CBSE Class–10 Mathematics

NCERT solution

Chapter - 5

Arithmetic Progressions - Exercise 5.3

1. Find the sum of the following AP's.

(i) 2, 7, 12... to 10 terms

(ii) –37, –33, –29... to 12 terms

(iii) 0.6, 1.7, 2.8... to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ **to 11 terms**

Ans. (i) 2, 7, 12... to 10 terms

Here First term = a = 2, Common difference = d = 7 – 2 = 5 and n = 10

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{10}{2}\left[4 + (10-1)5\right] = 5(4+45) = 5 \times 49 = 245$$

(ii) –37, –33, –29... to 12 terms

Here First term = a = –37, Common difference = d = –33 – (–37) = 4

And n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{12}{2}\left[-74 + (12-1)4\right] = 6(-74+44) = 6 \times (-30) = -180$$

(iii) 0.6, 1.7, 2.8... to 100 terms

Here First term = a = 0.6, Common difference = d = 1.7 – 0.6 = 1.1

And n = 100

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP,

$$Sn = \frac{100}{2}\left[1.2 + (100-1)\,1.1\right] = 50\,(1.2 + 108.9) = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}...$ to 11 terms

Here First term =a = $\frac{1}{15}$ Common difference = d = $\frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right) = \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + ... + 84$

(ii) 34 + 32 + 30 + ... + 10

(iii) –5 + (–8) + (–11) + ... + (–230)

Ans. (i) $7 + 10\frac{1}{2} + 14 + ... + 84$

Here First term = a = 7, Common difference = d = $\frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2} = 3.5$

And Last term = l = 84

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$[7 + (n - 1)(3.5)] = 84$

$\Rightarrow 7 + (3.5)n - 3.5 = 84$

$\Rightarrow 3.5n = 84 + 3.5 - 7$

$\Rightarrow 3.5n = 80.5$

$\Rightarrow n = 23$

Therefore, there are 23 terms in the given AP.

It means n = 23.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{23} = \frac{23}{2}(7 + 84)$$

$$\Rightarrow S_{23} = \frac{23}{2} \times 91 = 1046.5$$

(ii) 34 + 32 + 30 + ... + 10

Here First term = a = 34, Common difference = d = 32 – 34 = –2

And Last term = l = 10

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$[34 + (n - 1)(-2)] = 10$

$\Rightarrow 34 - 2n + 2 = 10$

$\Rightarrow -2n = -26 \Rightarrow n = 13$

Therefore, there are 13 terms in the given AP.

It means n = 13.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 286$$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Here First term = a = –5, Common difference = d = –8 – (–5) = –8 + 5 = –3

And Last term = $l = -230$

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$[-5 + (n - 1)(-3)] = -230$

$\Rightarrow -5 - 3n + 3 = -230$

$\Rightarrow -3n = -228 \Rightarrow n = 76$

Therefore, there are 76 terms in the given AP.

It means n = 76.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$S_{76} = \frac{76}{2}(-5-230) = 38 \times (-235) = -8930$

3. In an AP

(i) given a = 5, d = 3, $a_n = 50$, find n and S_n.

(ii) given a = 7, $a_{13} = 35$, find d and S_{13}.

(iii) given $a_{12} = 37$, d = 3, find a and S_{12}.

(iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

(v) given d = 5, $S_9 = 75$, find a and a_9.

(vi) given a = 2, d = 8, $S_n = 90$, find n and a_n.

(vii) given a = 8, $a_n = 62, S_n = 210$, find n and d.

(viii) given $a_n = 4$, d = 2, $S_n = -14$, find n and a.

(ix) given a = 3, n = 8, S = 192, find d.

(x) given l = 28, S = 144, and there are total of 9 terms. Find a.

