CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 13 Limits and Derivative

Exercise 13.1

Evaluate the following limits in Exercises 1 to 22.

1. $\lim\limits_{x \to 3} x+3$

Ans. $\lim\limits_{x \to 3} x+3 =$ 3 + 3 = 6

2. $\lim\limits_{x \to \pi}\left(x - \frac{22}{7}\right)$

Ans. $\lim\limits_{x \to \pi}\left(x - \frac{22}{7}\right) = \left(\pi - \frac{22}{7}\right)$

3. $\lim\limits_{r \to 1} \pi r^2$

Ans. $\lim\limits_{r \to 1} \pi r^2 = \pi \times (1)^2 = \pi$

4. $\lim\limits_{x \to 4} \frac{4x+3}{x-2}$

Ans. $\lim\limits_{x \to 4} \frac{4x+3}{x-2} = \frac{4 \times 4+3}{4-2} = \frac{19}{2}$

5. $\lim\limits_{x \to -1} \frac{x^{10}+x^5+1}{x-1}$

Ans. $\lim\limits_{x \to -1} \frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = \frac{-1}{2}$

6. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Ans. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$ is of the form $\frac{0}{0}$

Put $x+1=y$, now as $x\to 0, y\to 1$

$\therefore \lim_{x\to 0}\left(\frac{(x+1)^5-1}{x}\right) = \lim_{y\to 1}\left(\frac{y^5-1}{y-1}\right) = \lim_{y\to 1}\left(\frac{y^5-1^5}{y-1}\right)$

$= 5\cdot 1^{5-1} = 5\cdot 1 = 5$ since $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}$

$\therefore \lim_{x\to 0}\frac{(x+1)^5-1}{x}=5$

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7. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$

Ans. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}\left[\frac{0}{0}\text{ form}\right]$

$= \lim_{x\to 2}\frac{(x-2)(3x+5)}{(x+2)(x-2)}$

$= \lim_{x\to 2}\frac{(3x+5)}{(x+2)} = \frac{6+5}{2+2} = \frac{11}{4}$

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8. $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

Ans. $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$ is of the form $\frac{0}{0}$

$$\therefore \lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}=\lim_{x\to 3}\frac{(x^2+9)(x+3)(x-3)}{(x-3)(2x+1)}$$

$$=\lim_{x\to 3}\frac{(x^2+9)(x+3)}{(2x+1)}$$

$$=\frac{(9+9)(3+3)}{(2\times 3+1)}=\frac{18\times 6}{7}=\frac{108}{7}$$

9. $\lim_{x\to 0}\frac{ax+b}{cx+1}$

Ans. $\lim_{x\to 0}\frac{ax+b}{cx+1}=\frac{a\times 0+b}{c\times 0+1}=b$

10. $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Ans. $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$ is of the form $\frac{0}{0}$

$$\therefore \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=\lim_{z\to 1}\frac{\left(z^{\frac{1}{6}}\right)^2-(1)^2}{z^{\frac{1}{6}}-1}$$

$$=\lim_{z\to 1}\frac{\left(z^{\frac{1}{6}}+1\right)\left(z^{\frac{1}{6}}-1\right)}{z^{\frac{1}{6}}-1}$$

= $\lim_{z \to 1}\left(z^{\frac{1}{6}}+1\right)$

= $(1)^{\frac{1}{6}}+1$ = 1 + 1 = 2

11. $\lim_{x \to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$, $a+b+c \neq 0$

Ans. $\lim_{x \to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$

= $\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}$

= $\frac{a+b+c}{c+b+a}$ = 1

12. $\lim_{x \to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Ans. $\lim_{x \to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ = $\lim_{x \to -2}\frac{\frac{x+2}{2x}}{x+2}$

= $\lim_{x \to -2}\frac{x+2}{2x}\times\frac{1}{x+2}$

= $\lim_{x \to -2}\frac{1}{2x}$ = $\frac{1}{2\times(-2)}=\frac{-1}{4}$

13. $\lim_{x \to 0}\frac{\sin ax}{bx}$

Ans. $\lim_{x\to 0}\frac{\sin ax}{bx}$

$= \lim_{x\to 0}\left(\frac{\sin ax}{ax}\times\frac{ax}{bx}\right)$

$= \frac{a}{b}\lim_{x\to 0}\frac{\sin ax}{ax}$

$= \frac{a}{b}\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right)$ $[x \to 0 \Rightarrow ax \to 0]$ and $\left[\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$

