CBSE Class–10 Mathematics

NCERT solution

Chapter - 4

Quadratic Equations -Exercise 4.3

1. Find the roots of the following quadratic equations if they exist by the method of completing square.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) $2x^2 - 7x + 3 = 0$

First we divide equation by 2 to make coefficient of x^2 equal to 1,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

We divide middle term of the equation by 2*x*, we get $\frac{7}{2}x \times \frac{1}{2x} = \frac{7}{4}$

We add and subtract square of $\frac{7}{4}$ from the equation $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$,

=> $x^2 - \frac{7}{2}x + \frac{3}{2} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = 0$

$\Rightarrow x^2 + \left(\frac{7}{4}\right)^2 - \frac{7}{2}x + \frac{3}{2} - \left(\frac{7}{4}\right)^2 = 0$

$\left\{(a-b)^2 = a^2 + b^2 - 2ab\right\}$

$$\Rightarrow \left(x-\frac{7}{4}\right)^2+\frac{24-49}{16}=0$$

$$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{49-24}{16}$$

$$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{25}{16}$$

Taking Square root on both sides,

$$\Rightarrow x-\frac{7}{4}=\pm\frac{5}{4}$$

$$\Rightarrow x=\frac{5}{4}+\frac{7}{4}=\frac{12}{4}=3 \ and \ x=-\frac{5}{4}+\frac{7}{4}=\frac{2}{4}=\frac{1}{2}$$

Therefore, $x=\frac{1}{2},3$

(ii) $2x^2+x-4=0$

Dividing equation by 2,

$$x^2+\frac{x}{2}-2=0$$

Following procedure of completing square,

$$\Rightarrow x^2+\frac{x}{2}-2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$$

$$\Rightarrow x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-2-\frac{1}{16}=0$$

$$\left\{(a+b)^2=a^2+b^2+2ab\right\}$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

Taking square root on both sides,

$$\Rightarrow x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{\sqrt{33}-1}{4} \textit{ and } x=-\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{-\sqrt{33}-1}{4}$$

Therefore, $x=\frac{\sqrt{33}-1}{4},\frac{-\sqrt{33}-1}{4}$

(iii) $4x^2+4\sqrt{3}x+3=0$

Dividing equation by 4,

$$x^2+\sqrt{3}x+\frac{3}{4}=0$$

Following the procedure of completing square,

$$\Rightarrow x^2+\sqrt{3}x+\frac{3}{4}+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2=0$$

$$\Rightarrow x^2+\left(\frac{\sqrt{3}}{2}\right)^2+\sqrt{3}x+\frac{3}{4}-\frac{3}{4}=0$$

$$\left\{(a+b)^2=a^2+b^2+\ 2ab\right\}$$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2=0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0$

$\Rightarrow x+\frac{\sqrt{3}}{2}=0, x+\frac{\sqrt{3}}{2}=0$

$\Rightarrow x=-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

(iv) $2x^2+x+4=0$

Dividing equation by 2,

$x^2+\frac{x}{2}+2=0$

Following the procedure of completing square,

$\Rightarrow x^2+\frac{x}{2}+2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow x^2+\left(\frac{1}{4}\right)^2+\frac{x}{2}+2-\left(\frac{1}{4}\right)^2=0$

$\left\{(a+b)^2=a^2+b^2+2ab\right\}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2+2-\frac{1}{16}=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2=\frac{1-32}{16}=\frac{-31}{16}$

Right hand side does not exist because square root of negative number does not exist.