Ans. (i) Given a = 5, d = 3, $a_n = 50$, find n and S_n.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_n = 5 + (n - 1)(3)$

$\Rightarrow 50 = 5 + 3n - 3$

$\Rightarrow 48 = 3n \Rightarrow n = 16$

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_{16}=\frac{16}{2}[10+(16-1)3]=8(10+45)=8\times 55=440$$

Therefore, n = 16 and $S_n = 440$

(ii) Given a = 7, $a_{13} = 35$, find d and S_{13}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{13} = 7 + (13 − 1) (d)

⇒ 35 = 7 + 12d

⇒ 28 = 12d⇒ d = $\frac{7}{3}$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{13}=\frac{13}{2}\left[14+(13-1)\frac{7}{3}\right]=\frac{13}{2}(14+28)=\frac{13}{2}\times 42=273$$

Therefore, d = $\frac{7}{3}$ and $S_{13}=273$

(iii) Given $a_{12} = 37$, d = 3, find a and S_{12}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{12} = a + (12 − 1) 3

⇒ 37 = a + 33 ⇒ a = 4

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{12}=\frac{12}{2}[8+(12-1)3]=6(8+33)=6\times 41=246$$

Therefore, a = 4 and S_{12} = 246

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$a_3 = a + (3 - 1)(d)$

$\Rightarrow 15 = a + 2d$

$\Rightarrow a = 15 - 2d$... (1)

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$\Rightarrow 125 = 5(2a + 9d) = 10a + 45d$

Putting (1) in the above equation,

$125 = 5[2(15 - 2d) + 9d] = 5(30 - 4d + 9d)$

$\Rightarrow 125 = 150 + 25d$

$\Rightarrow 125 – 150 = 25d$

$\Rightarrow -25 = 25d \Rightarrow d = -1$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$a_{10} = a + (10 - 1)d$

Putting value of d and equation (1) in the above equation,

$a_{10} = 15 - 2d + 9d = 15 + 7d$

$= 15 + 7(-1) = 15 – 7 = 8$

Therefore, d = −1 and a_{10} = 8

(v) Given d = 5, $S_9 = 75, find\ a\ and\ a_9$.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP,

$$S_9 = \frac{9}{2}\left[2a+(9-1)5\right]$$

$\Rightarrow 75 = \frac{9}{2}\left[2a+40\right]$

⇒ 150 = 18a + 360

⇒ −210 = 18a

⇒ $a = \frac{-35}{3}$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_9 = \frac{-35}{3}$ + (9 − 1) (5)

$= \frac{-35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given a = 2, d = 8, $S_n = 90, find\ n\ and\ a_n$.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP,

$$90 = \frac{n}{2}\left[4+(n-1)8\right]$$

$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$

$\Rightarrow 90 = \frac{n}{2}[8n - 4]$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$

$\Rightarrow (n - 5)(2n + 9) = 0$

$\Rightarrow n = 5, -9/2$

We discard negative value of n because here n cannot be in negative or fraction.

The value of n must be a positive integer.

Therefore, $n = 5$

Using formula $a_n = a + (n - 1)d$,to find nth term of arithmetic progression,

$a_5 = 2 + (5 - 1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$

(vii) Given $a = 8$, $a_n = 62, S_n = 210$, find n and d.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$62 = 8 + (n - 1)(d) = 8 + nd - d$

$\Rightarrow 62 = 8 + nd - d$

$\Rightarrow nd - d = 54$

$\Rightarrow nd = 54 + d$... (1)

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$210 = \frac{n}{2}[16 + (n-1)d] = \frac{n}{2}(16 + nd - d)$$

Putting equation (1) in the above equation,

$$210 = \frac{n}{2}[16 + 54 + d - d] = \frac{n}{2} \times 70$$

$\Rightarrow n = \frac{210 \times 2}{70} = 6 \Rightarrow n = 6$

Putting value of n in equation (1),

$6d = 54 + d \Rightarrow d = \frac{54}{5}$

Therefore, $n = 6$ and $d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$4 = a + (n - 1)(2) = a + 2n - 2$

$\Rightarrow 4 = a + 2n - 2$

$\Rightarrow 6 = a + 2n$

$\Rightarrow a = 6 - 2n$... (1)

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a + (n-1)2] = \frac{n}{2}(2a + 2n - 2)$$

$$\Rightarrow -14 = \frac{n}{2}(2a + 2n - 2)$$

Putting equation (1) in the above equation, we get

$-28 = n\,[2\,(6 - 2n) + 2n - 2]$

$\Rightarrow -28 = n\,(12 - 4n + 2n - 2)$

$\Rightarrow -28 = n\,(10 - 2n)$

$\Rightarrow 2n^2 - 10n - 28 = 0$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n\,(n - 7) + 2\,(n - 7) = 0$

$\Rightarrow (n + 2)\,(n - 7) = 0$

$\Rightarrow n = -2, 7$

Here, we cannot have negative value of n.

