CBSE Class–10 Mathematics

NCERT solution

Chapter - 10

Circles - Exercise 10.2

**In Q 1 to 3, choose the correct option and give justification.**

**1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:**
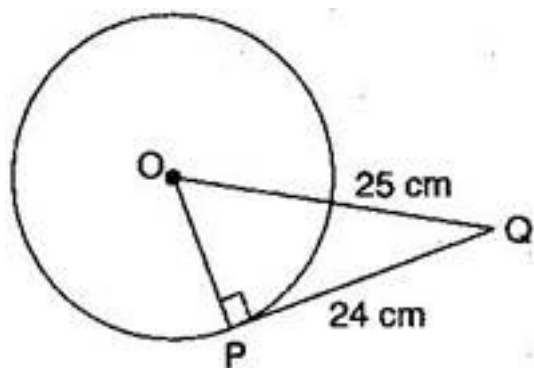
**(A) 7 cm  (B) 12 cm  (C) 15 cm  (D) 24.5 cm**

**Ans. (A)**

$\because$ $\angle$ OPQ = $90°$

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\therefore$ In right triangle OPQ,

$$OQ^2 = OP^2 + PQ^2$$
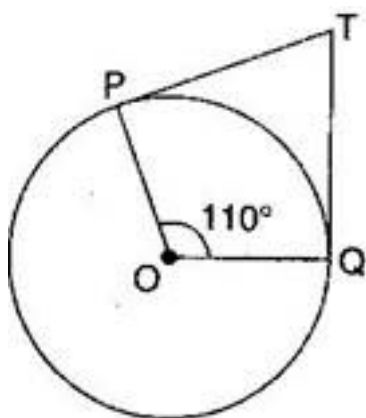
[By Pythagoras theorem]

$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$\Rightarrow OP^2$ = 625 – 576 = 49

$\Rightarrow$ OP = 7 cm

**2. In figure, if TP and TQ are the two tangents to a circle with centre O so that** $\angle$ **POQ =** $110°$**, then** $\angle$ **PTQ is equal to:**



**(A)** $60°$ **(B)** $70°$ **(C)** $80°$ **(D)** $90°$

**Ans. (B)**

$\angle$ POQ = $110°$, $\angle$ OPT = $90°$ and $\angle$ OQT = $90°$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ,

$\angle$ POQ + $\angle$ OPT + $\angle$ OQT + $\angle$ PTQ = $360°$

[Angle sum property of quadrilateral]

$\Rightarrow 110° + 90° + 90°$ + $\angle$ PTQ = $360°$

$\Rightarrow 290°$ + $\angle$ PTQ = $360°$

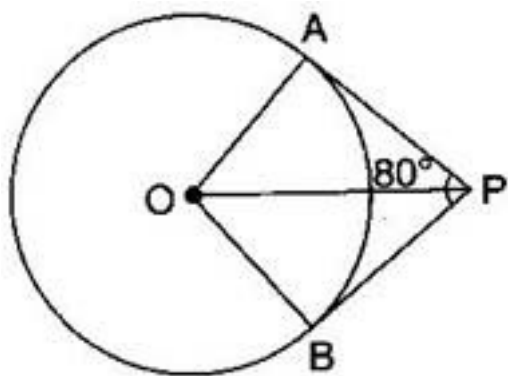$\Rightarrow \angle PTQ = 360^{0} - 290^{0}$

$\Rightarrow \angle$ PTQ = $70°$

**3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of $80°$, then $\angle$ POA is equal to:**

**(A)** $50°$ **(B)** $60°$ **(C)** $70°$ **(D)** $80°$

**Ans. (A)**

$\because$ $\angle$ OAP = $90°$

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\angle$ OPA = $\frac{1}{2}\angle$ BPA = $\frac{1}{2} \times 80° = 40°$

[Centre lies on the bisector of the

angle between the two tangents]

In $\triangle$ OPA,

$\angle$ OAP + $\angle$ OPA + $\angle$ POA = $180°$

[Angle sum property of a triangle]

$\Rightarrow 90° + 40°$ + $\angle$ POA = $180°$
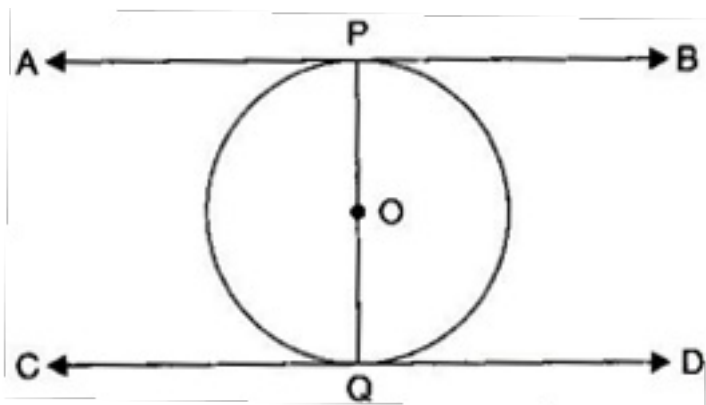
$\Rightarrow 130°$ + $\angle$ POA = $180°$

