

CBSE Class-10 Mathematics

NCERT solution

Chapter - 4

Quadratic Equations - Exercise 4.4

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Ans. (i) $2x^2 - 3x + 5 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get $a = 2$, $b = -3$ and $c = 5$

$$\text{Discriminant} = b^2 - 4ac = (-3)^2 - 4(2)(5)$$

$$= 9 - 40 = -31$$

Discriminant is less than 0 which means equation has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

$$\text{Discriminant} = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$$

$$= 48 - 48 = 0$$

Discriminant is equal to zero which means equations has equal real roots.

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{2\sqrt{3}}{3}$$

Because, equation has two equal roots, it means $x = \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $ax^2 + bx + c = 0$,

We get $a = 2$, $b = -6$, and $c = 3$

$$\text{Discriminant} = b^2 - 4ac = (-6)^2 - 4(2)(3)$$

$$= 36 - 24 = 12$$

Value of discriminant is greater than zero.

Therefore, equation has distinct and real roots.

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{3}}{2}$$

$$\Rightarrow x = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$$

2. Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Ans. (i) $2x^2 + kx + 3 = 0$

We know that quadratic equation has two equal roots only when the value of discriminant is equal to zero.

Comparing equation $2x^2 + kx + 3 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$

Discriminant = $b^2 - 4ac = k^2 - 4(2)(3) = k^2 - 24$

Putting discriminant equal to zero

$$k^2 - 24 = 0 \Rightarrow k^2 = 24$$

$$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$$

$$\Rightarrow k = 2\sqrt{6}, -2\sqrt{6}$$

(ii) $kx(x - 2) + 6 = 0$

$$\Rightarrow kx^2 - 2kx + 6 = 0$$

Comparing quadratic equation $kx^2 - 2kx + 6 = 0$ with general form $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$

Discriminant = $b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$$= 4k^2 - 24k$$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$$4k^2 - 24k = 0$$

$$\Rightarrow 4k(k - 6) = 0 \Rightarrow k = 0, 6$$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Therefore, in equation $kx^2 - 2kx + 6 = 0$, we cannot have $k = 0$.

Therefore, we discard $k = 0$.

Hence the answer is $k = 6$.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800m^2 ? If so, find its length and breadth.

Ans. Let breadth of rectangular mango grove = x metres

Let length of rectangular mango grove = $2x$ metres

$$\text{Area of rectangle} = \text{length} \times \text{breadth} = x \times 2x = 2x^2 \text{ m}^2$$

According to given condition:

$$\Rightarrow 2x^2 = 800$$

$$\Rightarrow 2x^2 - 800 = 0$$

$$\Rightarrow x^2 - 400 = 0$$

Comparing equation $x^2 - 400 = 0$ with general form of quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = 0$ and $c = -400$

$$\text{Discriminant} = b^2 - 4ac = (0)^2 - 4(1)(-400) = 1600$$

Discriminant is greater than 0 means that equation has two distinct real roots.

Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{0 \pm \sqrt{1600}}{2 \times 1} = \frac{\pm 40}{2} = \pm 20$$

$$\Rightarrow x = 20, -20$$

We discard negative value of x because breadth of rectangle cannot be in negative.

Therefore, $x =$ breadth of rectangle = 20 metres

Length of rectangle = $2x = 2 \times 20 = 40$ metres

4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans. Let age of first friend = x years

then age of second friend = $(20 - x)$ years

Four years ago, age of first friend = $(x - 4)$ years

Four years ago, age of second friend = $(20 - x) - 4 = (16 - x)$ years

According to given condition,

$$\Rightarrow (x - 4)(16 - x) = 48$$

$$\Rightarrow 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow 20x - x^2 - 112 = 0$$

$$\Rightarrow x^2 - 20x + 112 = 0$$

Comparing equation, $x^2 - 20x + 112 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$

$$\text{Discriminant} = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$$

Discriminant is less than zero which means we have no real roots for this equation.

Therefore, the give situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 metres and area 400 m^2 . If so, find its length and breadth.

Ans. Let length of park = x metres

We are given area of rectangular park = 400 m^2

Therefore, breadth of park = $\frac{400}{x}$ metres {Area of rectangle = length \times breadth}

Perimeter of rectangular park = $2(\text{length} + \text{breadth}) = 2\left(x + \frac{400}{x}\right)$ metres

We are given perimeter of rectangle = 80 metres

According to condition:

$$\Rightarrow 2\left(x + \frac{400}{x}\right) = 80$$

$$\Rightarrow 2\left(\frac{x^2 + 400}{x}\right) = 80$$

$$\Rightarrow 2x^2 + 800 = 80x$$

$$\Rightarrow 2x^2 - 80x + 800 = 0$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

Comparing equation, $x^2 - 40x + 400 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$

$$\text{Discriminant} = b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$$

Discriminant is equal to 0.

Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area $400m^2$.

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{40 \pm \sqrt{0}}{2} = \frac{40}{2} = 20$$

Here, both the roots are equal to 20.

Therefore, length of rectangular park = 20 metres

$$\text{Breadth of rectangular park} = \frac{400}{x} = \frac{400}{20} = 20 \text{ m}$$