Therefore, we discard negative value of n which means $n = 7$.

Putting value of n in equation (1), we get

$a = 6 - 2n = 6 - 2\,(7) = 6 - 14 = -8$

Therefore, $n = 7$ and $a = -8$

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

Using formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$192 = \frac{8}{2}[6 + (8 - 1)d] = 4(6 + 7d)$

$\Rightarrow 192 = 24 + 28d$

$\Rightarrow 168 = 28d \Rightarrow d = 6$

(x) Given $l = 28$, $S = 144$, and there are total of 9 terms. Find a.

Applying formula, $S_n = \frac{n}{2}[a + l]$, to find sum of n terms, we get

$144 = \frac{9}{2}[a + 28]$

$\Rightarrow 288 = 9[a + 28]$

$\Rightarrow 32 = a + 28 \Rightarrow a = 4$

4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

Ans. First term = a = 9, Common difference = d = 17 – 9 = 8, $S_n = 636$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$636 = \frac{n}{2}[18 + (n - 1)(8)]$

$\Rightarrow 1272 = n(18 + 8n - 8)$

$\Rightarrow 1272 = 18n + 8n^2 - 8n$

$\Rightarrow 8n^2 + 10n - 1272 = 0$

$\Rightarrow 4n^2 + 5n - 636 = 0$

Comparing equation $4n^2 + 5n - 636 = 0$ with general form $an^2 + bn + c = 0$, we get

a = 4, b = 5 and c = −636

Applying quadratic formula, $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and putting values of a, b and c, we get

$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{8}$

$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$

$\Rightarrow n = \frac{-5 \pm 101}{8}$

$\Rightarrow n = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{106}{8}$

We discard negative value of n here because n cannot be in negative, n can only be a positive integer.

Therefore, n = 12

Therefore, 12 terms of the given sequence make sum equal to 636.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. First term = a = 5, Last term = l = 45, $S_n = 400$

Applying formula, $S_n = \frac{n}{2}\left[a + l\right]$ to find sum of n terms of AP, we get

$400 = \frac{n}{2}\left[5 + 45\right]$

$\Rightarrow \frac{400}{50} = \frac{n}{2} \Rightarrow$ n = 16

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP and putting value of n, we get

$$400 = \frac{16}{2}[10 + (16-1)d]$$

$\Rightarrow 400 = 8\ (10 + 15d)$

$\Rightarrow 400 = 80 + 120d$

$\Rightarrow 320 = 120d$

$\Rightarrow d = \frac{320}{120} = \frac{8}{3}$

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6. The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?

Ans. First term = a = 17, Last term = l = 350 and Common difference = d = 9

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$350 = 17 + (n - 1)\ (9)$

$\Rightarrow 350 = 17 + 9n - 9$

$\Rightarrow 342 = 9n \Rightarrow n = 38$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$S_{38} = \frac{38}{2}[34 + (38-1)9]$$

$\Rightarrow S_{38} = 19\ (34 + 333) = 6973$

Therefore, there are 38 terms and their sum is equal to 6973.

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7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Ans. It is given that 22nd term is equal to 149 $\Rightarrow a_{22} = 149$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

149 = a + (22 − 1) (7)

⇒ 149 = a + 147⇒ a = 2

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP and putting value of a, we get

$$S_{22} = \frac{22}{2}[4+(22-1)7]$$

⇒ S_{22} = 11 (4 + 147)

⇒ S_{22} = 1661

Therefore, sum of first 22 terms of AP is equal to 1661.

