CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.2

1. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

= $\dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

= $\dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4}$ =1

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

= $2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

= $2 + \dfrac{3}{4} - \dfrac{3}{4}$ = 2

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \cos ec30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)}$

$= \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2(3-1)}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \cos ec60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$= \dfrac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \dfrac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4}$

$= \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \dfrac{3\sqrt{3}-4}{3\sqrt{3}-4}$

= $\dfrac{27+16-24\sqrt{3}}{27-16}$ [Since $(a+b)(a-b)=a^2-b^2$]

= $\dfrac{43-24\sqrt{3}}{11}$

(v) $\dfrac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 30^\circ}$

= $\dfrac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

= $\dfrac{5\times\frac{1}{4}+4\times\frac{4}{3}-1}{\frac{1}{4}+\frac{3}{4}}$ = $\dfrac{\frac{5}{4}+\frac{16}{3}-1}{\frac{4}{4}}$

= $\dfrac{15+64-12}{12}$ = $\dfrac{67}{12}$

2. Choose the correct option and justify:

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ =

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ =

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A** =

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ =

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D) None of these

Ans. (i) (A) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

= $\frac{2}{\sqrt{3}} \times \frac{3}{3+1}$ = $\frac{\sqrt{3}}{2}$ = $\sin 60^\circ$

(ii) (D) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ = $\frac{1-1}{1+1}$ = $\frac{0}{2}$ = 0

(iii) (A) Since A = 0, then

$\sin 2A = \sin 0^\circ = 0$ and $2\sin A = 2\sin 0^\circ$

= $2 \times 0 = 0$

$\therefore \sin 2A = \sin A$ when A = 0

(iv) (C) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

= $\frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ = $\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

= $\frac{2}{\sqrt{3}} \times \frac{3}{3-1}$ = $\sqrt{3} = \tan 60^\circ$

3. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^\circ < A+B \le 90^\circ$; $A > B$, **find A and B.**

Ans. $\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^\circ$

$\Rightarrow$ A + B = 60° ....(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A - B) = \tan 30^\circ$

$\Rightarrow$ A – B = 30° ....(ii)

On adding eq. (i) and (ii), we get,

2A = $90^\circ \Rightarrow$ A = 45°

On Subtracting eq. (i) and eq. (ii), we get

2B = $30^\circ \Rightarrow$ B = 15°

4. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0^\circ$.

Ans. (i) False, because, let A = 60° and B = 30°

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

(ii) True, because it is clear from the table below:

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin\theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(v) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

= $\frac{1}{0}$ i.e. undefined.