CBSE Class–10 Mathematics

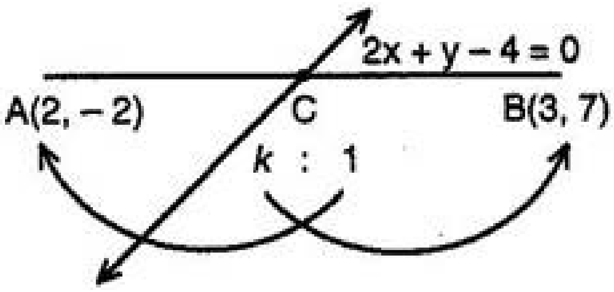
NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.4

1. Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points A$\left(2,-2\right)$ and B$\left(3,7\right)$.

Ans. Let the line $2x+y-4=0$ divides the line segment joining A$\left(2,-2\right)$ and B (3, 7) in the ratio $k:1$ at point C. Then, the coordinates of C are $\left(\frac{3k+2}{k+1},\frac{7k-2}{k+1}\right)$.



But C lies on $2x+y-4=0$, therefore

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Hence, the required ratio if 2: 9 internally.

2. Find a relation between x and y if the points $\left(x,y\right),\left(1,2\right)$ and $\left(7,0\right)$ are collinear.

Ans. The points A$\left(x,y\right)$, B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$

$\Rightarrow 2x-y+7y-14=0$

$\Rightarrow 2x+6y-14=0$

$\Rightarrow x+3y-7=0$

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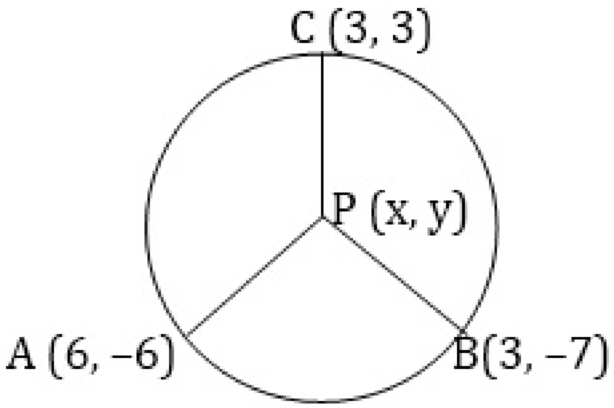
3. Find the centre of a circle passing through the points $(6,-6),(3,-7)$ and $(3,3)$.

Ans. Let P(x,y), be the centre of the circle passing through the points A$(6,-6)$, B$(3,-7)$ and C (3, 3). Then AP = BP = CP.

Taking AP = BP



$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-6)^2+(y+6)^2=(x-3)^2+(y+7)^2$

$\Rightarrow x^2-12x+36+y^2+12y+36=x^2-6x+9+y^2+14y+49$

$\Rightarrow -12x+6x+12y-14y+72-58=0$

$\Rightarrow -6x-2y+14=0$

$\Rightarrow 3x+y-7=0$ ....(i)

Again, taking BP = CP

$\Rightarrow BP^2 = CP^2$

$\Rightarrow (x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2-6x+9+y^2+14y+49=x^2-6x+9+y^2-6y+9$

$\Rightarrow -6x+6x+14y+6y+58-18=0$

$\Rightarrow 20y+40=0$

$\Rightarrow y=-2$

Putting the value of y in eq. (i),

$3x+y-7=0$

$\Rightarrow 3x=9$

$\Rightarrow x=3$

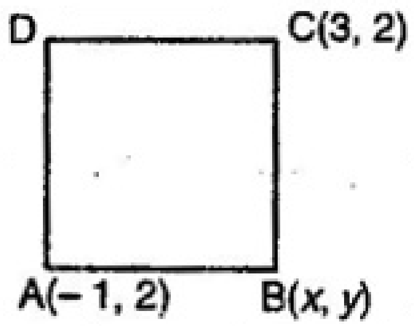
Hence, the centre of the circle is $(3,-2)$.

4. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.

Ans. Let ABCD be a square and B(x,y) be the unknown vertex.