$= \frac{a}{b}\times 1 = \frac{a}{b}$

14. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}, a, b \neq 0$

Ans. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$

$= \lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)ax}{\left(\frac{\sin bx}{bx}\right)bx} = \frac{a}{b}\lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)}{\left(\frac{\sin bx}{bx}\right)}$

$= \frac{a}{b}\cdot\frac{\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx\to 0}\left(\frac{\sin bx}{bx}\right)}$ since $\left[\begin{matrix} x \to 0 \Rightarrow ax \to 0 \\ x \to 0 \Rightarrow bx \to 0\end{matrix}\right]$

$= \frac{a}{b}\cdot\frac{1}{1} = \frac{a}{b}$ $\left[\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$

15. $\lim_{x\to\pi}\frac{\sin(\pi - x)}{\pi(\pi - x)}$

Ans. $\lim_{x\to\pi}\frac{\sin(\pi - x)}{\pi(\pi - x)}\left[\frac{0}{0}\text{ form}\right]$

Put $x = \pi + y$ now as $x \to \pi, y \to 0$

$\therefore \lim\limits_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \lim\limits_{y \to 0} \frac{\sin(\pi - \pi - y)}{\pi(\pi - \pi - y)}$

$= \lim\limits_{y \to 0} \frac{\sin(-y)}{-\pi y} = \lim\limits_{y \to 0} \frac{-\sin y}{-\pi y} \quad [\sin(-\theta) = -\sin\theta]$

$= \frac{1}{\pi} \lim\limits_{y \to 0} \frac{\sin y}{y} \quad \left[\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$

$= \frac{1}{\pi} \times 1 = \frac{1}{\pi}$

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16. $\lim\limits_{x \to 0} \frac{\cos x}{\pi - x}$

Ans. $\lim\limits_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

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17. $\lim\limits_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

Ans. $\lim\limits_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$ is of the form $\frac{0}{0}$

$\therefore \lim\limits_{x \to 0} \left(\frac{\cos 2x - 1}{\cos x - 1}\right) = \lim\limits_{x \to 0} \frac{(2\cos^2 x - 1) - 1}{\cos x - 1} \quad \left[\cos 2\theta = 2\cos^2\theta - 1\right]$

$= \lim\limits_{x \to 0} \frac{2(\cos^2 x - 1)}{\cos x - 1}$

$= 2 \lim\limits_{x \to 0} \frac{(\cos x - 1)(\cos x + 1)}{(\cos x - 1)}$

$= 2 \lim\limits_{x \to 0} (\cos x + 1) = 2(1 + 1) = 2 \times 2 = 4$

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18. $\lim\limits_{x \to 0} \frac{ax + x\cos x}{b \sin x}$

Ans. $\lim\limits_{x\to 0}\dfrac{ax+x\cos x}{b\sin x}\left[\dfrac{0}{0}\text{ form}\right]$

= $\frac{1}{b}\lim\limits_{x\to 0}\frac{x(a+\cos x)}{\sin x}$

= $\frac{1}{b}\lim\limits_{x\to 0}\frac{x}{\sin x}\cdot\lim\limits_{x\to 0}(a+\cos x)$

= $\frac{1}{b}\cdot\frac{\lim\limits_{x\to 0}(a+\cos x)}{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)}$ $\left[\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$

= $\frac{1}{b}\times\frac{a+1}{1}=\frac{a+1}{b}$

19. $\lim\limits_{x\to 0}x\sec x$

Ans. $\lim\limits_{x\to 0}x\sec x$ = $\lim\limits_{x\to 0}x\dfrac{1}{\cos x}$

= $\lim\limits_{x\to 0}\dfrac{x}{\cos x}$ = $\dfrac{0}{1}$ = 0

20. $\lim\limits_{x\to 0}\dfrac{\sin ax+bx}{ax+\sin bx}, a, b, a+b\neq 0$

Ans. $\lim\limits_{x\to 0}\dfrac{\sin ax+bx}{ax+\sin bx}\left[\dfrac{0}{0}\text{ form}\right]$

