CBSE Class–10 Mathematics

NCERT solution

Chapter - 6

Triangles - Exercise 6.5

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i)7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii)50 cm, 80 cm, 100 cm

(iv)13 cm, 12 cm, 5 cm

Ans. (i) Let a = 7 cm, b = 24 cm and c = 25 cm

Here the larger side is c = 25 cm.

We have, $a^2 + b^2 = 7^2 + 24^2 = 49 + 576 = 625 = c^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 25 cm

(ii) Let a = 3 cm, b = 8 cm and c = 6 cm

Here the larger side is b = 8 cm.

We have, $a^2 + c^2 = 3^2 + 6^2 = 9 + 36 = 45 \neq b^2$

So, the triangle with the given sides is not a right triangle.

(iii) Let a = 50 cm, b = 80 cm and c = 100 cm

Here the larger side is c = 100 cm.

We have, $a^2 + b^2 = 50^2 + 80^2 = 2500 + 6400 = 8900 \neq c^2$

So, the triangle with the given sides is not a right triangle.

(iv) Let a = 13 cm, b = 12 cm and c = 5 cm

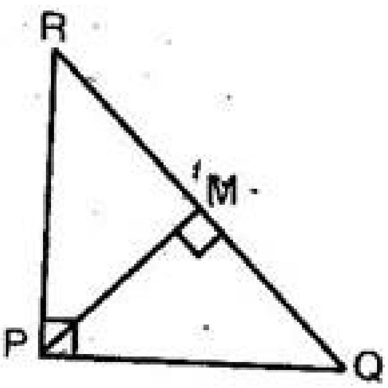
Here the larger side is a = 13 cm.

We have, $b^2+c^2=12^2+5^2=144+25=169=a^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 13 cm

2. PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that PM^2 = QM x MR.

Ans. Given: PQR is a triangle right angles at P and PM $\perp$ QR



To Prove: PM^2 = QM.MR

Proof: Since PM $\perp$ QR

$\therefore \Delta QMP \sim \Delta PMR$

$$\Rightarrow \frac{PM}{QM}=\frac{MR}{PM}$$

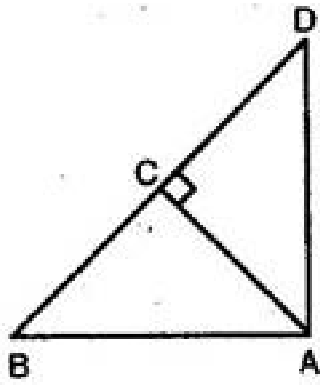
$$\Rightarrow PM^2 = QM.MR$$

3. In the given figure, ABD is a triangle right angled at A and AC $\perp$ BD. Show that:

(i) AB^2 = BC.BD

(ii) AC^2 = BC.DC

(iii) AD^2 = BD.CD



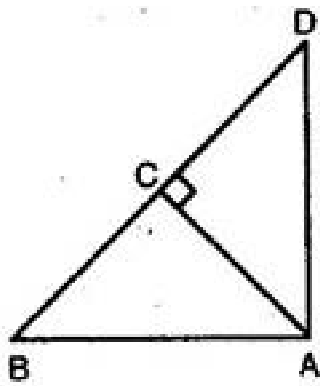
Ans. Given: ABD is a triangle right angled at A and AC $\perp$ BD.

To Prove: **(i)** AB^2 = BC.BD, **(ii)** $AC^2 = BC.DC$, **(iii)** $AD^2 = BD.CD$

Proof:**(i)** Since AC $\perp$ BD

$\therefore$ $\triangle$CBA $\sim$ $\triangle$ CAD and each triangle is similar to $\triangle$ABD

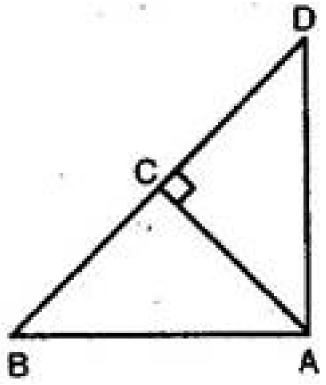
$\therefore$ $\triangle$ABD $\sim$ $\triangle$ CBA



$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$

$\Rightarrow AB^2 = BC.BD$

(ii) Since $\triangle$ABC $\sim$ $\triangle$ DAC



$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC}$

$\Rightarrow AC^2 = BC.DC$

(iii) Since $\Delta CAD \sim \Delta ABD$

$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$

$\Rightarrow AD^2 = BD.CD$

4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Ans. Since ABC is an isosceles right triangle, right angled at C.