AB = BC

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow 2x+1=-6x+9$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(i)

In ΔABC, $AB^2+BC^2=AC^2$

$\Rightarrow (x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2=(3+1)^2+(2-2)^2$

$\Rightarrow 2(y-2)^2+\left[(x+1)^2+(x-3)^2\right]=16+0$

$\Rightarrow 2(y-2)^2+\left[(1+1)^2+(1-3)^2\right]=16$

$\Rightarrow 2\left(y^2-4y+4\right)+8=16$

$\Rightarrow y^2-4y+4=4$

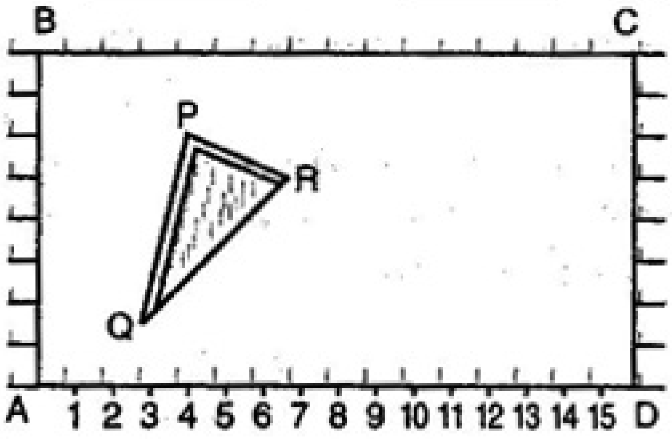
$\Rightarrow y(y-4)=0$

$\Rightarrow y$ = 0 or 4

Hence, the required vertices of the square are (1, 0) and (1, 4).

5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted

on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of ΔPQR if C is the origin? Also calculate the area of the triangle in these cases. What do you observe?

Ans. (i) Taking A as the origin, AD and AB as the coordinate axes. Clearly, the points P, Q and R are (4, 6), (3, 2) and (6, 5) respectively.

(ii) Taking C as the origin, CB and CD as the coordinate axes. Clearly, the points P, Q and R are given by (12, 2), (13, 6) and (10, 3) respectively.

We know that the area of the triangle = $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\therefore$ Area of ΔPQR (First case) = $\frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$

= $\frac{1}{2}\left[4(-3)+3(-1)+6(4)\right]$

= $\frac{1}{2}\left[-12-3+24\right]$ = $\frac{9}{2}$ sq. units

And Area of ΔPQR (Second case) = $\frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$

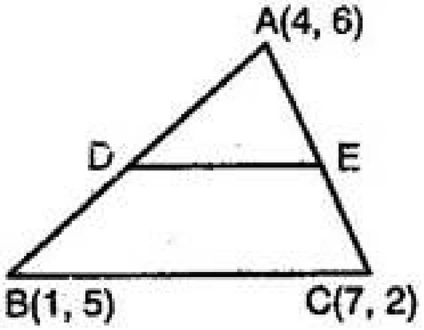
= $\frac{1}{2}\left[12(3)+13(1)+10(-4)\right]$

= $\frac{1}{2}\left[36+13-40\right]$ = $\frac{9}{2}$ sq. units

Hence, the areas are same in both the cases.

6. The vertices of a $\triangle$ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the $\triangle$ADE and compare it with the area of $\triangle$ABC.

Ans. Since, $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$



$\therefore$ DE $\parallel$ BC[By Thales theorem]

$\therefore$ $\triangle$ADE ~ $\triangle$ABC

$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area}(\Delta ABC)}=\frac{AD^2}{AB^2}$

$=\left(\frac{AD}{AB}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}$ ....(i)

Now, Area ($\triangle$ABC) = $\frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$

= $\frac{1}{2}[12-4+7]=\frac{15}{2}$ sq. units..........(ii)

From eq. (i) and (ii),

Area ($\triangle$ADE) = $\frac{1}{16}\times$ Area ($\triangle$ABC) = $\frac{1}{16}\times\frac{15}{2}=\frac{15}{32}$ sq. units

$\therefore$ Area ($\triangle$ADE): Area ($\triangle$ABC) = 1: 16

7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of $\triangle$ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1.

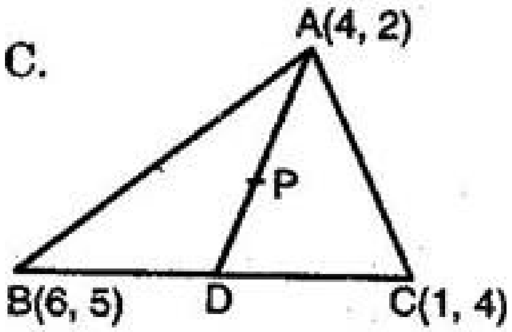
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR : RF = 2 : 1.

(iv) What do you observe?

(Note: The point which is common to all the three medians is called *centroid* and this point divides each median in the ratio 2: 1)

(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of $\triangle$ABC, find the coordinates of the centroid of the triangle.

