CBSE Class–10 Mathematics

NCERT solution

Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.7

1. The age of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Ans. Let the age of Ani and Biju be x years and y years respectively.

Age of Dharam = 2x years and Age of Cathy = $\frac{y}{2}$ years

According to question, x – y = 3… (1)

And $2x-\frac{y}{2}=30$

$\Rightarrow$ 4x – y = 60… (2)

Subtracting (1) from (2), we obtain:

3x = 60 – 3 = 57

$\Rightarrow$ x = Age of Ani = 19 years

Age of Biju = 19 – 3 = 16 years

Again, According to question, y – x = 3… (3)

And $2x-\frac{y}{2}=30$

$\Rightarrow$ 4x – y = 60… (4)

Adding (3) and (4), we obtain:

3x = 63

$\Rightarrow$ x = 21

Age of Ani = 21 years

Age of Biju = 21 + 3 = 24 years

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]

Ans. Let the money with the first person and second person be Rs x and Rs y respectively.
According to the question,

$x + 100 = 2(y - 100)$

$\Rightarrow x + 100 = 2y - 200$

$\Rightarrow x - 2y = -300$... (1)

Again, $6(x - 10) = (y + 10)$

$\Rightarrow 6x - 60 = y + 10$

$\Rightarrow 6x - y = 70$... (2)

Multiplying equation (2) by 2, we obtain:

$12x - 2y = 140$... (3)

Subtracting equation (1) from equation (3), we obtain:

$11x = 140 + 300$

$\Rightarrow 11x = 440$

$\Rightarrow x = 40$

Putting the value of x in equation (1), we obtain:

$40 - 2y = -300$

$\Rightarrow 40 + 300 = 2y$

$\Rightarrow$ 2y = 340

$\Rightarrow$ y = 170

Thus, the two friends had Rs 40 and Rs 170 with them.

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Ans. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Since Speed = $\dfrac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$

$\Rightarrow x = \dfrac{d}{t}$

$\Rightarrow d = xt$ ... (1)

According to the question

$x + 10 = \dfrac{d}{t-2}$

$\Rightarrow (x+10)(t-2) = d$

$\Rightarrow xt + 10t - 2x - 20 = d$ [Since, $xt = d$]

$\Rightarrow -2x + 10t = 20$ ......(2)[Using eq. (1)]

Again, $x - 10 = \dfrac{d}{t+3}$

$\Rightarrow (x-10)(t+3) = d$

$\Rightarrow xt - 10t + 3x - 30 = d$

$\Rightarrow$ $3x - 10t = 30$ ......(3)[Using eq. (1)] [Since, $xt = d$]

Adding equations (2) and (3), we obtain:
x = 50
Substituting the value of x in equation (2), we obtain:
$(-2)\times(50)+10t=20$

$\Rightarrow$ –100 + 10t = 20

$\Rightarrow$ 10t = 120

$\Rightarrow$ t = 12

From equation (1), we obtain:

d = xt = 50×12 = 600

Thus, the distance covered by the train is 600 km.

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4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans. Let the number of rows be x and number of students in a row be y.
Total number of students in the class = Number of rows x Number of students in a row = xy

According to the question,

Total number of students = (x – 1) (y + 3)

$\Rightarrow$ xy = (x – 1) (y + 3)

$\Rightarrow$ xy = xy – y + 3x – 3

$\Rightarrow$ 3x – y – 3 = 0

$\Rightarrow$ 3x – y = 3... (1)

Total number of students = (x + 2) (y – 3)

$\Rightarrow$ xy = xy + 2y – 3x – 6

$\Rightarrow$ 3x – 2y = –6... (2)

Subtracting equation (2) from (1), we obtain:

y = 9

Substituting the value of y in equation (1), we obtain:

3x – 9 = 3

$\Rightarrow$ 3x = 9 + 3 = 12

$\Rightarrow$ x = 4

Number of rows = x = 4

Number of students in a row = y = 9

Hence, Total number of students in a class = xy = $4 \times 9 = 36$

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5. In a $\triangle$ABC, $\angle$ C = 3 $\angle$ B = 2($\angle$ A + $\angle$ B). Find three angles.

