CBSE Class–10 Mathematics

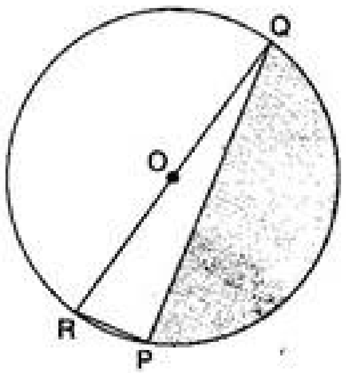
NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. Find the area of the shaded region in figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Ans. In the given figure, $\angle$RPQ = 90° [Angle in semi-circle is 90°]

$\therefore\ RQ^2 = PR^2 + PQ^2$

= $(7)^2 + (24)^2$ = 49 + 576 = 625

$\Rightarrow$ RQ = 25 cm

$\Rightarrow$ Diameter of the circle = 25 cm

$\therefore$ Radius of the circle = $\frac{25}{2}$ cm

Area of the semicircle = $\frac{1}{2}\pi r^2$

= $\frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$ = $\frac{6875}{28} cm^2$

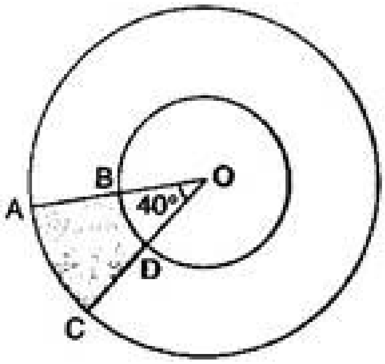
Area of right triangle RPQ = $\frac{1}{2}$ x PQ x PR

= $\frac{1}{2} \times 24 \times 7$ = $84\ cm^2$

Area of shaded region = Area of semicircle – Area of right triangle RPQ

= $\frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$ = $\frac{4523}{28}\ cm^2$

2. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle$ AOC = $40°$.



Ans. Area of shaded region = Area of sector OAC – Area of sector OBD

= $\frac{40°}{360°} \times \frac{22}{7} \times (14)^2 - \frac{40°}{360°} \times \frac{22}{7} \times (7)^2$

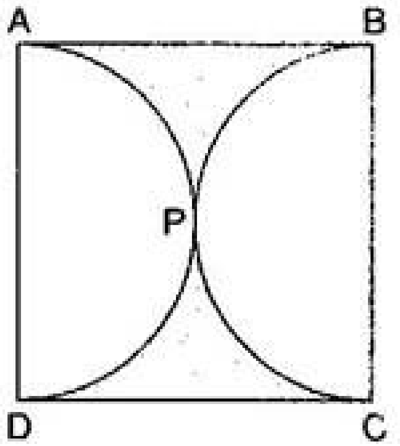
= $\frac{40°}{360°} \times \frac{22}{7}\left[(14)^2 - (7)^2\right]$

= $\frac{40°}{360°} \times \frac{22}{7}(14-7)(14+7)$

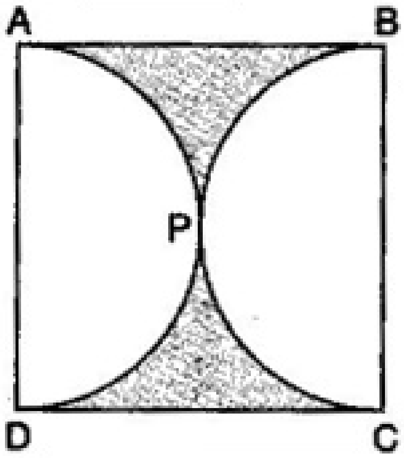
= $\frac{40°}{360°} \times \frac{22}{7} \times 7 \times 21$

= $\frac{154}{3}\ cm^2$

3. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Ans. Area of shaded region



= Area of square ABCD – (Area of semicircle APD + Area of semicircle BPC)

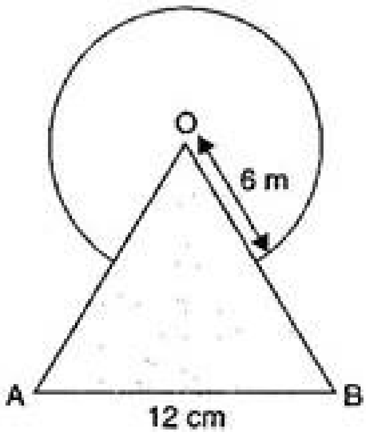
$= 14 \times 14 - \left[\frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2\right]$

