CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.1

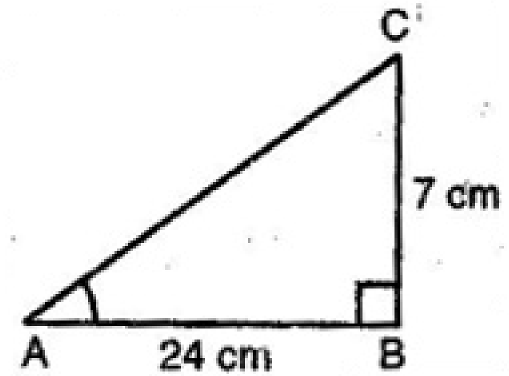
1. In $\triangle$ ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$

Ans. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,



Let AC = 24k and BC = 7k

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

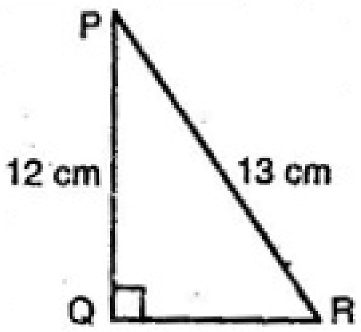
= $(24)^2 + (7)^2$ = 576 + 49 = 625

$\Rightarrow$ AC = 25 cm

(i) $\sin \mathrm{A} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}, \cos \mathrm{A} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}$

(ii) $\sin \mathrm{C} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}, \cos \mathrm{C} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}$

2. In adjoining figure, find $\tan P - \cot R$ **:**



Ans. In triangle PQR, Using Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (13)^2 = (12)^2 + QR^2$$

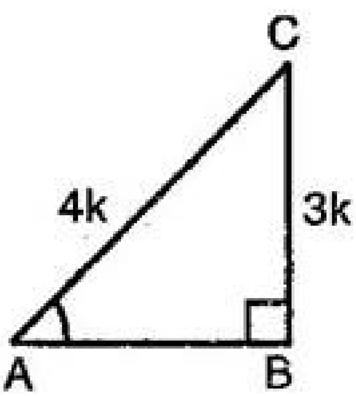
$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow$ QR = 5 cm

$$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{13} - \frac{5}{13} = 0$$

3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$



Let BC = $3k$ and AC = $4k$

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2-(BC)^2}$ = $\sqrt{(4k)^2-(3k)^2}$

= $\sqrt{16k^2-9k^2}$ = $k\sqrt{7}$

$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

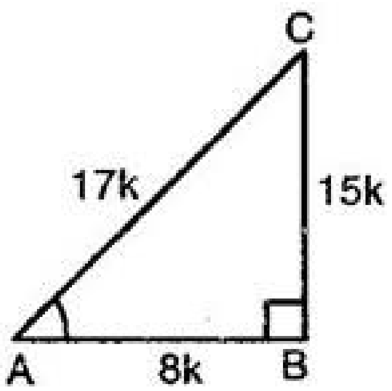
$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

4. Given $15\cot A = 8$, find $\sin A$ and $\sec A$

Ans. Given: A triangle ABC in which $\angle$B = 90°

$15\cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$



Let AB = $8k$ and BC = $15k$

Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2+(BC)^2}$

= $\sqrt{(8k)^2+(15k)^2}$

= $\sqrt{64k^2+225k^2}$

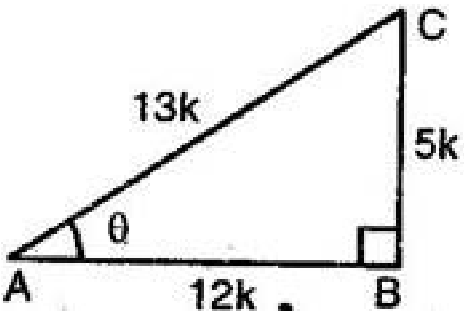
= $\sqrt{289k^2}$ = $17k$

$\therefore\ \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. Given $\sec\theta = \frac{13}{12}$, **calculate all other trigonometric ratios.**

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2 - (AB)^2}$

= $\sqrt{(13k)^2 - (12k)^2}$

= $\sqrt{169k^2 - 144k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore\ \sin\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

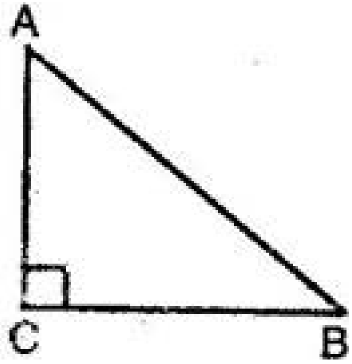
$\tan\theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\cos ec\theta = \frac{\text{H}}{\text{P}} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$

6. If $\angle$ And $\angle$ B are acute angles such that $\cos A = \cos B$, then show that $\angle$ A = $\angle$ B.

Ans. In right triangle ABC,



$\cos A = \frac{\text{AC}}{\text{AB}}$ and $\cos B = \frac{\text{BC}}{\text{AB}}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{\text{AC}}{\text{AB}} = \frac{\text{BC}}{\text{AB}}$