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8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. It is given that second and third term of AP are 14 and 18 respectively.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

14 = a + (2 − 1) d

⇒ 14 = a + d … (1)

And, 18 = a + (3 − 1) d

⇒ 18 = a + 2d … (2)

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$

Putting value of a in equation (2), we get

$18 = 14 – d + 2d$

$\Rightarrow d = 4$

Therefore, common difference $d = 4$

Putting value of d in equation (1), we get

$18 = a + 2\ (4)$

$\Rightarrow a = 10$

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{51} = \frac{51}{2}\left[20+(51-1)d\right] = \frac{51}{2}(20+200) = \frac{51}{2}\times 220 = 51\times 110 = 5610$$

Therefore, sum of first 51 terms of an AP is equal to 5610.

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9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans. It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$49 = \frac{7}{2}\left[2a+(7-1)d\right]49$$

$\Rightarrow 98 = 7\ (2a + 6d)$

$\Rightarrow 7 = a + 3d \Rightarrow a = 7 - 3d$ … (1)

And, $289 = \frac{17}{2}\left[2a + (17-1)d\right]$

$\Rightarrow 578 = 17\,(2a + 16d)$

$\Rightarrow 34 = 2a + 16d$

$\Rightarrow 17 = a + 8d$

Putting equation (1) in the above equation, we get

$17 = 7 - 3d + 8d$

$\Rightarrow 10 = 5d \Rightarrow d = 2$

Putting value of d in equation (1), we get

$a = 7 - 3d = 7 - 3\,(2) = 7 - 6 = 1$

Again applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$S_n = \frac{n}{2}\left[2(1) + (n-1)2\right]$

$\Rightarrow S_n = \frac{n}{2}\left[2 + 2n - 2\right]$

$\Rightarrow S_n = \frac{n}{2} \times 2n \Rightarrow S_n = n^2$

Therefore, sum of n terms of AP is equal to n^2.

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10. Show that $a_1, a_2 \ldots a_n$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) We need to show that $a_1, a_2 ... a_n$ form an AP where $a_n = 3 + 4n$

Let us calculate values of $a_1, a_2, a_3 ...$ using $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3+4=7$

$a_2 = 3 + 4 (2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3+12=15$

$a_4 = 3 + 4 (4) = 3 + 16 = 19$

So, the sequence is of the form 7, 11, 15, 19 ...

Let us check difference between consecutive terms of this sequence.

11 – 7 = 4, 15 – 11 = 4, 19 – 15 = 4

Therefore, the difference between consecutive terms is constant which means terms a_1, a_2 ... a_n form an AP.

We have sequence 7, 11, 15, 19 ...

First term = a = 7 and Common difference = d = 4

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[14 + (15-1)4] = \frac{15}{2}(14 + 56) = \frac{15}{2} \times 70 = 15 \times 35 = 525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 ... a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 ...$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1)$ = 9-5 = 4 a_2= 9 – 5 (2) = 9 – 10 = −1

$a_3 = 9 - 5(3)$ = 9-15=-6 a_4= 9 – 5 (4) = 9 – 20 = −11

So, the sequence is of the form 4, −1, −6, −11 ...

Let us check difference between consecutive terms of this sequence.

–1 – (4) = –5,–6 – (–1)

= –6 + 1 = –5,–11 – (–6)

= –11 + 6 = –5

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence 4, −1, −6, −11 ...

First term = a = 4 and Common difference = d = –5

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$$S_{15} = \frac{15}{2}[8 + (15-1)(-5)] = \frac{15}{2}(8-70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to –465.

11. If the sum of the first n terms of an AP is $(4n - n^2)$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Ans. It is given that the sum of n terms of an AP is equal to $(4n - n^2)$

It means $S_n = 4n - n^2$

Let us calculate $S_1 and\ S_2$ using $S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2$ = 4 – 1 = 3

$S_2 = 4(2) - (2)^2$ = 8 – 4 = 4

First term = a = S_1 = 3 … (1)

Let us find common difference now.

We can write any AP in the form of general terms like a , a + d, a + 2d …

We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$

Therefore, we can say that a + (a + d) = 4

Putting value of a from equation (1), we get

2a + d = 4

⇒ 2 (3) + d = 4

⇒ 6 + d = 4

⇒ d = –2

Using formula a_n = a + (n – 1) d, to find n^{th} term of arithmetic progression,

Second term of AP = a_2 = a + (2 – 1) d = 3 + (2 – 1) (–2) = 3 – 2 = 1

Third term of AP = a_3 = a + (3 – 1) d = 3 + (3 – 1) (–2) = 3 – 4 = –1

Tenth term of AP = a_{10} = a + (10 – 1) d = 3 + (10 – 1) (–2) = 3 – 18 = –15

n^{th} term of AP = a_n = a + (n – 1) d = 3 + (n – 1) (–2) = 3 – 2n + 2 = 5 – 2n

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12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, 24 … 40 terms.