Ans. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of $\triangle$ABC.



(i) Since AD is the median of $\triangle$ABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since P divides AD in the ratio 2: 1

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{7}{2}+1\times 4}{2+1}, \frac{2\times\frac{9}{2}+1\times 2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Since BE is the median of ΔABC.

$\therefore$ E is the mid-point of AD.

$\therefore$ Its coordinates are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Since Q divides BE in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Since CF is the median of ΔABC.

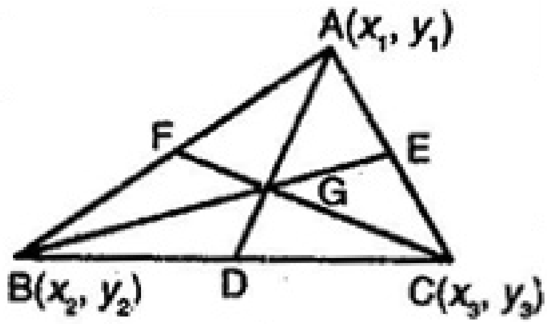
$\therefore$ F is the mid-point of AB.

$\therefore$ Its coordinates are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Since R divides CF in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) We observe that the points P, Q and R coincide, i.e., the medians AD, BE and CF are concurrent at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.



(v) According to the question, D, E, and F are the mid-points of BC, CA and AB respectively.

$\therefore$ Coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Coordinates of a point dividing AD in the ratio 2: 1 are

$$\left(\frac{1.x_1+2\left(\frac{x_2+x_2}{2}\right)}{1+2}, \frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$.

$\therefore$ The coordinates of a point dividing BE in the ratio 2: 1 are

$$\left(\frac{1.x_2+2\left(\frac{x_1+x_3}{2}\right)}{1+2}, \frac{1.y_2+2\left(\frac{y_1+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly, the coordinates of a point dividing CF in the ratio 2: 1 are

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Thus, the point $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 2: 1.

$\therefore$ The median of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

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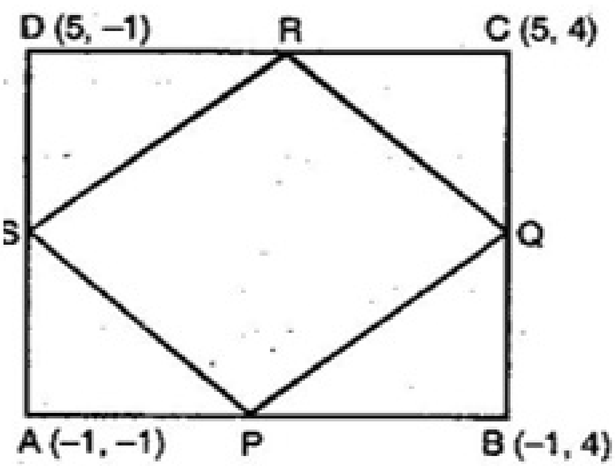
8. ABCD is a rectangle formed by joining points A$(-1,-1)$, B$(-1,4)$, C$(5,4)$ and D $(5,-1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? Or a rhombus? Justify your answer.

Ans. Since P is mid-point of AB, therefore, the coordinates of P are $\left(-1, \frac{3}{2}\right)$.

Similarly, the coordinates of Q are (2, 4)., the coordinates of R are $\left(5, \frac{3}{2}\right)$ and the coordinates of S are $(2, -1)$.

Using distance formula, PQ = $\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$



= $\sqrt{9+\frac{25}{4}}$ = $\sqrt{\frac{61}{4}}$

QR = $\sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2}$ = $\sqrt{9+\frac{25}{4}}$ = $\sqrt{\frac{61}{4}}$

RS = $\sqrt{(2-5)^2+\left(-1-\frac{3}{2}\right)^2}$ = $\sqrt{9+\frac{25}{4}}$ = $\sqrt{\frac{61}{4}}$

SP = $\sqrt{(-1-2)^2+\left(\frac{3}{2}+1\right)^2}$ = $\sqrt{9+\frac{25}{4}}$ = $\sqrt{\frac{61}{4}}$

$\Rightarrow$ PQ = QR = RS = SP

Now, PR = $\sqrt{(5+1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2}$ = $\sqrt{36}$ = 6

And SQ = $\sqrt{(2-2)^2+(4+1)^2}$ = $\sqrt{25}$ = 5

$\Rightarrow$ PR $\neq$ SQ

Since all the sides are equal but the diagonals are not equal.

$\therefore$ PQRS is a rhombus.