$= 196 - \left[\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right]$

$= 196 - \frac{22}{7} \times 7 \times 7$

= 196 – 154 = $42\ cm^2$

4. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans. Area of shaded region

= Area of circle + Area of equilateral triangle OAB – Area common to the circle and the triangle (Area of sector)

$= \pi r^2 + \frac{\sqrt{3}}{4}(a)^2 - \frac{\theta}{360^\circ} \times \pi r^2$

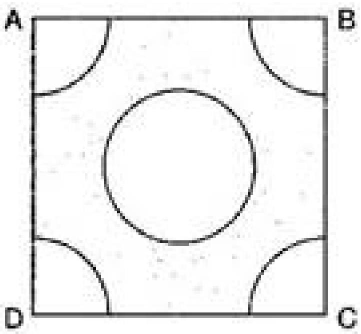
$= \pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - \frac{60^\circ}{360^\circ} \times \pi(6)^2$

$= 36\pi + 36\sqrt{3} - 6\pi$

$= 30\pi + 36\sqrt{3} = 30 \times \frac{22}{7} + 36\sqrt{3}$

$= \left(\frac{660}{7} + 36\sqrt{3}\right) cm^2$

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the figure.



Ans. Area of remaining portion of the square

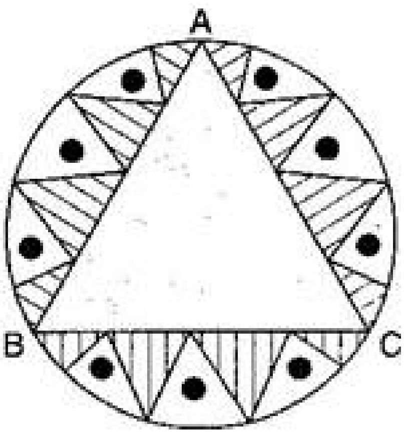
= Area of square – [(4 $\times$ Area of a quadrant + Area of a circle)]

$$= 4\times4-\left[4\times\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times(1)^2+\frac{22}{7}\times\left(\frac{2}{2}\right)^2\right]$$

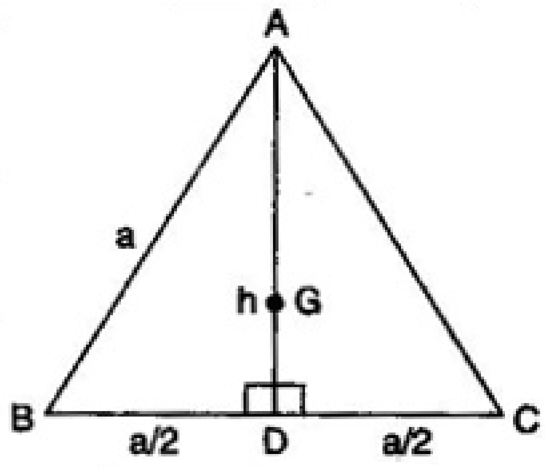
$$= 16 - \left[\frac{22}{7}\left(4\times\frac{1}{4}+1\right)\right]$$

$$= 16-2\times\frac{22}{7} = \frac{68}{7}cm^2$$

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design (shaded region).



Ans. Area of design = Area of circular table cover – Area of the equilateral triangle ABC



= $\pi(32)^2 - \frac{\sqrt{3}}{4}a^2$.........(i)

$\because$ G is the centroid of the equilateral triangle.

$\therefore$ radius of the circumscribed circle = $\frac{2}{3}h$ cm

According to the question, $\frac{2}{3}h = 32$

$\Rightarrow h$ = 48 cm

Now, $a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow a^2 = h^2 + \frac{a^2}{4}$

$\Rightarrow a^2 - \frac{a^2}{4} = h^2$

$\Rightarrow \frac{3a^2}{4} = h^2$

$\Rightarrow a^2 = \frac{4h^2}{3}$

$\Rightarrow a^2 = \frac{4(48)^2}{3}$ = 3072

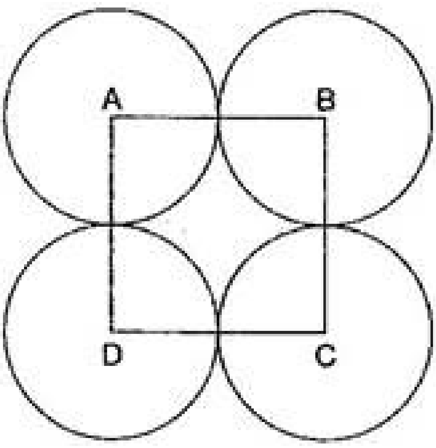
$\Rightarrow a = \sqrt{3072}$ cm

$\therefore$ Required area = $\pi(32)^2 - \frac{\sqrt{3}}{4} \times 3072$ [From eq. (i)]