$\Rightarrow \angle$ POA = $50°$

**4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

**Ans. Given**: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.



**To Prove**: AB ∥ CD

**Proof**: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

∴ ∠ OPA = $90°$..........(i)

[The tangent at any point of a circle is ⊥ to the radius through the point of contact]

∵ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

∴ ∠ OQD = $90°$..........(ii)

[The tangent at any point of a circle is ⊥ to the radius through the point of contact]

From eq. (i) and (ii), ∠ OPA = ∠ OQD

But these form a pair of equal alternate angles also,

∴ AB || CD

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**5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.**

**Ans.** Let AB be the tangent drawn at the point P on the circle with O.

If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, AB ⊥ OP ⇒ ∠OPB = 90⁰

Also, ∠QPB = 90⁰ [By construction]
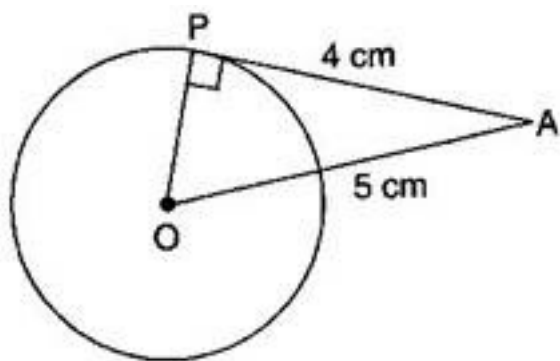
Therefore, ∠QPB = ∠OPB, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

**6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.**

**Ans.** We know that the tangent at any point of a circle is ⊥ to the radius through the point of contact.

∴ ∠ OPA = 90°

∴ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (OP)^2 + (4)^2$
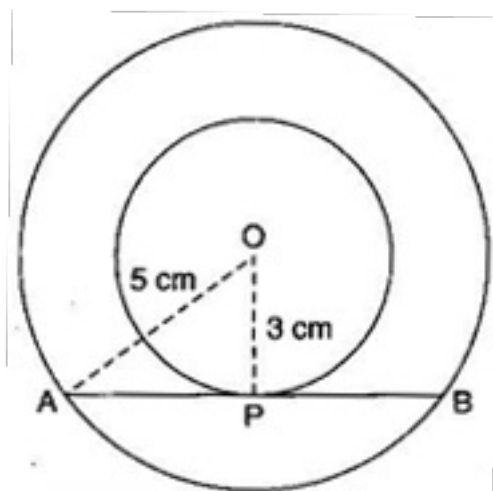
$\Rightarrow 25 = (OP)^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3$ cm

**7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

**Ans.** Let O be the common centre of the two concentric circles.

Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = $90°$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ OA$^2$ = OP$^2$ + AP$^2$

[By Pythagoras theorem]

$\Rightarrow \left(5\right)^{2} = \left(3\right)^{2} + AP^{2}$

$\Rightarrow 25 = 9 + AP^{2}$

$\Rightarrow AP^{2}$ = 16

$\Rightarrow$ AP = 4 cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

$\Rightarrow$ AB = AP + BP
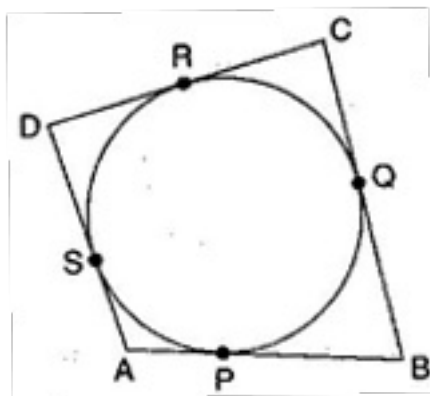
= AP + AP = 2AP

= $2 \times 4$ = 8 cm

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**8. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:**

**AB + CD = AD + BC**

**Ans.** We know that the tangents from an external point to a circle are equal.

∴ AP = AS ..........(i)

BP = BQ ..........(ii)

CR = CQ ..........(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

⟹ AB + CD = (AS + DS) + (BQ + CQ)