$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$ [BC = AC, given]

$\Rightarrow AB^2 = 2AC^2$

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Ans. Since ABC is an isosceles right triangle with AC = BC and $AB^2 = 2AC^2$

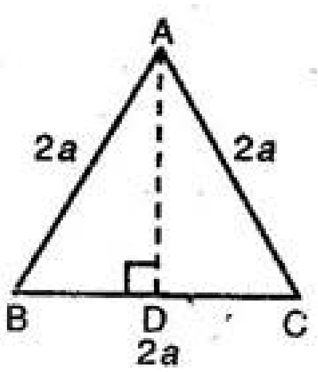
$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$ [BC = AC, given]

$\therefore$ ΔABC is right angled at C.

6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Ans. Let ABC be an equilateral triangle of side $2a$ units.



Draw AD $\perp$ BC. Then, D is the midpoint of BC.

$\Rightarrow$ BD = $\frac{1}{2}$ BC = $\frac{1}{2} \times 2a = a$

Since, ABD is a right triangle, right triangle at D.

$\therefore AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + (a)^2$

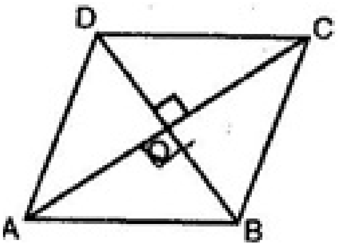
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

$\therefore$ Each of its altitude = $\sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of squares of its diagonals.

Ans. Let the diagonals AC and BD of rhombus ABCD intersect each other at O. Since the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$ and AO = CO, BO = OD



Since AOB is a right triangle, right angled at O.

$\therefore\ AB^2 = OA^2 + OB^2$

$\Rightarrow\ AB^2 = \left(\frac{1}{2}\text{AC}\right)^2 + \left(\frac{1}{2}\text{BD}\right)^2$

[∵ OA = OC and OB = OD]

$\Rightarrow\ 4AB^2 = AC^2 + BD^2$ ....(1)

Similarly, we have $4BC^2 = AC^2 + BD^2$ ....(2)

$4CD^2 = AC^2 + BD^2$ ....(3)

$4AD^2 = AC^2 + BD^2$ ....(4)

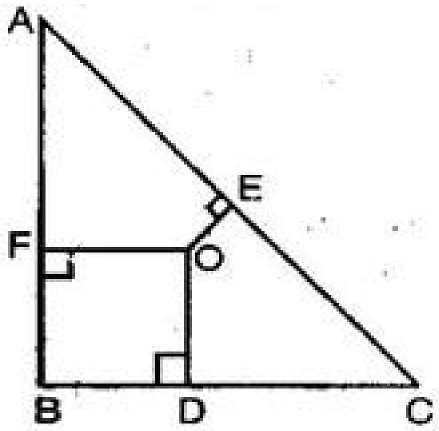
Adding all these results, we get

$4\left(AB^2 + BC^2 + CD^2 + DA^2\right) = 4\left(AC^2 + BD^2\right)$

$\Rightarrow\ AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

8. In the given figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC,

OE $\perp$ AC and OF $\perp$ AB. Show that:



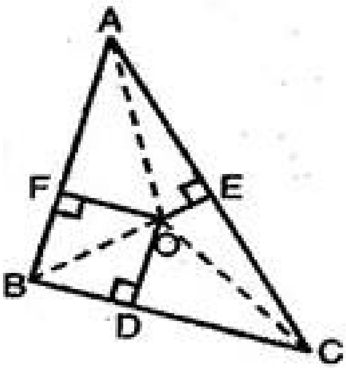
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Ans. Join AO, BO and CO.

