefore$ Length of string she has out= 3 m

Length of the string pulled at the rate of 5 cm/sec in 12 seconds



= (5 x 12) cm = 60 cm = 0.60 m

∴ Remaining string left out = 3 – 0.6 = 2.4 m

II. To find: The length of PB

$PB^2 = PC^2 - BC^2$

$= (2.4)^2 - (1.8)^2$

= 5.76 – 3.24 = 2.52

$\Rightarrow PB = \sqrt{2.52}$ = 1.59 (approx.)

Hence, the horizontal distance of the fly from Nazima after 12 seconds

= 1.59 + 1.2 = 2.79 m (approx.)