Ans. $\angle$ C = 3 $\angle$ B = 2($\angle$ A + $\angle$ B)

Taking 3 $\angle$ B = 2($\angle$ A + $\angle$ B)

$\Rightarrow$ $\angle$ B = 2 $\angle$ A

$\Rightarrow$ 2 $\angle$ A – $\angle$ B = 0 ....(1)

We know that the sum of the measures of all angles of a triangle is 180°.

$\angle$ A + $\angle$ B + $\angle$ C = $180°$

$\Rightarrow$ $\angle$ A + $\angle$ B + 3 $\angle$ B = $180°$

$\Rightarrow$ $\angle$ A + 4 $\angle$ B = $180°$ ....(2)

Multiplying equation (1) by 4, we obtain:

8 $\angle$ A – 4 $\angle$ B = 0 ....(3)

Adding equations (2) and (3), we get

$9\angle A = 180^\circ$

$\Rightarrow \angle A = 20^\circ$

From eq. (2), we get,

$20^\circ + 4\angle B = 180^\circ$

$\Rightarrow \angle B = 40^\circ$

And $\angle C = 3 \times 40^\circ = 120^\circ$

Hence the measures of $\angle A$, $\angle B$ and $\angle C$ are $20^\circ, 40^\circ$ *and* 120° respectively.

6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinate of the vertices of the triangle formed by these lines and the $y-$axis.

Ans. $5x - y = 5$

$\Rightarrow y = 5x - 5$

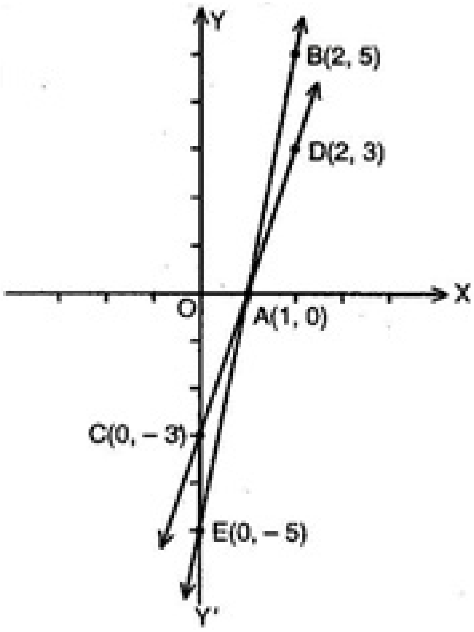
Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	–5	0	5

$3x - y = 3$

$\Rightarrow y = 3x - 3$

x	0	1	2
y	–3	0	3



It can be observed that the required triangle is ΔABC.
The coordinates of its vertices are A (1, 0), B (0, –3), C (0, –5).

7. Solve the following pair of linear equations:

(i) $px + py = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Ans. (i) $px+qy=p-q$ ... (1)

$qx-py=p+q$ ... (2)

Multiplying equation (1) by p and equation (2) by q, we obtain:

$p^2x+pqy=p^2-pq$ ... (3)

$q^2x-pqy=pq+q^2$ ... (4)

Adding equations (3) and (4), we obtain:

$p^2x+q^2x=p^2+q^2$

$\Rightarrow\left(p^2+q^2\right)x=p^2+q^2$

$\Rightarrow x=\dfrac{p^2+q^2}{p^2+q^2}$ = 1

Substituting the value of x in equation (1), we obtain:

$p(1)+qy=p-q$

$\Rightarrow qy=-q \Rightarrow y=-1$

Hence the required solution is x = 1 and y = –1.