= $\frac{22}{7} \times 1024 - 768\sqrt{3}$

= $\left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$

7. In figure ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Ans. Area of shaded region = Area of square – 4 $\times$ Area of sector

= $14 \times 14 - 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$

= $196 - \frac{22}{7} \times 7 \times 7$ = 196 – 154 = $42\ cm^2$

8. Figure depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge.

(ii) the area of the track.

Ans. (i) Distance around the track along its inner edge

$= 106+106+2\times\left[\frac{180^\circ}{360^\circ}\times 2\times\frac{22}{7}\times\left(\frac{60}{2}\right)\right]$

$= 212+2\times\left[\frac{1}{2}\times 2\times\frac{22}{7}\times\frac{60}{2}\right]$

$= 212+60\times\frac{22}{7} = 212+\frac{1320}{7} = \frac{2804}{7}$ m

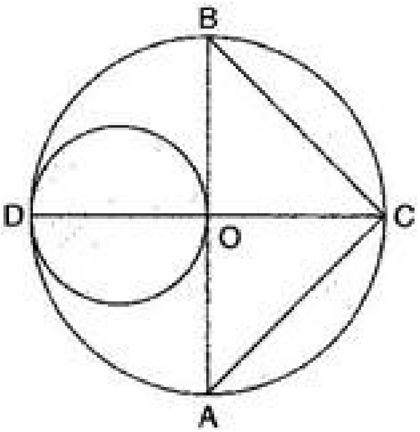
(ii) Area of track = $106\times 10+106\times 10+2\times\left[\frac{1}{2}\times\frac{22}{7}(30+10)^2-\frac{1}{2}\times\frac{22}{7}(30)^2\right]$

$= 1060+1060+\frac{22}{7}\left[(40)^2-(30)^2\right]$

$= 2120+\frac{22}{7}(40+30)(40-30)$

$= 2120+\frac{22}{7}\times 700 = 4320\ m^2$

9. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, Find the area of the shaded region.



Ans. Area of shaded region = Area of circle with diameter OD + Area of semicircle ACB – Area of $\triangle$ ACB

= $\frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7} \times (7)^2 - \left(\frac{1}{2} \times 7 \times 7 + \frac{1}{2} \times 7 \times 7\right)$

= $\frac{77}{2} + 187 - 49$ = $\frac{133}{2}$ = $66.5\ cm^2$

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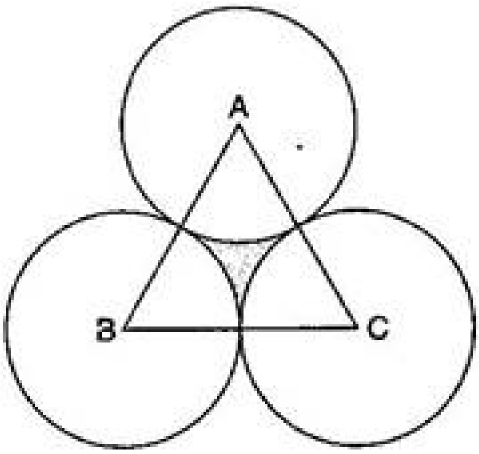
10. The area of an equilateral triangle ABC is $17320.5\ \text{cm}^2$. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.\right.$



