CBSE Class–10 Mathematics

NCERT solution

Chapter - 6

Triangles - Exercise 6.4

1. Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Ans. We have, $\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta DEF)} = \frac{BC^2}{EF^2}$

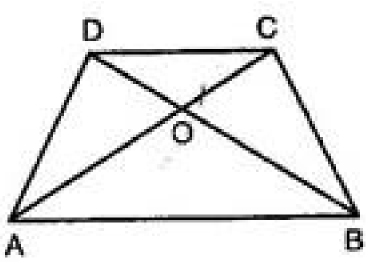
$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow BC = \left(\frac{8}{11} \times 15.4\right) \text{cm} = 11.2 \text{ cm}$$

2. Diagonals of a trapezium ABCD with AB $\parallel$ DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Ans. In $\triangle$s AOB and COD, we have,



$\angle$ AOB = $\angle$ COD[Vertically opposite angles]

$\angle$ OAB = $\angle$ OCD[Alternate angles]

By AA-criterion of similarity,

$\therefore$ Δ AOB ~ Δ COD

$$\therefore \frac{\text{Area }(\Delta AOB)}{\text{Area }(\Delta COD)}=\frac{AB^2}{DC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta AOB)}{\text{Area }(\Delta COD)}=\frac{(2DC)^2}{DC^2}=\frac{4}{1}$$

Hence, Area (Δ AOB) : Area (Δ COD) = 4 : 1

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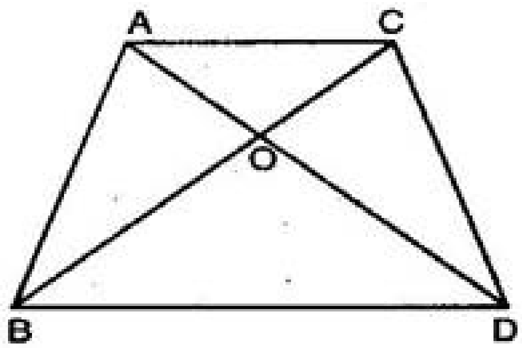
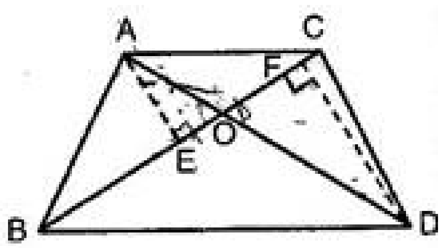
3. In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$.



Ans. Given: Two Δ s ABC and DBC which stand on the same base but on the opposite sides of BC.



To Prove: $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)}=\frac{AO}{DO}$

Construction: Draw AE $\perp$ BC and DF $\perp$ BC.

Proof: In $\triangle$s AOE and DOF, we have, $\angle$AEO = $\angle$DFO = 90°

and $\angle$AOE = $\angle$DOF[Vertically opposite)

$\therefore$ $\triangle$AOE ~ $\triangle$DOF[By AA-criterion]

$\therefore \dfrac{AE}{DF} = \dfrac{AO}{OD}$ ....(i)

Now, $\dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF}$

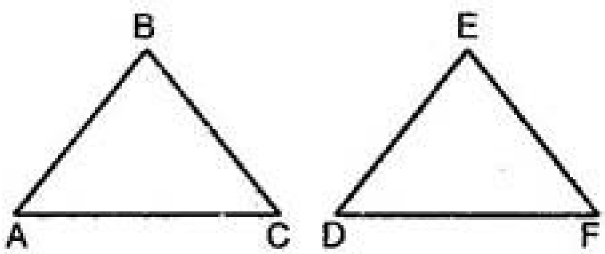
$\Rightarrow \dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{AE}{DF}$

$\Rightarrow \dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{AO}{OD}$ [using eq. (i)]

4. If the areas of two similar triangles are equal, prove that they are congruent.

Ans. Given: Two $\triangle$s ABC and DEF such that $\triangle$ABC ~ $\triangle$DEF

And Area($\triangle$ABC) = Area ($\triangle$DEF)



To Prove: $\triangle$ABC $\cong$ $\triangle$DEF

Proof: $\triangle$ABC ~ $\triangle$DEF

$\therefore$ $\angle$A = $\angle$D, $\angle$B = $\angle$E, $\angle$C = $\angle$F

And $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

To establish $\Delta ABC \cong \Delta DEF$, it is sufficient to prove that, AB = DE, BC = EF and AC = DF

Now, Area(Δ ABC) = Area (Δ DEF)

$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)}=1$

$\Rightarrow \frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=1$

$\Rightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=1$

$\Rightarrow$ AB = DE, BC = EF, AC = DF

Hence, $\Delta ABC \cong \Delta DEF$

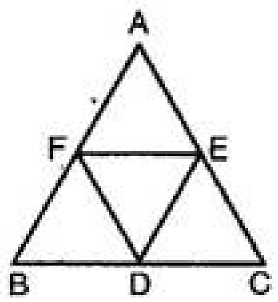
5. D, E and F are respectively the midpoints of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Ans. Since D and E are the midpoints of the sides BC and CA of ΔABC respectively.