Therefore, we want to find sum of 40 terms of sequence of the form:

6, 12, 18, 24 … 40 terms

Here, first term = a = 6 and Common difference = d = 12 – 6 = 6, n = 40

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{40} = \frac{40}{2}\left[12 + (40-1)6\right]$$

$= 20\ (12 + 39 \times 6)$

= 20 (12 + 234)

$= 20 \times 246 = 4920$

13. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are 8,16, 24, 32 … 15 terms

First term = a = 8 and Common difference = d = 16 – 8 = 8, n = 15

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}\left[16 + (15-1)8\right] = \frac{15}{2}(16 + 14 \times 8) = \frac{15}{2}(16 + 112) = \frac{15}{2} \times 128 = 15 \times 64 = 960$$

14. Find the sum of the odd numbers between 0 and 50.

Ans. The odd numbers between 0 and 50 are 1, 3, 5, 7 … 49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 – 1 = 2, Last term = l = 49

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n = a + (n - 1) d$, to find nth term of arithmetic progression, we get

$49 = 1 + (n - 1) 2$

$\Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 50 = 2n \Rightarrow n = 25$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$$S_{25} = \frac{25}{2}(1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

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15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250

Penalty for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form200, 250, 300, 350 … 30 terms

First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30-1)50]$$

$\Rightarrow S_n = 15\ (400\ +\ 29\ \times\ 50)$

$\Rightarrow S_n$ = 15 (400 + 1450) = 27750

Therefore, penalty for 30 days is Rs. 27750.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize =Rs (a − 20)

Let value of third prize = Rs (a − 40)

So, we have sequence of the form:

a, a − 20, a − 40, a – 60 …

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a – 20) – a = –20

n = 7 (Because there are total of seven prizes)

$S_7 =\ Rs\ \ 700$ {given}

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_7 = \frac{7}{2}\left[2a + (7-1)(-20)\right]$$

$$\Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$$

⇒ 200 = 2a – 120

⇒ 320 = 2a ⇒ a = 160

Therefore, value of first prize = Rs 160

Value of second prize = 160 – 20 = Rs 140

Value of third prize = 140 – 20 = Rs 120

Value of fourth prize = 120 – 20 = Rs 100

Value of fifth prize = 100 – 20 = Rs 80

Value of sixth prize = 80 – 20 = Rs 60

Value of seventh prize = 60 – 20 = Rs 40

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17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$

The number of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$

Therefore, we have sequence of the form 3, 6, 9 … 12 terms

To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 … 12 terms.

First term = a = 3, Common difference = d= 6 – 3 = 3 and n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP , we get

$$S_{12} = \frac{12}{2}[6 + (12-1)3] = 6(6+33) = 6 \times 39 = 234$$

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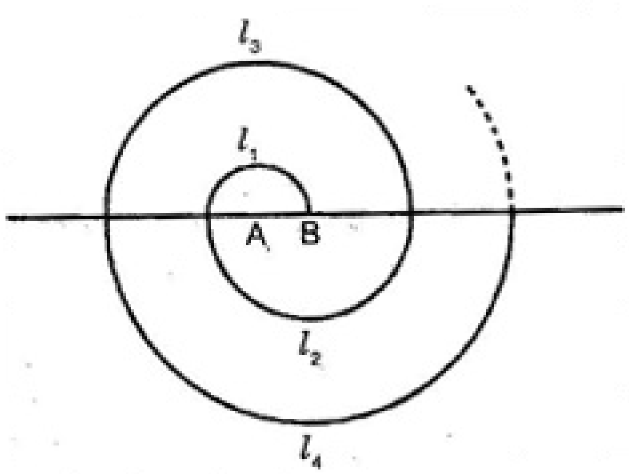
18. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles.