Ans. Given: Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ = 17320.5

$\Rightarrow a^2 = \frac{17320.5 \times 4}{\sqrt{3}}$

$\Rightarrow a^2 = \dfrac{17320.5 \times 4}{1.73205}$

$\Rightarrow a^2 = 40000$

$\Rightarrow a = 200$ cm

Area of shaded region = Area of ΔABC - Area of 3 sectors

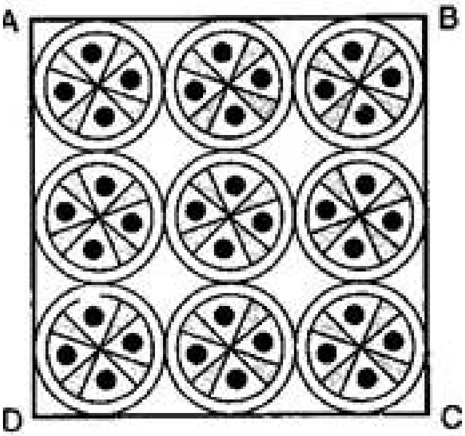
$= 17320.5 - 3\left[\dfrac{60^\circ}{360^\circ} \times 3.14 \times \left(\dfrac{200}{2}\right)^2\right]$

$= 17320.5 - 3\left[\frac{1}{6} \times 3.14 \times 100 \times 100\right]$

= 17320.5 - 3 x 5233.33

= 17320.5 – 15700 = $1620.5\ cm^2$

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.



Ans. Radius of each design = 7 cm, then Diamter = 7 x 2 = 14 cm

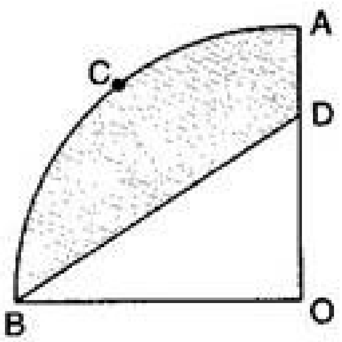
Therefore, side of square = 14 + 14 + 14 = 42 cm

Area of remaining portion of handkerchief = Area of square ABCD – Area of 9 circular designs

= $42 \times 42 - 9 \times \frac{22}{7} \times 7 \times 7$

= 1764 – 1386 = $378\ cm^2$

12. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the:



(i) quadrant OACB

(ii) shaded region

Ans. (i) Area of quadrant OACB = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (3.5)^2$

= $\frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$ = $\frac{77}{8}\ cm^2$

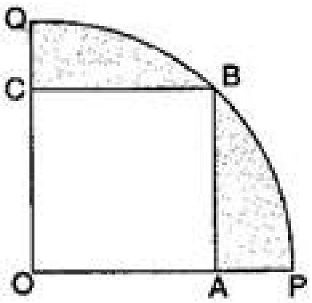
(ii) Area of shaded region = Area of quadrant OACB – Area of ΔOBD

= $\frac{77}{8} - \frac{1}{2} \times \text{OB} \times \text{OD}$

= $\frac{77}{8} - \frac{3.5 \times 2}{2}$

= $\frac{77}{8} - \frac{35}{10}$ = $\frac{49}{8}$ cm^2

13. In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. $(\text{Use } \pi = 3.14)$



Ans. OB = $\sqrt{OA^2 + AB^2}$ [Using Pythagoras theorem]

= $\sqrt{OA^2 + OA^2}$

= $\sqrt{2}$ OA = $\sqrt{2} \times 20 = 20\sqrt{2}$ cm

Area of shaded region = Area of quadrant OPBQ – Area of square OABC

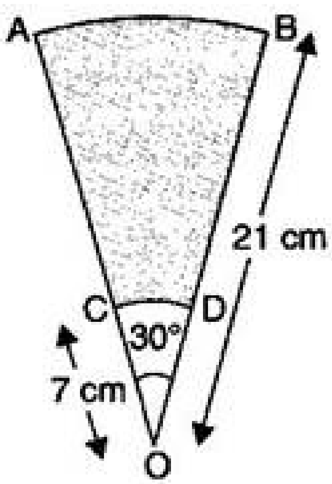
= $\frac{90^\circ}{360^\circ} \times 3.14\left(20\sqrt{2}\right)^2 - 20 \times 20$

= $\frac{1}{4} \times 3.14 \times 800 - 400$

= $200 \times 3.14 - 400$

= $228\ cm^2$

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If $\angle$ AOB = 30°, find the area of the shaded region.