$\therefore$ DE $\|$ BA $\Rightarrow$ DE $\|$ FA ....(i)

Since D and F are the midpoints of the sides BC and AB of ΔABC respectively.

$\therefore$ DF $\|$ CA $\Rightarrow$ DE $\|$ AE ....(ii)



From (i) and (ii), we can say that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in $\triangle$s DEF and ABC, we have

$\angle$FDE = $\angle$A[opposite angles of $\|$gm AFDE]

And $\angle$DEF = $\angle$B[opposite angles of $\|$gm BDEF]

$\therefore$ By AA-criterion of similarity, we have $\triangle$DEF $\sim$ $\triangle$ABC

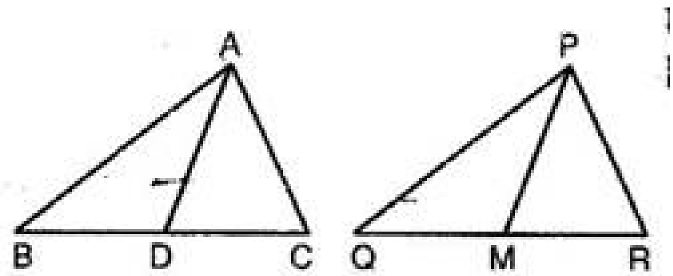
$$\Rightarrow \frac{\text{Area }(\Delta DEF)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{\left(\frac{1}{2}AB\right)^2}{AB^2}\frac{1}{4}$$

[$\because$ DE = $\frac{1}{2}$AB]

Hence, Area ($\triangle$DEF): Area ($\triangle$ABC) = 1 : 4

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Given: $\triangle$ABC $\sim$ $\triangle$PQR, AD and PM are the medians of $\triangle$s ABC and PQR respectively.



To Prove: $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\triangle$ABC $\sim$ $\triangle$PQR

$$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AB^2}{PQ^2} \text{ ....(1)}$$

But, $\frac{AB}{PQ} = \frac{AD}{PM}$ ....(2)

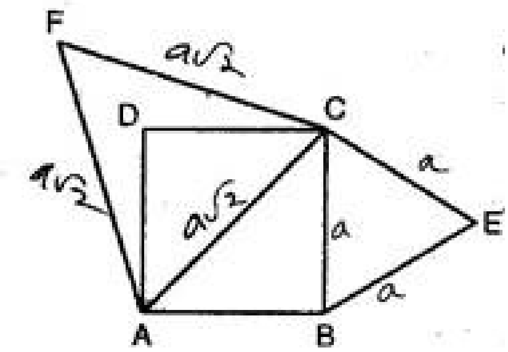
$\therefore$ From eq. (1) and (2), we have,

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$$

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.

Ans. Given: A square ABCD,

Equilateral Δs BCE and ACF have been drawn on side BC and the diagonal AC respectively.



To Prove: Area (ΔBCE) = $\frac{1}{2}$ Area (ΔACF)

Proof: $\Delta BCE \sim \Delta ACF$

[Being equilateral so similar by AAA criterion of

similarity]

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$

[$\because$ Diagonal = $\sqrt{2}$ side $\Rightarrow$ AC = $\sqrt{2}$ BC]

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{1}{2}$$

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. The ratio of the areas of triangles ABC and BDE is:

(A) 2: 1

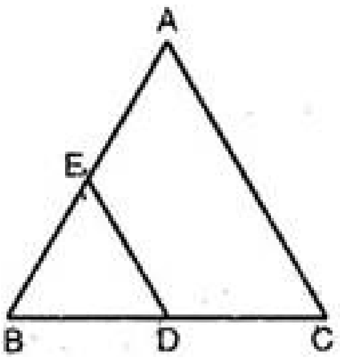
(B) 1: 2

(C) 4: 1

(D) 1: 4

Ans. (C) Since ΔABC and ΔBDE are equilateral, they are equiangular and hence,

$\Delta ABC \sim \Delta BDE$



$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{(2BD)^2}{BD^2}$$

[$\because$ D is the midpoint of BC]

$$\Rightarrow \frac{\text{Area}\,(\Delta ABC)}{\text{Area}\,(\Delta BDE)} = \frac{4}{1}$$

$\therefore$ (C) is the correct answer.

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9. Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio:

(A) 2: 3

(B) 4: 9

(C) 81: 16

(D) 16: 81

Ans. (D) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. Therefore,

Ratio of areas = $\frac{(4)^2}{(9)^2} = \frac{16}{81}$

$\therefore$ (D) is the correct answer.