Ans. Length of semi–circle = $\frac{\text{Circumference of circle}}{2} = \frac{2\pi r}{2} = \pi r$

Length of semi-circle of radii 0.5 cm = π (0.5) cm

Length of semi-circle of radii 1.0 cm = π (1.0) cm

Length of semi-circle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π (0.5), π (1.0), π (1.5) … 13 terms {There are total of thirteen semi–circles}.

To find total length of the spiral, we need to find sum of the sequence π (0.5), π (1.0), π (1.5) … 13 terms

Total length of spiral = π (0.5) + π (1.0) + π (1.5) … 13 terms

⇒ Total length of spiral = π (0.5 + 1.0 + 1.5) … 13 terms … (1)

Sequence 0.5, 1.0, 1.5 … 13 terms is an arithmetic progression.

Let us find the sum of this sequence.

First term = a = 0.5, Common difference = 1.0 – 0.5 = 0.5 and n = 13

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

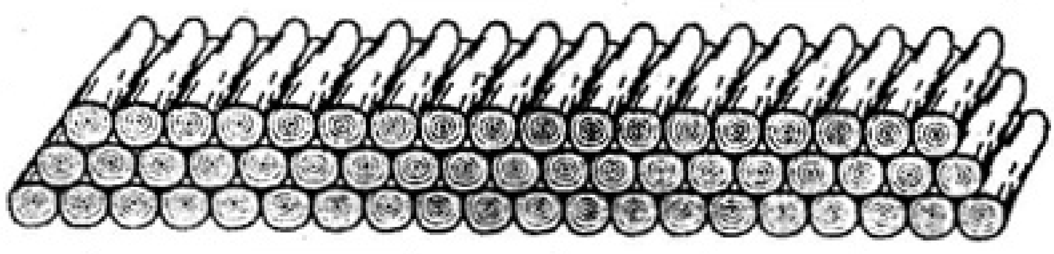
$$S_{13} = \frac{13}{2}\left[1+(13-1)0.5\right] = 6.5(1+6) = 6.5\times 7 = 45.5$$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 … 13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral= π (0.5 + 1.5 + 2.0 + … 13 terms) = π (45.5) = 143 cm

19. 200 logs are stacked in the following manner:20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 …

First term = a = 20, Common difference = d = 19 – 20 = –1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$200 = \frac{n}{2}\left[40+(n-1)(-1)\right]$$

⇒ 400 = n (40 – n + 1)

⇒ 400 = $40n - n^2 + n$

⇒ $n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

⇒ n^2 − 25n − 16n + 400 = 0

⇒ n (n − 25) – 16 (n − 25) = 0

⇒ (n − 25) (n − 16)

⇒ n = 25, 16

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 …

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

200 = 8 (20 + l)

⇒ 200 = 160 + 8I

⇒ 40 = 8l ⇒ l = 5

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

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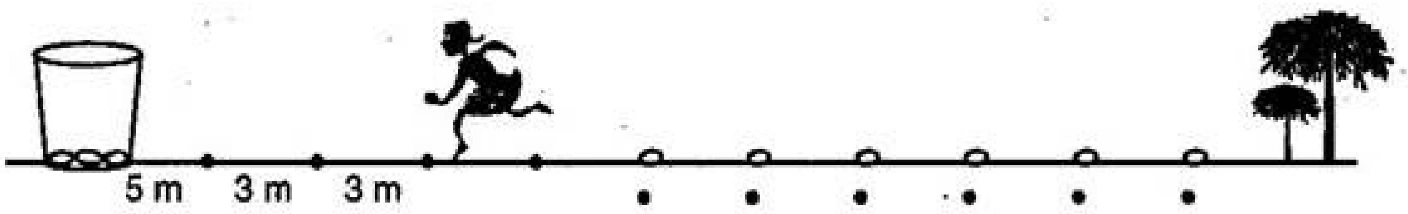
20. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket = 5×2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket = 8×2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = 11×2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22 … 10 terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

10 + 16 + 22 + … 10 terms

First term = a = 10, Common difference = d = 16 – 10 = 6

n = 10{There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}\left[20 + (10-1)6\right] = 5(20+54) = 5 \times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.