(i) In right Δs OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$, $OB^2 = BD^2 + OD^2$ and $OC^2 = CE^2 + OE^2$



Adding all these, we get

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) In right Δs ODB and ODC, we have

$OB^2 = BD^2 + OD^2$ and $OC^2 = OD^2 + CD^2$

$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2$ ....(1)

Similarly, we have $OB^2 - OC^2 = BD^2 - CD^2$ ....(2)

and $OB^2 - OC^2 = BD^2 - CD^2$ ....(3)

Adding equations (1), (2) and (3), we get

$= \left(OB^2 - OC^2\right) + \left(OC^2 - OA^2\right) + \left(OA^2 - OB^2\right)$

$= \left(BD^2 - CD^2\right) + \left(CE^2 - AE^2\right) + \left(AF^2 - BF^2\right)$

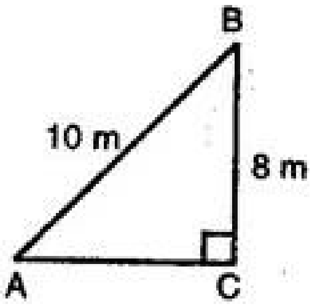
$\Rightarrow \left(BD^2 + CE^2 + AF^2\right) - \left(AE^2 + CD^2 + BF^2\right) = 0$

$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$

9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Ans. Let AB be the ladder, B be the window and CB be the wall. Then, ABC is a

right triangle, right angled at C.



$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow 10^2 = AC^2 + 8^2$

$\Rightarrow AC^2 = 100 - 64$

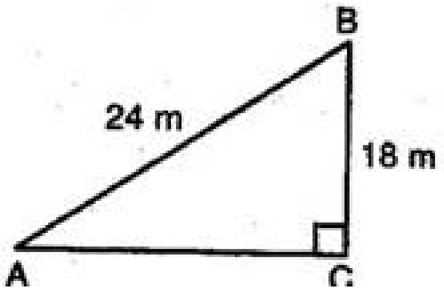
$\Rightarrow AC^2 = 36$

$\Rightarrow$ AC = 6

Hence, the foot of the ladder is at a distance 6 m from the base of the wall.

10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other hand. How far from the base of the pole should the stake be driven so that the wire will be taut?

Ans. Let AB (= 24m) be a guy wire attached to a vertical pole. BC of height 18 m. To keep the wire taut, let it be fixed to a stake at A. Then, ABC is a right triangle, right angled at C.



$\therefore\ AB^2 = AC^2 + BC^2$

$\Rightarrow 24^2 = AC^2 + 18^2$

$\Rightarrow AC^2 = 576 - 324$

$\Rightarrow AC^2 = 252$

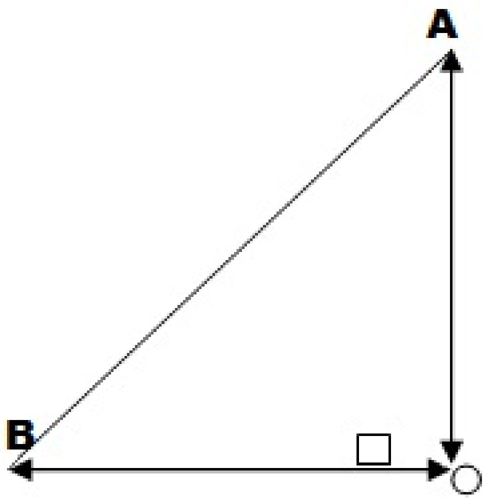
$\Rightarrow AC = 6\sqrt{7}$

Hence, the stake may be placed at a distance of $6\sqrt{7}$ m from the base of the pole.