Ans. Area of shaded region = Area of sector OAB – Area of sector OCD

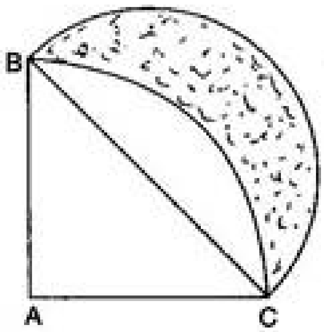
= $\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 - \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$

= $\frac{1}{12} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$

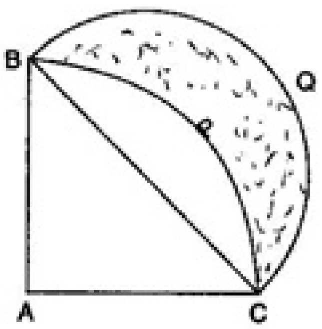
= $\frac{231}{2} - \frac{77}{6}$ = $\frac{693-77}{6}$

= $\frac{616}{6}$ = $\frac{308}{3}$ cm^2

15. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Ans. In right triangle BAC, $BC^2 = AB^2 + AC^2$ [Pythagoras theorem]



$\Rightarrow BC^2 = (14)^2 + (14)^2 = 2(14)^2$

$\Rightarrow$ BC = $14\sqrt{2}$ cm

$\therefore$ Radius of the semicircle = $\dfrac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

$\therefore$ Required area = Area BPCQB

= Area BCQB – Area BCPB

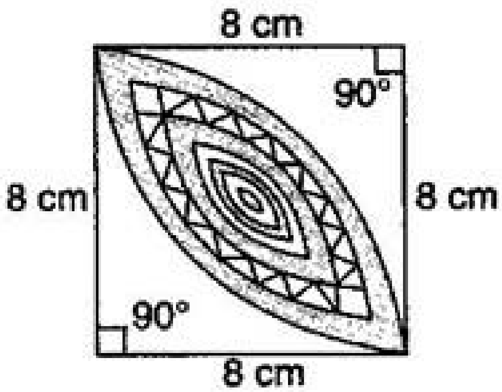
= Area BCQB – (Area BACPB – Area ΔBAC)

= $\dfrac{180^\circ}{360^\circ}\times\dfrac{22}{7}\left(7\sqrt{2}\right)^2 - \left[\dfrac{90^\circ}{360^\circ}\times\dfrac{22}{7}\times(14)^2 - \dfrac{14\times14}{2}\right]$

= $\dfrac{1}{2}\times\dfrac{22}{7}\times98 - \left(\dfrac{1}{4}\times\dfrac{22}{7}\times196 - 98\right)$

= 154 – (154 – 98) = $98\ cm^2$

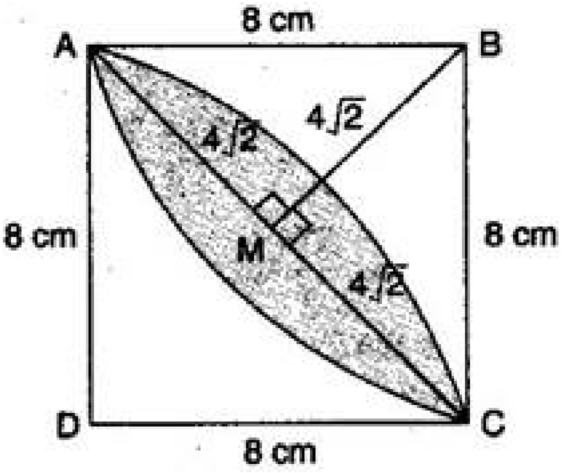
16. Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each.



Ans. In right triangle ADC, $AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$\Rightarrow AC^2 = (8)^2 + (8)^2 = 2(8)^2$

$\Rightarrow$ AC = $\sqrt{128} = 8\sqrt{2}$ cm



Draw BM $\perp$ AC.

Then AM = MC = $\frac{1}{2}$ AC = $\frac{1}{2} \times 8\sqrt{2}$ = $4\sqrt{2}$ cm

In right triangle AMB,

$AB^2 = AM^2 + BM^2$ [Pythagoras theorem]

$\Rightarrow (8)^2 = \left(4\sqrt{2}\right)^2 + BM^2$

$\Rightarrow BM^2 = 64 - 32 = 32$

$\Rightarrow$ BM = $4\sqrt{2}$ cm

$\therefore$ Area of ΔABC = $\frac{1}{2} \times AC \times BM$

= $\frac{8\sqrt{2} \times 4\sqrt{2}}{2}$ = $32\ cm^2$

$\therefore$ Half Area of shaded region

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 - 32$

= $16 \times \frac{22}{7} - 32$ = $\frac{128}{7} cm^2$

$\therefore$ Area of designed region

= $2 \times \frac{128}{7}$ = $\frac{256}{7} cm^2$