⟹ AB + CD = AD + BC

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**9. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that** $\angle$ **AOB = 90°.**
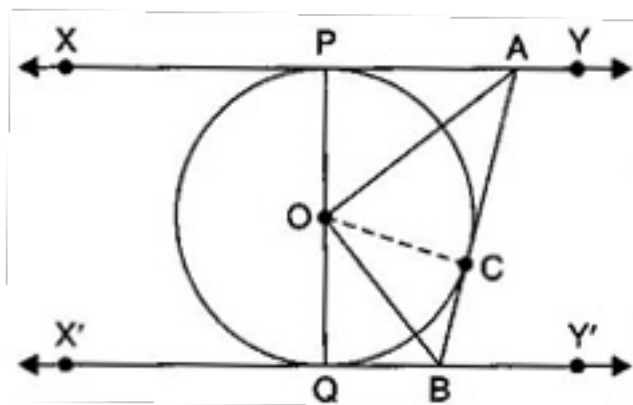
**Ans. Given**: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

**To Prove**: $\angle$ AOB = $90°$

**Construction**: Join OC

**Proof**: $\angle$ OPA = $90°$..........(i)



$\angle$ OCA = $90°$..........(ii)

[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle$ OPA = $\angle$ OCA = $90^{0}$

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore$ $\triangle$ OPA $\cong$ $\triangle$ OCA

[RHS congruence criterion]

$\therefore$ $\angle$ OAP = $\angle$ OAC [By C.P.C.T.]

$\Rightarrow$ $\angle$ OAC = $\frac{1}{2}$ $\angle$ PAB ..........(iii)

Similarly, $\angle$ OBQ = $\angle$ OBC

$\Rightarrow$ $\angle$ OBC = $\frac{1}{2}$ $\angle$ QBA ..........(iv)

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$\therefore$ $\angle$ PAB + $\angle$ QBA = $180°$

[Sum of the consecutive interior angles on the same side of the transversal is $180°$]

$\Rightarrow$ $\frac{1}{2}$ $\angle$ PAB + $\frac{1}{2}$ $\angle$ QBA

= $\frac{1}{2} \times 180°$ ..........(v)

$\Rightarrow$ $\angle$ OAC + $\angle$ OBC = $90°$

[From eq. (iii) & (iv)]

In $\triangle$ AOB,

$\angle$ OAC + $\angle$ OBC + $\angle$ AOB = $180°$

[Angel sum property of a triangle]

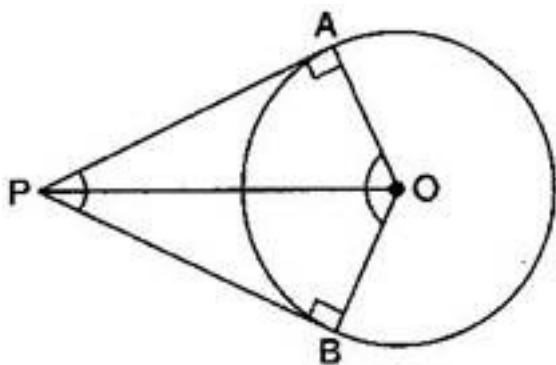$\Rightarrow$ $90°$ + $\angle$ AOB = $180°$ [From eq. (v)]

$\Rightarrow$ $\angle$ AOB = $90°$

Hence proved.

**10. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.**

**Ans.** $\angle$ OAP = $90°$ ..........(i)

$\angle$ OBP = $90°$ ..........(ii)



[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore$ $\angle$ APB + $\angle$ AOB + $\angle$ OAP + $\angle$ OBP = $360°$

[Angle sum property of a quadrilateral]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB + $90°$ + $90°$ = $360°$

[From eq. (i) & (ii)]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB = $180°$

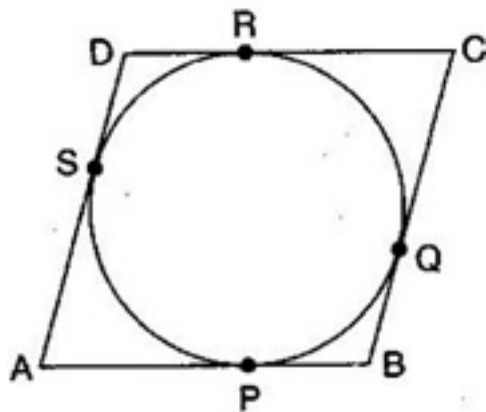$\therefore$ $\angle$ APB and $\angle$ AOB are supplementary.