Dividing numerator and denominator by ax,

= $\lim\limits_{x\to 0}\dfrac{\dfrac{\sin ax}{ax}+\dfrac{bx}{ax}}{\dfrac{ax}{ax}+\dfrac{\sin bx}{ax}}$

$= \lim\limits_{x\to 0} \dfrac{\left(\frac{\sin ax}{ax}\right)+\frac{bx}{ax}}{1+\left(\frac{\sin bx}{bx}\right)\frac{bx}{ax}}$

$= \dfrac{\lim\limits_{ax\to 0}\left(\frac{\sin ax}{ax}\right)+\frac{b}{a}\lim\limits_{ax\to 0} 1}{1+\lim\limits_{bx\to 0}\left(\frac{\sin bx}{bx}\right)\cdot\frac{b}{a}\lim\limits_{bx\to 0} 1}$ $\left[\begin{matrix} x\to 0 \Rightarrow ax \to 0 \\ x \to 0 \Rightarrow bx \to 0\end{matrix}\right]$

$= \dfrac{1+\frac{b}{a}}{1+\frac{b}{a}} = 1$ $\left[\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$

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21. $\lim\limits_{x\to 0}\left(\operatorname{cosec} x - \cot x\right)$

Ans. Given: $\lim\limits_{x\to 0}\left(\operatorname{cosec} x - \cot x\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right) = \lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin x}$

$= \lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \lim\limits_{x\to 0}\tan\dfrac{x}{2} = 0$

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22. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\tan 2x}{x-\frac{\pi}{2}}$

Ans. Given: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\tan 2x}{x-\frac{\pi}{2}}\left[\dfrac{0}{0}\text{ form}\right]$

Put $x = \dfrac{\pi}{2} + y$ now as $x \to \dfrac{\pi}{2}, y \to 0$

= $\lim\limits_{y \to 0} \dfrac{\tan 2\left(\dfrac{\pi}{2}+y\right)}{\dfrac{\pi}{2}+y-\dfrac{\pi}{2}}$

= $\lim\limits_{y \to 0} \dfrac{\tan(\pi+2y)}{y}$ = $\lim\limits_{y \to 0} \dfrac{\tan 2y}{y}$ $[\tan(\pi+\theta)=\tan\theta]$

= $\lim\limits_{y \to 0} \dfrac{\tan 2y}{2y} \times 2$

=$2\lim\limits_{y \to 0} \frac{\tan 2y}{2y}$ $\left[\lim\limits_{\theta \to 0} \frac{\tan\theta}{\theta}=1\right]$

= $2 \times 1 = 2$

23. Find $\lim\limits_{x \to 0} f(x)$ **and** $\lim\limits_{x \to 1} f(x)$ **where** $f(x)=\begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x>0 \end{cases}$

Ans. Given: $f(x)=\begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x>0 \end{cases}$

For x>0 Right hand limit = $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} 3(x+1) = 3(0+1) = 3$

For x<0 Left hand limit = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} (2x+3) = 2(0)+3 = 3$

As $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x) = 3$, we have $\lim\limits_{x \to 0} f(x) = 3$

For x>1 Right hand limit = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1} 3(x+1) = 3(1+1) = 6$

For x<1 Left hand limit = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1} 3(x+1) = 3(1+1) = 6$

As $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^-} f(x) = 6$, we have $\lim\limits_{x \to 1} f(x) = 6$

24. Find $\lim_{x\to1} f(x)$ **where** $f(x)=\begin{cases}x^2-1 & x\le1\\ -x^2-1 & x>1\end{cases}$

Ans. Given: $f(x)=\begin{cases}x^2-1 & x\le1\\ -x^2-1 & x>1\end{cases}$

For x>1 Right hand limit = $\lim_{x\to1^+} f(x)=\lim_{x\to1}(-x^2-1)=-1-1=-2$

For x<1 Left hand limit = $\lim_{x\to1^-} f(x)=\lim_{x\to1}(x^2-1)=1-1=0$

As $\lim_{x\to1^+} f(x)\ne\lim_{x\to1^-} f(x)$,we have $\lim_{x\to1} f(x)$ does not exist