11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Ans. Let the first aeroplane starts from O and goes upto A towards north where

OA = $\left(1000\times\frac{3}{2}\right)$ km = 1500 km



Let the second aeroplane starts from O at the same time and goes upto

B towards west where

OB = $\left(1200\times\frac{3}{2}\right)$ km = 1800 km

According to the question the required distance = BA

In right angled triangle ABC, by Pythagoras theorem, we have,

$AB^2 = OA^2 + OB^2$

= $(1500)^2 + (1800)^2$

= 2250000 + 3240000

= 5490000 = $9\times61\times100\times100$

$\Rightarrow$ AB = $300\sqrt{61}$ km

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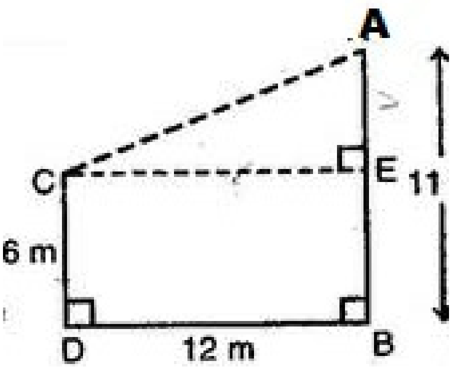
12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Ans. Let AB = 11 m and CD = 6 m be the two poles such that BD = 12 m



Draw CE $\perp$ AB and join AC.

$\therefore$ CE = DB = 12 m

AE = AB – BE = AB – CD = (11 – 6)m = 5 m

In right angled triangle ACE, by Pythagoras theorem, we have

$AC^2 = CE^2 + AE^2 = 12^2 + 5^2$

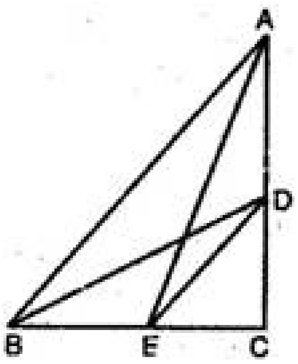
= 144 + 25 = 169

$\Rightarrow$ AC = 13 m

Hence, the distance between the tops of the two poles is 13 m.

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Ans. In right angled Δs ACE and DCB, we have



$AE^2 = AC^2 + CE^2$ and $BD^2 = DC^2 + BC^2$

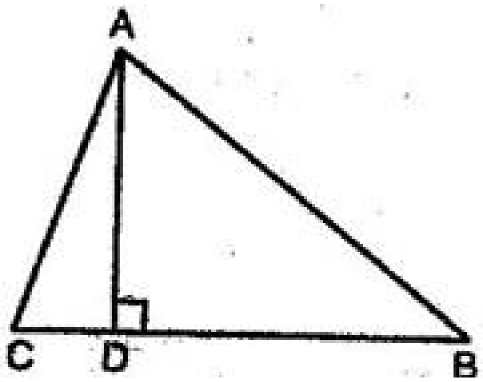
$\Rightarrow AE^2 + BD^2 = (AC^2 + CE^2) + (DC^2 + BC^2)$

$\Rightarrow AE^2 + BD^2 = (AC^2 + BC^2) + (DC^2 + CE^2)$

$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$

[By Pythagoras theorem, $AC^2 + BC^2 = AB^2$ and $DC^2 + CE^2 = DE^2$]

14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Ans. We have, DB = 3CD

Now, BC = DB + CD

$\Rightarrow$ BC = 3CD + CD

$\Rightarrow$ BC = 4CD

$\therefore$ CD = $\frac{1}{4}$ BC and DB = 3CD = $\frac{3}{4}$ BC ……….(1)

Since, Δ ABD is a right triangle, right angled at D. Therefore by Pythagoras theorem, we have,

$AB^2 = AD^2 + DB^2$ ……….(2)

Similarly, from ΔACD, we have, $AC^2 = AD^2 + CD^2$ ……….(3)

From eq. (2) and (3) $AB^2 - AC^2 = DB^2 - CD^2$

$\Rightarrow AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$ [Using eq.(1)]

$\Rightarrow AB^2 - AC^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2$

$\Rightarrow AB^2 - AC^2 = \frac{1}{2}BC^2$

$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

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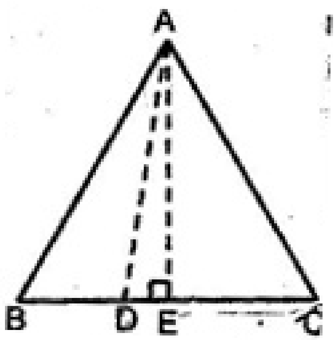
15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

Ans. Let ABC be an equilateral triangle and let D be a point on BC such that BD = $\frac{1}{3}$ BC



Draw AE $\perp$ BC, Join AD.