Therefore, there is no solution for quadratic equation $2x^2+x+4=0$

2. Find the roots of the following Quadratic Equations by applying quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2+4\sqrt{3}x+3=0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) $2x^2 - 7x + 3 = 0$

Comparing quadratic equation $2x^2 - 7x + 3 = 0$ with general form $ax^2+bx+c = 0$, we get a = 2, b = -7 and c = 3

Putting these values in quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{7\pm\sqrt{(-7)^2-4(2)(3)}}{2\times 2}$$

$$\Rightarrow x=\frac{7\pm\sqrt{49-24}}{4}$$

$$\Rightarrow x=\frac{7\pm 5}{4}$$

$$\Rightarrow x=\frac{7+5}{4},\frac{7-5}{4}$$

⇒ x = 3, ½

(ii) $2x^2 + x - 4 = 0$

Comparing quadratic equation $2x^2 + x - 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = -4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow x = \frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2 \times 4}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2}$$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

Comparing quadratic equation $2x^2 + x + 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get a = 2, b = 1 and c = 4

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

But, square root of negative number is not defined.

Therefore, Quadratic Equation $2x^2 + x + 4 = 0$ has no solution.

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3. Find the roots of the following equations:

(i) $\frac{x-1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Ans. (i) $x - \frac{1}{x} = 3 \text{ where } x \neq 0$

$$\Rightarrow \frac{x^2 - 1}{x} = 3$$

$$\Rightarrow x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

Comparing equation $x^2 - 3x - 1 = 0$ with general form $ax^2 + bx + c = 0$, We get a = 1, b = −3 and c = −1

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$x=\dfrac{3\pm\sqrt{(-3)^2-4\times1\times(-1)}}{2\times1}$

$\Rightarrow x=\dfrac{3\pm\sqrt{13}}{2}$

$\Rightarrow x=\dfrac{3+\sqrt{13}}{2},\dfrac{3-\sqrt{13}}{2}$

(ii) $\dfrac{1}{x+4}-\dfrac{1}{x-7}=\dfrac{11}{30}\ where\ x\neq-4,7$

$\Rightarrow \dfrac{(x-7)-(x+4)}{(x+4)(x-7)}=\dfrac{11}{30}$

$\Rightarrow \dfrac{-11}{(x+4)(x-7)}=\dfrac{11}{30}$

$\Rightarrow -30=x^2-7x+4x-28$

$\Rightarrow x^2-3x+2=0$

Comparing equation $x^2-3x+2=0$ with general form $ax^2+bx+c=0$,

We get a = 1, b = −3 and c = 2

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$x=\dfrac{3\pm\sqrt{(-3)^2-4(1)(2)}}{2\times1}$

$\Rightarrow x=\dfrac{3\pm\sqrt{1}}{2}$

$\Rightarrow\ x = \frac{3+\sqrt{1}}{2}, \frac{3-\sqrt{1}}{2}$

$\Rightarrow x = 2, 1$

4. The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans. Let present age of Rehman = x years

Age of Rehman 3 years ago = $(x - 3)$ years.

Age of Rehman after 5 years = $(x + 5)$ years

According to the given condition:

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow 3(2x + 2) = (x - 3)(x + 5)$

$\Rightarrow 6x + 6 = x^2 - 3x + 5x - 15$

$\Rightarrow x^2 - 4x - 15 - 6 = 0$

$\Rightarrow x^2 - 4x - 21 = 0$

Comparing quadratic equation $x^2 - 4x - 21 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -4$ and $c = -21$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-21)}}{2 \times 1}$$

$\Rightarrow x = \frac{4 \pm \sqrt{16+84}}{2}$

$\Rightarrow x = \frac{4 \pm \sqrt{100}}{2} = \frac{4 \pm 10}{2}$

$\Rightarrow x = \frac{4+10}{2}, \frac{4-10}{2}$

$\Rightarrow x = 7, -3$

We discard $x=-3$. Since age cannot be in negative.

Therefore, present age of Rehman is 7 years.

