CBSE Class–10 Mathematics

NCERT solution

Chapter - 2

Polynomials - Exercise 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following.

(i) $\mathbf{p}(\mathbf{x}) = x^3 - 3x^2 + 5x - 3$, $\mathbf{g}(\mathbf{x}) = x^2 - 2$

(ii) $\mathbf{p}(\mathbf{x}) = x^4 - 3x^2 + 4x + 5$, $\mathbf{g}(\mathbf{x}) = x^2 - x + 1$

(iii) $\mathbf{p}(\mathbf{x}) = x^4 - 5x + 6$, $\mathbf{g}(\mathbf{x}) = 2 - x^2$

Ans. (i)

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-2) & x^3 - 3x^2 + 5x - 3 \\
 & \pm x^3 \qquad \mp 2x \\
\hline
 & -3x^2 + 7x - 3 \\
 & \mp 3x^2 \qquad \pm 6 \\
\hline
 & 7x - 9
\end{array}
$$

Therefore, quotient = x – 3 and Remainder = 7x – 9

(ii)

$$
\begin{array}{r l}
 & \qquad\qquad x^2 + x - 3 \\
\hline
x^2 - x + 1) & x^4 - 3x^2 + 4x + 5 \\
 & \pm x^4 \pm x^2 \qquad \mp x^3 \\
\hline
 & -4x^2 + 4x + 5 + x^3 \\
 & \mp x^2 \pm x \qquad \pm x^3 \\
\hline
 & -3x^2 + 3x + 5 \\
 & \mp 3x^2 \pm 3x \mp 3 \\
\hline
 & \qquad\qquad 8
\end{array}
$$

Therefore, quotient = $x^2 + x - 3$ and, Remainder = 8

(iii)

$$
\begin{array}{r l}
 & \qquad\qquad -x^2 - 2 \\
\hline
-x^2 + 2) & x^4 - 5x + 6 \\
 & \pm x^4 \qquad \mp 2x^2 \\
\hline
 & -5x + 6 + 2x^2 \\
 & \qquad \mp 4 \pm 2x^2 \\
\hline
 & -5x + 10
\end{array}
$$

Therefore, quotient = $-x^2 - 2$ and, Remainder = $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Ans. (i)

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2-3 & \cancel{2t^4}+3t^3-2t^2-9t-12 \\
 & \pm\cancel{2t^4} \quad \mp 6t^2 \\
\hline
 & +\cancel{3t^3}+4t^2-\cancel{9t}-12 \\
 & \pm\cancel{3t^3} \quad \mp\cancel{9t} \\
\hline
 & +\cancel{4t^2} \quad -\cancel{12} \\
 & \pm\cancel{4t^2} \quad \mp\cancel{12} \\
\hline
 & 0
\end{array}
$$

$\because$ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(ii)

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & \pm 3x^4 \pm 9x^3 \pm 3x^2 \\
\hline
 & -4x^3-10x^2+2x+2 \\
 & \mp 4x^3 \mp 12x^2 \mp 4x \\
\hline
 & +2x^2+6x+2 \\
 & \pm 2x^2 \pm 6x \pm 2 \\
\hline
 & 0
\end{array}
$$

$\because$ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(iii)

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & \pm x^5 \mp 3x^3 \pm x^2 \\
\hline
 & -x^3 + 3x + 1 \\
 & \mp x^3 \pm 3x \mp 1 \\
\hline
 & 2
\end{array}
$$

∵ Remainder ≠0

Hence first polynomial is not factor of second polynomial.

3. Obtain all other zeroes of $\left(3x^4 + 6x^3 - 2x^2 - 10x - 5\right)$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Ans. Two zeroes of $\left(3x^4 + 6x^3 - 2x^2 - 10x - 5\right)$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ which means that $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3} = 3x^2 - 5$ is a factor of $\left(3x^4 + 6x^3 - 2x^2 - 10x - 5\right)$.

Applying Division Algorithm to find more factors we get:

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad 3x^2+6x+3 \\ x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & \underline{\pm 3x^4 \quad\quad \mp 5x^2} \\ & \quad\quad +6x^3+3x^2-10x-5 \\ & \quad\quad \underline{\pm 6x^3 \quad\quad \mp 10x} \\ & \quad\quad\quad\quad +3x^2 \quad -5 \\ & \quad\quad\quad\quad \underline{\pm 3x^2 \quad \mp 5} \\ & \quad\quad\quad\quad\quad\quad 0 \end{array}$$

We have $p(x) = g(x) \times q(x)$.

$\Rightarrow \left(3x^4+6x^3-2x^2-10x-5\right)$

$= \left(3x^2-5\right)\left(x^2+2x+1\right)$

$= \left(3x^2-5\right)(x+1)(x+1)$

Therefore, other two zeroes of $\left(3x^4+6x^3-2x^2-10x-5\right)$ are −1 and −1.

---

4. On dividing $\left(x^3-3x^2+x+2\right)$ by a polynomial g(x), the quotient and remainder were (x-2) and (-2x+4) respectively. Find g(x).

Ans. Let $p(x) = x^3-3x^2+x+2$, q(x) = (x – 2) and r(x) = (–2x+4)

According to Polynomial Division Algorithm, we have

p(x) = g(x).q(x) + r(x)

$\Rightarrow x^3-3x^2+x+2 = g(x).(x-2)-2x+4$

$\Rightarrow x^3-3x^2+x+2+2x-4 = g(x).(x-2)$

$\Rightarrow x^3-3x^2+3x-2 = g(x).(x-2)$

$\Rightarrow g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x-2}$

So, Dividing $\left(x^3 - 3x^2 + 3x - 2\right)$ by (x−2), we get

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x-2) & x^3 - 3x^2 + 3x - 2 \\ & \pm x^3 \mp 2x^2 \\ \hline & -x^2 + 3x - 2 \\ & \mp x^2 \pm 2x \\ \hline & x - 2 \\ & \pm x \mp 2 \\ \hline & 0 \end{array}$$

Therefore, we have $g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x-2} = x^2 - x + 1$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Ans. (i) Let $p(x) = 3x^2 + 3x + 6$, g(x) = 3

$$\begin{array}{r}
x^2+x+2 \\
\hline
3)\ 3x^2+3x+6 \\
\pm 3x^2 \qquad\quad \\
\hline
+3x+6 \\
\pm 3x \quad\ \\
\hline
+6 \\
\pm 6 \\
\hline
0
\end{array}$$

So, we can see in this example that deg p(x) = deg q(x) = 2

(ii) Let $p(x) = x^3+5$ and $g(x) = x^2-1$

$$\begin{array}{r}
x \\
\hline
x^2-1)\ x^3+5 \\
\pm x^3 \quad \mp x \\
\hline
x+5
\end{array}$$

We can see in this example that deg q(x) = deg r(x) = 1

(iii) Let $p(x) = x^2+5x-3$, g(x) = x+3

$$\begin{array}{r}
x+2 \\
\hline
x+3)\ x^2+5x-3 \\
\pm x^2 \pm 3x \quad \\
\hline
+2x-3 \\
\pm 2x \pm 6 \\
\hline
-9
\end{array}$$

We can see in this example that deg r(x) = 0