CBSE Class–10 Mathematics

NCERT solution

Chapter - 2

Polynomials - Exercise 2.4

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2; \frac{1}{2}, 1, -2$

(ii) $x^3-4x^2+5x-2; 2, 1, 1$

Ans. (i) Comparing the given polynomial with ax^3+bx^2+cx+d, we get

$a=2, b=1, c=-5$ and $d=2$.

$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$

= $\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$ = $\frac{1+1-10+8}{4}=\frac{0}{4}$ = 0

$p(1)=2(1)^3+(1)^2-5(1)+2$

= $2+1-5+2$ = 0

$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$

= $2(-8)+4+10+2$ = $-16+16$ = 0

$\therefore \frac{1}{2}, 1$ and -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, $\alpha+\beta+\gamma$

= $\frac{1}{2}+1+(-2)=\frac{1+2-4}{2}=\frac{-1}{2}=\frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha$

= $\left(\frac{1}{2}\right)(1)+(1)(-2)+(-2)\left(\frac{1}{2}\right)$

= $\frac{1}{2}-2-1$ = $\frac{-5}{2}=\frac{c}{a}$

And $\alpha\beta\gamma$ = $\frac{1}{2}\times1\times(-2)=-1=\frac{-2}{2}=\frac{-d}{a}$

(ii) Comparing the given polynomial with ax^3+bx^2+cx+d, we get

$a=1, b=-4, c=5$ and $d=-2.$

$p(2)=(2)^3-4(2)^2+5(2)-2$

= $8-16+10-2$ = 0

$p(1)=(1)^3-4(1)^2+5(1)-2$

= $1-4+5-2$ = 0

$\therefore$ $2, 1$ and 1 are the zeroes of $x^3-4x^2+5x-2.$

Now, $\alpha+\beta+\gamma$

= $2+1+1=4=\frac{-(-4)}{1}=\frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha$ = $(2)(1)+(1)(1)+(1)(2)$

= $2+1+2$ = $\frac{5}{1}=\frac{c}{a}$

And $\alpha\beta\gamma$ = $2\times1\times1=2=\frac{-(-2)}{1}=\frac{-d}{a}$

2. Find a cubic polynomial with the sum of the product of its zeroes taken two at a time and the product of its zeroes are $2, -7, -14$ respectively.

Ans. Let the cubic polynomial be ax^3+bx^2+cx+d and its zeroes be α, β and γ.

Then $\alpha+\beta+\gamma$ = 2 = $\frac{-(-2)}{1}=\frac{-b}{a}$ and $\alpha\beta+\beta\gamma+\gamma\alpha$ = $-7=\frac{-7}{1}=\frac{c}{a}$

And $\alpha\beta\gamma$ = $-14=\frac{-14}{1}=\frac{d}{a}$

Here, $a=1, b=-2, c=-7$ and $d=14$

Hence, cubic polynomial will be $x^3-2x^2-7x+14$.

3. If the zeroes of the polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find a and b.

Ans. Since $(a-b), a, (a+b)$ are the zeroes of the polynomial x^3-3x^2+x+1.

$\therefore\ \alpha+\beta+\gamma$ = $a-b+b+a+b=\frac{-(-3)}{1}=3$

$\Rightarrow 3a=3$

$\Rightarrow\ a=1$

And $\alpha\beta+\beta\gamma+\gamma\alpha$

= $(a-b)a+a(a+b)+(a+b)(a-b)=\frac{1}{1}=1$

$\Rightarrow\ a^2-ab+a^2+ab+a^2-b^2=1$

$\Rightarrow\ 3a^2-b^2=1$

$\Rightarrow 3(1)^2 - b^2 = 1 \ [\because a = 1]$

$\Rightarrow 3 - b^2 = 1 \qquad \Rightarrow \qquad b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

Hence $a = 1$ and $b = \pm\sqrt{2}$.

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4. If the two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Ans. Since $2 \pm \sqrt{3}$ are two zeroes of the polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$.

Let $x = 2 \pm \sqrt{3} \Rightarrow x - 2 = \pm\sqrt{3}$

Squaring both sides, $x^2 - 4x + 4 = 3$

$\Rightarrow x^2 - 4x + 1 = 0$

Now we divide $p(x)$ by $x^2 - 4x + 1$ to obtain other zeroes.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & \underline{\pm x^4 \mp 4x^3 \pm x^2} \\
 & -2x^3 - 27x^2 + 138x \\
 & \underline{\mp 2x^3 \pm 8x^2 \mp 2x} \\
 & -35x^2 + 140x - 35 \\
 & \underline{\mp 35x^2 \pm 140x \mp 35} \\
 & 0
\end{array}
$$

$\therefore p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

= $\left(x^2-4x+1\right)\left(x^2-7x+5x-35\right)$

= $\left(x^2-4x+1\right)\left[x(x-7)+5(x-7)\right]$

= $\left(x^2-4x+1\right)(x+5)(x-7)$

$\Rightarrow (x+5)$ and $(x-7)$ are the other factors of $p(x)$.

$\therefore -5$ and 7 are other zeroes of the given polynomial.

5. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$, find k and a.

Ans. Let us divide $x^4-6x^3+16x^2-25x+10$ by x^2-2x+k.

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\
 & \pm x^4 \mp 2x^3 \pm kx^2 \\
\cline{2-2}
 & -4x^3+(16-k)x^2-25x+10 \\
 & \mp 4x^3 \pm 8x^2 \mp 4kx \\
\cline{2-2}
 & (8-k)\,x^2+(4k-25)x+10 \\
 & \pm(8-k)x^2 \mp 2(8-k)x \pm (8-k)k \\
\cline{2-2}
 & (2k-9)x-(8-k)k+10
\end{array}
$$

$\therefore$ Remainder = $(2k-9)x-(8-k)k+10$

On comparing this remainder with given remainder, i.e. $x+a$,

$2k-9=1 \Rightarrow 2k=10$

$\Rightarrow k=5$

And $-(8-k)k+10=a$

$\Rightarrow a=-(8-5)5+10=-5$