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5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = $30 - x$

If, she had got 2 marks more in Mathematics, her marks would be = $x + 2$

If, she had got 3 marks less in English, her marks in English would be = $30 - x - 3 = 27 - x$

According to given condition:

=> $(x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -25$ and $c = 156$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(156)}}{2 \times 1}$

$\Rightarrow x = \frac{25 \pm \sqrt{625 - 624}}{2}$

$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$

$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$

$\Rightarrow x = 13, 12$

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = 30 – x = 30 – 13 = 17

Or Shefali's marks in English = 30 – x = 30 – 12 = 18

Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

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6. The diagonal of a rectangular field is 60 metres more than the shorter side. If, the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans. Let shorter side of rectangle = $x$metres

Let diagonal of rectangle = (x + 60) metres

Let longer side of rectangle = (x + 30) metres

According to pythagoras theorem,

=> $(x+60)^2 = (x+30)^2 + x^2$

$\Rightarrow x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$

$\Rightarrow x^2 - 60x - 2700 = 0$

Comparing equation $x^2 - 60x - 2700 = 0$ with standard form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -60$ and $c = -2700$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{60 \pm \sqrt{(-60)^2 - 4(1)(-2700)}}{2 \times 1}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{3600 + 10800}}{2}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{14400}}{2} = \frac{60 \pm 120}{2}$$

$$\Rightarrow x = \frac{60 + 120}{2}, \frac{60 - 120}{2}$$

$\Rightarrow x = 90, -30$

We ignore –30. Since length cannot be in negative.

Therefore, $x = 90$ which means length of shorter side = 90 metres

And length of longer side = $x + 30 = 90 + 30 = 120$ metres

Therefore, length of sides are 90 and 120 in metres.

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7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans. Let the larger number be x, then square of smaller number be $8x$ and square of larger number be x^2.

According to condition:

$x^2 - 8x = 180$

$\Rightarrow \quad x^2 - 8x - 180 = 0$

$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0$

$\Rightarrow \quad x(x - 18) + 10(x - 18) = 0$

$\Rightarrow \quad (x - 18)(x + 10) = 0$

$\Rightarrow \quad x - 18 = 0 \quad \text{or} \quad x + 10 = 0$

$\Rightarrow \quad x = 18 \quad x = -10$

When $x = 18$, then square of smaller number = 144

Then smaller number = ± 12

Therefore, two numbers are (12, 18) or (−12, 18)

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8. A train travels 360 km at a uniform speed. If, the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the speed of the train = x km/hr

If, speed had been 5 km/hr more, train would have taken 1 hour less.

So, according to this condition

=> $\frac{360}{x} = \frac{360}{x+5} + 1$

$\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$

$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right) = 1$

$\Rightarrow 360 \times 5 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 1800 = 0$

Comparing equation $x^2 + 5x - 1800 = 0$ with general equation $ax^2 + bx + c = 0$,

We get $a = 1$, $b = 5$ and $c = -1800$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1800)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 + 7200}}{2}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{7225}}{2} = \frac{-5 \pm 85}{2}$$

$$\Rightarrow x = \frac{-5 + 85}{2}, \frac{-5 - 85}{2}$$

$\Rightarrow x = 40, -45$

Since speed of train cannot be in negative. Therefore, we discard $x = -45$

Therefore, speed of train = 40 km/hr

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9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. Let time taken by tap of smaller diameter to fill the tank = x hours

Let time taken by tap of larger diameter to fill the tank = $(x - 10)$ hours

It means that tap of smaller diameter fills $\frac{1}{x}^{th}$ part of tank in 1 hour.... (1)

And, tap of larger diameter fills $\frac{1}{x-10}^{th}$ part of tank in 1 hour. ... (2)

When two taps are used together, they fill tank in $\frac{75}{8}$ hours.

In 1 hour, they fill $\frac{8}{75}^{th}$ part of tank $\left(\frac{1}{\frac{75}{8}} = \frac{8}{75}\right)$ ... (3)

From (1), (2) and (3),

=> $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

⇒ $\frac{x-10+x}{x(x-10)} = \frac{8}{75}$

⇒ $75(2x - 10) = 8\left(x^2 - 10x\right)$

⇒ $150x - 750 = 8x^2 - 80x$

⇒ $8x^2 - 80x - 150x + 750 = 0$

⇒ $4x^2 - 115x + 375 = 0$

Comparing equation $4x^2 - 115x + 375 = 0$ with general equation $ax^2 + bx + c = 0$,