**11. Prove that the parallelogram circumscribing a circle is a rhombus.**

**Ans. Given**: ABCD is a parallelogram circumscribing a circle.

**To Prove**: ABCD is a rhombus.

**Proof**: Since, the tangents from an external point to a circle are equal.

∴ AP = AS ..........(i)



BP = BQ ..........(ii)

CR = CQ ..........(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Longrightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Longrightarrow$ AB + CD = AD + BC

$\Longrightarrow$ AB + AB = AD + AD

[Opposite sides of ‖ gm are equal]
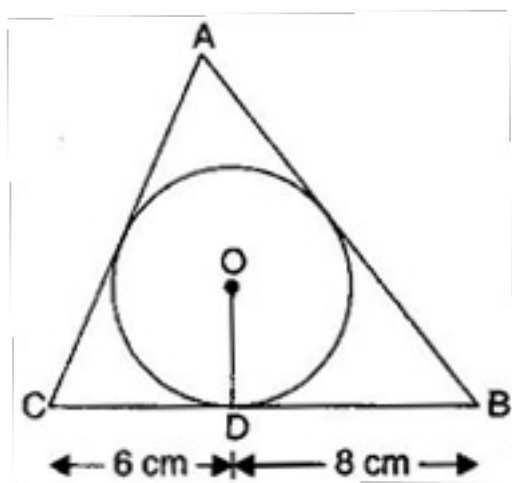
$\Longrightarrow$ 2AB = 2AD

$\Longrightarrow$ AB = AD

But AB = CD and AD = BC

[Opposite sides of ‖ gm]
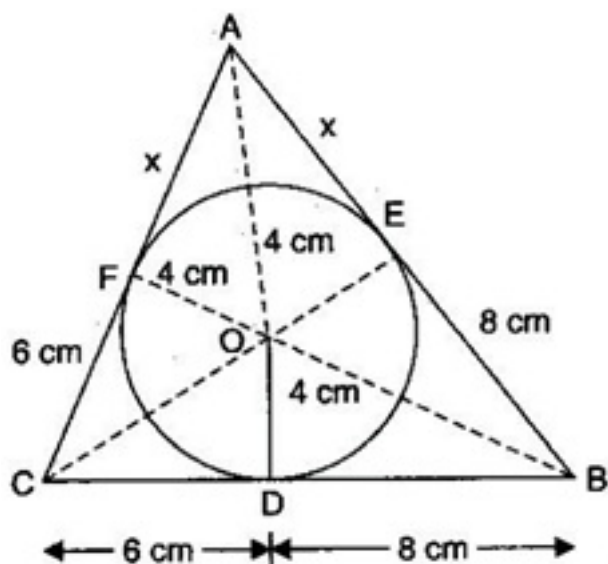
∴ AB = BC = CD = AD

∴ Parallelogram ABCD is a rhombus.

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**12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.**



**Ans.** Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

∴ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = $x$

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

∴ Semi-perimeter of $\triangle$ABC = $\dfrac{(x+6)+(x+8)+(6+8)}{2}$ = $(x+14)$ cm

∴ Area of $\triangle$ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(6)(8)}$ cm$^2$

Now, Area of $\triangle$ABC = Area of $\triangle$OBC + Area of $\triangle$OCA + Area of $\triangle$OAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $\dfrac{(6+8)4}{2} + \dfrac{(x+6)4}{2} + \dfrac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $28+2x+12+2x+16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8) = 16(x+14)^2$$

$$\Rightarrow 3x = x+14$$

$$\Rightarrow 2x = 14$$

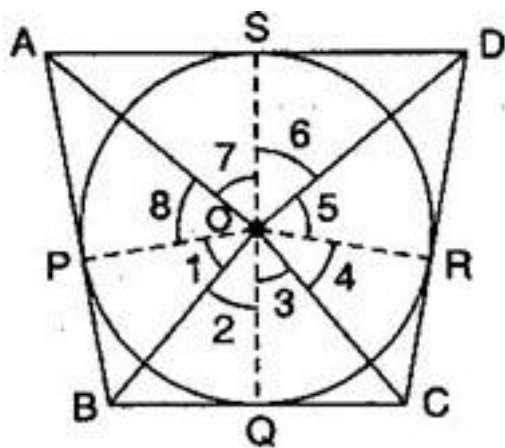$$\Rightarrow x = 7$$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

And AC = $x+6$ = 7 + 6 = 13 cm

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**13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

**Ans.** Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = $180°$ (ii) $\angle$ BOC + $\angle$ AOD = $180°$

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ...........(i)

CQ = CR

DR = DS

In $\triangle$ OBP and $\triangle$ OBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ $\triangle$ OPB $\cong$ $\triangle$ OBQ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$\Rightarrow 2\left(\angle 1 + \angle 4 + \angle 5 + \angle 8\right) = 360°$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

$\Rightarrow \left(\angle 1 + \angle 5\right) + \left(\angle 4 + \angle 8\right) = 180°$

$\Rightarrow$ $\angle$ AOB + $\angle$ COD = $180°$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180°$