$\Rightarrow$ AC = BC

$\Rightarrow$ $\angle$ A = $\angle$ B

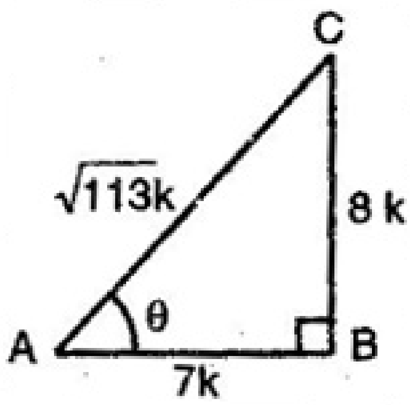
[Angles opposite to equal sides are equal]

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2+(AB)^2}$

= $\sqrt{(8k)^2+(7k)^2}$

= $\sqrt{64k^2+49k^2}$

= $\sqrt{113k^2}$ = $\sqrt{113}k$

$$\therefore \sin\theta=\frac{BC}{AC}=\frac{8k}{\sqrt{113}k}=\frac{8}{\sqrt{113}}$$

$$\cos\theta=\frac{AB}{AC}=\frac{7k}{\sqrt{113}k}=\frac{7}{\sqrt{113}}$$

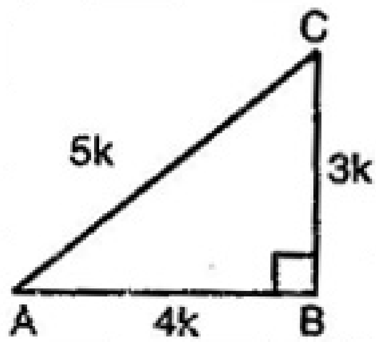
(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ = $\frac{1-\sin^2\theta}{1-\cos^2\theta}$

= $\frac{1-\frac{64}{113}}{1-\frac{49}{113}}$ = $\frac{113-64}{113-49}$ = $\frac{49}{64}$

(ii) $\cot^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta}$ = $\dfrac{49/113}{64/113}$ = $\dfrac{49}{64}$

8. If $3\cot A = 4$, **check whether** $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Ans. Consider a triangle ABC in which $\angle$ B = 90°.



And $3\cot A = 4$

$\Rightarrow \cot A = \dfrac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(\mathrm{BC})^2 + (\mathrm{AB})^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin A = \dfrac{\mathrm{BC}}{\mathrm{AC}} = \dfrac{3k}{5k} = \dfrac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

= $\frac{16-9}{16+9}$ = $\frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A$ = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25}$ = $\frac{7}{25}$

$\because$ L.H.S. = R.H.S.

$\therefore$ $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\cos^2 A - \sin^2 A$

---

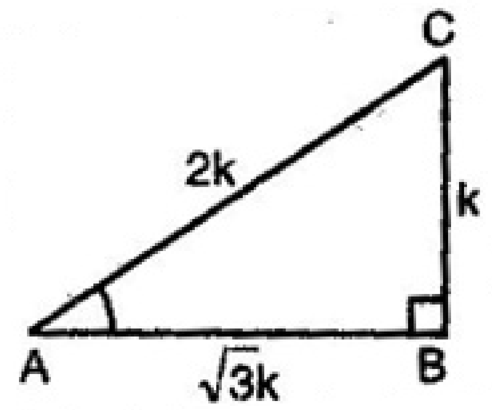
9. In $\triangle$ABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Ans. Consider a triangle ABC in which $\angle$B = 90°.

Let BC = k and AB = $\sqrt{3}k$



Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$ = $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

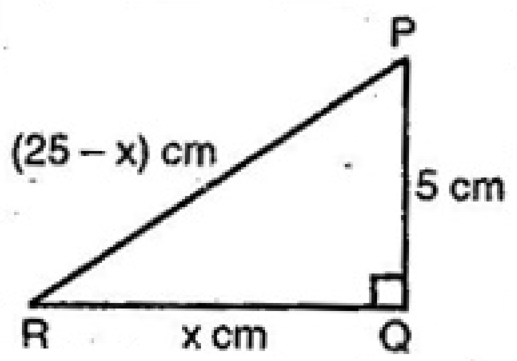
= $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ = 1

(ii) $\cos A\cos C-\sin A\sin C$ = $\frac{\sqrt{3}}{2}\times\frac{1}{2}-\frac{1}{2}\times\frac{\sqrt{3}}{2}$

= $\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=0$

10. In $\triangle$PQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P$, $\cos P$ **and** $\tan P$.

Ans. In $\triangle$PQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm, then PR = $\left(25-x\right)$ cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow \left(25-x\right)^2=\left(x\right)^2+\left(5\right)^2$

$\Rightarrow 625-50x+x^2=x^2+25$

$\Rightarrow -50x=-600$

$\Rightarrow x=12$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1.