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25. Evaluate $\lim_{x\to0} f(x)$ **where** $f(x)=\begin{cases}\frac{|x|}{x} & x\ne0\\ 0 & x=0\end{cases}$

Ans. Given: $f(x)=\begin{cases}\frac{|x|}{x} & x\ne0\\ 0 & x=0\end{cases}$

We have $|x|=x$ when x is positive

$\therefore$ For x>0 Right hand limit = $\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{|x|}{x}=\lim_{x\to0}\frac{x}{x}=1$

We have $|x|=-x$ when x is negative

$\therefore$ For x<0 Left hand limit = $\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\frac{|x|}{x}=\lim_{x\to0}\frac{-x}{x}=-1$

As $\lim_{x\to0^+} f(x)\ne\lim_{x\to0^-} f(x)$,we have $\lim_{x\to0} f(x)$ does not exist

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26. Find $\lim_{x\to 0} f(x)$ **where** $f(x)=\begin{cases}\frac{x}{|x|} & x\neq 0\\ 0 & x=0\end{cases}$

Ans. Given: $f(x)=\begin{cases}\frac{x}{|x|} & x\neq 0\\ 0 & x=0\end{cases}$

We have $|x|=x$ when x is positive

$\therefore$ For x>0 Right hand limit = $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{x}{|x|}=\lim_{x\to 0}\frac{x}{x}=1$

We have $|x|=-x$ when x is negative

$\therefore$ For x<0 Left hand limit = $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{x}{|x|}=\lim_{x\to 0}\frac{x}{-x}=-1$

As $\lim_{x\to 0^+} f(x)\neq \lim_{x\to 0^-} f(x)$,we have $\lim_{x\to 0} f(x)$ does not exist

27. Find $\lim_{x\to 5} f(x)$ **where** $f(x)=|x|-5$.

Ans. Given: $f(x)=|x|-5$

L.H.L. $\lim_{x\to 5^-} f(x)=\lim_{x\to 5^-}|x|-5$

Putting $x=5-h$ as $x\to 5, h\to 0$

$\therefore \lim_{h\to 0}|5-h|-5 = \lim_{h\to 0} 5-h-5$

$= \lim_{h\to 0}(-h) = 0$

R.H.L. $\lim_{x\to 5^+} f(x)=\lim_{x\to 5^+}|x|-5$

Putting $x=5+h$ as $x\to 5, h\to 0$

$\therefore \lim_{h\to 0} |5+h|-5 = \lim_{h\to 0} 5+h-5$

$= \lim_{h\to 0} h = 0$

Here, L.H.L. = R.H.L.

Therefore, this limit exists at $x=5$ and $\lim_{x\to 5} f(x)=0$

28. Suppose $f(x)=\begin{cases} a+bx & x<1 \\ 4 & x=1 \\ b-ax & x>1 \end{cases}$ **and if** $\lim_{x\to 1} f(x)=f(1)$ **what are possible values of** a **and** b **?**

Ans. Given: $f(x)=\begin{cases} a+bx & x<1 \\ 4 & x=1 \\ b-ax & x>1 \end{cases}$ and $\lim_{x\to 1} f(x)=f(1)$

$\Rightarrow \lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=f(1)=4$

$\Rightarrow \lim_{x\to 1^-} f(x)=4$ and $\lim_{x\to 1^+} f(x)=4$

$\Rightarrow \lim_{x\to 1} a+bx=4$ and $\lim_{x\to 1} b-ax=4$

$\Rightarrow a+b=4$ and $b-a=4$

On solving these equation, we get $a=0$ and $b=4$

29. Let $a_1, a_2, \ldots\ldots a_n$ **be fixed real numbers and define a function** $f(x)=(x-a_1)(x-a_2)\ldots.(x-a_n)$. **What is** $\lim_{x\to a_1} f(x)$ **? For some** $a \neq a_1, a_2, \ldots\ldots a_n$, **compute** $\lim_{x\to a} f(x)$.