In $\triangle$s AEB and AEC, we have,

AB = AC[$\because$ $\triangle$ABC is equilateral]

$\angle$AEB = $\angle$AEC [$\because$ each 90°]

And AE = AE

$\therefore$ By SAS-criterion of similarity, we have

$\Delta AEB \sim \Delta AEC$

$\Rightarrow$ BE = EC

Thus, we have, BD = $\frac{1}{3}$ BC, DC = $\frac{2}{3}$ BC and BE = EC = $\frac{1}{3}$ BC ....(1)

Since, $\angle C = 60^\circ$

$\therefore$ ΔADC is an acute angle triangle.

$\therefore$ $AD^2 = AC^2 + DC^2 - 2DC \times EC$

= $AC^2 + \left(\frac{2}{3}BC\right)^2 - 2 \times \frac{2}{3}BC \times \frac{1}{2}BC$ [using eq.(1)]

$\Rightarrow$ $AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$

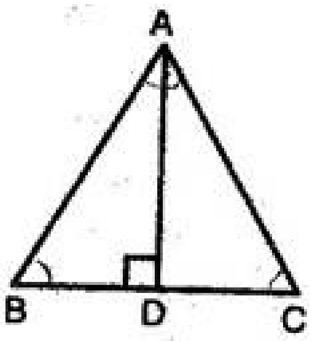
= $AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2$ [$\because$ AB = BC = AC]

$\Rightarrow$ $AD^2 = \frac{(9+4-6)AB^2}{9} = \frac{7}{9}AB^2$

$\Rightarrow$ $9AD^2 = 7AB^2$

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans. Let ABC be an equilateral triangle and let AD $\perp$ BC. In Δs ADB and ADC, we have,



AB = AC [Given]

$\angle$B = $\angle$C = 60° [Given]

And $\angle$ADB = $\angle$ADC[Each = 90°]

$\therefore$ $\triangle$ADB $\cong$ $\triangle$ADC[By RHS criterion of congruence]

$\Rightarrow$ BD = DC

$\Rightarrow$ BD = DC = $\frac{1}{2}$ BC

Since $\triangle$ADB is a right triangle, right angled at D, by Pythagoras theorem, we have,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$$

$$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2$$

$$\Rightarrow AB^2 = AD^2 + \frac{AB^2}{4} \quad [\because BC = AB]$$

$$\Rightarrow \frac{3}{4}AB^2 = AD^2$$

$$\Rightarrow 3AB^2 = 4AD^2$$

17. Tick the correct answer and justify: In $\triangle$ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. the angles A and B are respectively:

(A) 90° **and** 30°

(B) 90° **and** 60°

(C) 30° **and** 90°

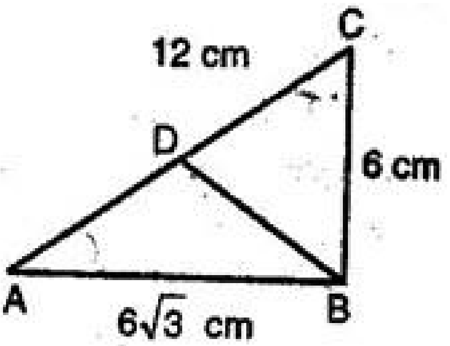
(D) 60° **and** 90°

Ans. (C) In $\triangle$ABC, we have, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

Now, $AB^2 + BC^2 = \left(6\sqrt{3}\right)^2 + (6)^2 = 36\times3+36 = 108+36 = 144 = (AC)^2$

Thus, $\triangle$ABC is a right triangle, right angled at B.

$\therefore$ $\angle$B = 90°



Let D be the midpoint of AC. We know that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

AD = BD = CD

$\Rightarrow$ CD = BD = 6 cm [$\because$ CD = $\frac{1}{2}$ AC]

Also, BC = 6 cm

$\therefore$ In $\triangle$BDC, we have, BD = CD = BC

$\Rightarrow$ ΔBDC is equilateral

$\Rightarrow$ $\angle$ACB = 60°

$\therefore$ $\angle$A = $180^\circ - (\angle B + \angle C) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

Thus, $\angle$A = 30° and $\angle$B = 90°