We get $a = 4$, $b = -115$ and $c = 375$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{115 \pm \sqrt{(-115)^2 - 4(4)(375)}}{2 \times 4}$$

$\Rightarrow x = \frac{115 \pm \sqrt{13225 - 6000}}{8}$

$\Rightarrow x = \frac{115 \pm \sqrt{7225}}{8}$

$\Rightarrow x = \frac{115 \pm 85}{8}$

$\Rightarrow x = \frac{115 + 85}{8}, \frac{115 - 85}{8}$

$\Rightarrow x = 25, 3.75$

Time taken by larger tap = $x - 10 = 3.75 - 10 = -6.25$ hours

Time cannot be in negative. Therefore, we ignore this value.

Time taken by larger tap = $x - 10 = 25 - 10 = 15$ hours

Therefore, time taken by larger tap is 15 hours and time taken by smaller tap is 25 hours.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If, the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.

Ans. Let average speed of passenger train = x km/h

Let average speed of express train = $(x + 11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given condition,

=> $\frac{132}{x} = \frac{132}{x+11} + 1$

$\Rightarrow 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$

$\Rightarrow 132\left(\frac{x+11-x}{x(x+11)}\right) = 1$

⇒ 132 (11) = x (x + 11)

$\Rightarrow 1452 = x^2 + 11x$

$\Rightarrow x^2 + 11x - 1452 = 0$

Comparing equation $x^2 + 11x - 1452 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get a = 1, b = 11 and c = −1452

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-1452)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{121 + 5808}}{2}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2}$$

$$\Rightarrow x = \frac{-11 \pm 77}{2}$$

$$\Rightarrow x = \frac{-11+77}{2}, \frac{-11-77}{2}$$

⇒ x = 33, −44

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h

And, speed of express train = x + 11 = 33 + 11 = 44 km/h

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11. Sum of areas of two squares is 468 m^2. If, the difference of their perimeters is 24 metres, find the sides of the two squares.

Ans. Let perimeter of first square = x metres

Let perimeter of second square = (x + 24) metres

Length of side of first square = $\frac{x}{4}$ metres {Perimeter of square = $4 \times$ *length of side*}

Length of side of second square = $\left(\frac{x+24}{4}\right)$ metres

Area of first square = side × side = $\frac{x}{4} \times \frac{x}{4} = \frac{x^2}{16} m^2$

Area of second square = $\left(\frac{x+24}{4}\right)^2 m^2$

According to given condition:

=> $\frac{x^2}{16} + \left(\frac{x+24}{4}\right)^2 = 468$

$\Rightarrow \frac{x^2}{16} + \frac{x^2 + 576 + 48x}{16} = 468$

$\Rightarrow \frac{x^2 + x^2 + 576 + 48x}{16} = 468$

$\Rightarrow 2x^2 + 576 + 48x = 468 \times 16$

$\Rightarrow 2x^2 + 48x + 576 = 7488$

$\Rightarrow 2x^2 + 48x - 6912 = 0$

$\Rightarrow x^2 + 24x - 3456 = 0$

Comparing equation $x^2 + 24x - 3456 = 0$ with standard form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = 24$ and $c = -3456$

Applying Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-3456)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{576 + 13824}}{2}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{14400}}{2} = \frac{-24 \pm 120}{2}$$

$$\Rightarrow x = \frac{-24 + 120}{2}, \frac{-24 - 120}{2}$$

$\Rightarrow x = 48, -72$

Perimeter of square cannot be in negative. Therefore, we discard $x = -72$.

Therefore, perimeter of first square = 48 metres

And, Perimeter of second square = $x + 24 = 48 + 24 = 72$ metres

$\Rightarrow$ Side of First square = $\dfrac{Perimeter}{4} = \dfrac{48}{4} = 12\ m$

And, Side of second Square = $\dfrac{Perimeter}{4} = \dfrac{72}{4} = 18\ m$