Ans. Given: $f(x)=(x-a_1)(x-a_2)\ldots.(x-a_n)$

Now $\lim_{x \to a_1} f(x) = \lim_{x \to a_1} (x-a_1)(x-a_2)....(x-a_n)$

= $(a_1-a_1)(a_1-a_2)....(a_1-a_n)$

= $0 \times (a_1-a_2)....(a_1-a_n)$ = 0

$\therefore \lim_{x \to a_1} f(x)$ = 0

Also $\lim_{x \to a} f(x) = \lim_{x \to a} (x-a_1)(x-a_2)....(x-a_n)$

= $(a-a_1)(a-a_2)....(a-a_n)$

$\therefore \lim_{x \to a} f(x)$ = $(a-a_1)(a-a_2)....(a-a_n)$

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30. If $f(x) = \begin{cases} |x|+1 & x<0 \\ 0 & x=0 \\ |x|-1 & x>0 \end{cases}$ **for what values of** a **does** $\lim_{x \to a} f(x)$ **exists?**

Ans. Given: $f(x) = \begin{cases} |x|+1 & x<0 \\ 0 & x=0 \\ |x|-1 & x>0 \end{cases} \Rightarrow f(x) = \begin{cases} -x+1 & x<0 \\ 0 & x=0 \\ x-1 & x>0 \end{cases}$

Consider $\lim_{x \to a} f(x)$

When $a = 0$

L.H.L.= $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} |x|+1$ = $\lim_{x \to 0}(-x+1) = (0+1) = 1$

Also R.H.L.= $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} |x|-1$ = $\lim_{x \to 0}(x-1) = (0-1) = -1$

Here, L.H.L. $\neq$ R.H.L.

Therefore, this limit does not exist at $a = 0$

When a > 0,

L.H.L.= $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a}(x-1) = a-1 \qquad [x \to a^- \Rightarrow 0 < x < a]$

Also R.H.L.= $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a}(x-1) = a-1 \quad [x \to a^+ \Rightarrow 0 < a < x]$

Here, L.H.L. = R.H.L.

Therefore, this limit exist at x=a when a > 0

When a > 0,

L.H.L.= $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a}(-x+1) = -a+1 \quad [x \to a^- \Rightarrow x < a < 0]$

Also R.H.L.= $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a}(-x+1) = -a+1 \quad [x \to a^+ \Rightarrow a < x < 0]$

Here, L.H.L. = R.H.L.

Therefore, this limit exist at x=a when a < 0

$\therefore \lim\limits_{x \to a} f(x)$ exists for all $a \neq 0$.

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31. If the function $f(x)$ satisfies $\lim\limits_{x \to 1} \frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim\limits_{x \to 1} f(x)$.

Ans. $\lim\limits_{x \to 1} \frac{f(x)-2}{x^2-1} = \pi$

$\Rightarrow \frac{\lim\limits_{x \to 1} f(x) - 2}{\lim\limits_{x \to 1}(x^2-1)} = \pi$

$\Rightarrow \lim\limits_{x \to 1} f(x) - 2 = \pi \cdot \lim\limits_{x \to 1}(x^2-1)$

Since $\lim\limits_{x \to 1}(x^2-1) = (1)^2 - 1 = 1 - 1 = 0$,we get

$\lim\limits_{x \to 1} f(x) - 2 = \pi.0 = 0$

$\Rightarrow \lim_{x \to 1} f(x) = 2$

32. If $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1 \\ nx^3 + m, & x > 1 \end{cases}$ **for what integer** m **and** n **does both** $\lim_{x \to 0} f(x)$ **and** $\lim_{x \to 1} f(x)$ **exist?**

Ans. Left hand limit = $\lim_{x \to 0^-} f(x) = \lim_{x \to 0} (mx^2 + n) = m \cdot 0 + n = n$

Right hand limit = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} (nx + m) = n \cdot 0 + m = m$

Thus $\lim_{x \to 0} f(x)$ exists only if m = n

Left hand limit = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (nx + m) = n \cdot 1 + m = n + m$

Right hand limit = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (nx^3 + m) = n \cdot 1 + m = n + m$

As $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = m + n$ we get $\lim_{x \to 1} f(x)$ exist for any integral value of m and n