(ii) $ax+by=c$ ... (1)

$bx+ay=1+c$ ... (2)

Multiplying equation (1) by a and equation (2) by b, we obtain:

$a^2x+aby=ac$ ... (3)

$b^2x+aby=b+bc$ ... (4)

Subtracting equation (4) from equation (3),

$\left(a^2-b^2\right)x=ac-bc-b$

$\Rightarrow x=\dfrac{c(a-b)-b}{a^2-b^2}$

Substituting the value of x in equation (1), we obtain:

$a\left\{\dfrac{c(a-b)-b}{a^2-b^2}\right\}+by=c$

$$\Rightarrow \frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$\Rightarrow by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$\Rightarrow by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$\Rightarrow by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$\Rightarrow y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

$\Rightarrow bx-ay=0$ ....(1)

$ax+by=a^2+b^2$ ....(2)

Multiplying equation (1) and (2) by b and a respectively, we obtain:

$b^2x-aby=0$ ....(3)

$a^2x+aby=a^3+ab^2$ ....(4)

Adding equations (3) and (4), we obtain:

$b^2x+a^2x=a^3+ab^2$

$\Rightarrow x(b^2+a^2)=a(a^2+b^2)$

$\Rightarrow x=a$

Substituting the value of x in equation (1), we obtain:

$b(a)-ay=0$

$\Rightarrow ab-ay=0$

$\Rightarrow\ y = b$

(iv) $\left(a-b\right)x+\left(a+b\right)y = a^2 - 2ab - b^2$ ... (1)

$\left(a+b\right)\left(x+y\right) = a^2 + b^2$

$\Rightarrow\ \left(a+b\right)x+\left(a+b\right)y = a^2 + b^2$ ....(2)

Subtracting equation (2) from (1), we obtain:

$\left(a-b\right)x-\left(a+b\right)x = \left(a^2 - 2ab - b^2\right) - \left(a^2 + b^2\right)$

$\Rightarrow\ \left(a-b-a-b\right)x = -2ab - 2b^2$

$\Rightarrow\ -2bx = -2b\left(a+b\right)$

$\Rightarrow\ x = a+b$

Substituting the value of x in equation (1), we obtain:

$\left(a-b\right)\left(a+b\right)+\left(a+b\right)y = a^2 - 2ab - b^2$

$\Rightarrow\ a^2 - b^2 + \left(a+b\right)y = a^2 - 2ab - b^2$

$\Rightarrow\ \left(a+b\right)y = -2ab$

$\Rightarrow\ y = \dfrac{-2ab}{a+b}$

(v)152x – 378y = –74... (1)

–378x + 152y = –604 ... (2)

Adding the equations (1) and (2), we obtain:

–226x – 226y = –678

$\Rightarrow$ x + y = 3 ....(3)

Subtracting the equation (2) from equation (1), we obtain:

530x – 530y = 530

$\Rightarrow$ x – y = 1 ....(4)

Adding equations (3) and (4), we obtain:

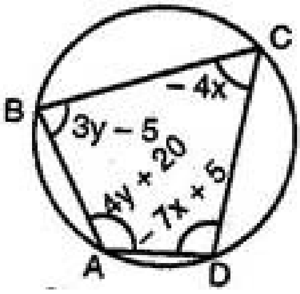
2x = 4

$\Rightarrow$ x = 2

Substituting the value of x in equation (3), we obtain:

y = 1

8. ABCD is a cyclic quadrilateral (see figure). Find the angles of the cyclic quadrilateral.



Ans. We know that the sum of the measures of opposite angles in a cyclic quadrilateral is $180°$.

$\therefore \angle A + \angle C = 180°$
$\Rightarrow 4y + 20 - 4x = 180°$
$\Rightarrow -4x + 4y = 160°$
$\Rightarrow x - y = -40°$ ....(1)
Also $\angle B + \angle D = 180°$
$\Rightarrow 3y - 5 - 7x + 5 = 180°$
$\Rightarrow -7x + 3y = 180°$.........(2)
Multiplying equation (1) by 3, we obtain:
$3x - 3y = -120°$ ....(3)
Adding equations (2) and (3), we obtain:
$-4x = 60° \Rightarrow x = -15°$
Substituting the value of x in equation (1), we obtain:
$-15 - y = -40°$
$\Rightarrow y = -15 + 40 = 25$
$\therefore \angle A = 4y + 20 = 4 \times 25 + 20 = 120°$
$\angle B = 3y - 5 = 3 \times 25 - 5 = 70°$
$\angle C = -4x = -4 \times (-15) = 60°$
$\angle D = -7x + 5 = -7(-15) + 5 = 110°$