CBSE Class–10 Mathematics
NCERT solution
Chapter - 1
Real Numbers - Exercise 1.3

1. Prove that $\sqrt{5}$ is irrational.

Ans. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers **a and b** ($b \neq 0$) such that $\sqrt{5} = \dfrac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ ... **(1)**

It means that 5 is factor of a^2

Hence, **5 is also factor of a** by Theorem. ... **(2)**

If, **5 is factor of a**, it means that we can write **a = 5c** for some integer **c**.

Substituting value of **a** in **(1)**,

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2.

Hence, **5 is also factor of b** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that **5** is factor of both **a and b**.

But, **a and b** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

2. Prove that $(3 + 2\sqrt{5})$ is irrational.

Ans. We will prove this by contradiction.

Let us suppose that $(3+2\sqrt{5})$ is rational.

It means that we have co-prime integers **a and b** ($b \neq 0$) such that

$$\frac{a}{b} = 3 + 2\sqrt{5} \implies \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\implies \frac{a-3b}{b} = 2\sqrt{5}$$

$$\implies \frac{a-3b}{2b} = \sqrt{5} \, ... \, \textbf{(1)}$$

a and **b** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore, our supposition is wrong. $(3+2\sqrt{5})$ cannot be rational.

Hence, $(3+2\sqrt{5})$ is irrational.

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3. Prove that the following are irrationals.

(i) $\dfrac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Ans. (i) We can prove $\dfrac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\dfrac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$) such that

$$\dfrac{1}{\sqrt{2}} = \dfrac{a}{b}$$

$$\Rightarrow \sqrt{2} = \dfrac{b}{a} \quad \text{... (1)}$$

R.H.S of **(1)** is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\dfrac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

(ii) We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$) such that

$$7\sqrt{5} = \dfrac{a}{b}$$

$$\Rightarrow \sqrt{5} = \dfrac{a}{7b} \quad \text{... (1)}$$

R.H.S of **(1)** is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

(iii) We will prove $6 + \sqrt{2}$ irrational by contradiction.

Let us suppose that ($6 + \sqrt{2}$) is rational.

It means that we have co-prime integers **a and b** ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a - 6b}{b} \quad \text{... (1)}$$

a and b are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6 + \sqrt{2}$) cannot be rational.

Hence, ($6 + \sqrt{2}$) is irrational.