CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 9 Sequences and Series

Miscellaneous Exercise

1. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A.P. is equal to twice the m^{th} terms.

Ans. Here, $a_{m+n} = a+(m+n-1)d$ .....(i)

and $a_{m-n} = a+(m-n-1)d$ .....(ii)

To prove: $a_{m+n} + a_{m-n} = 2a_m$

Adding eq. (i) and (ii), we get

$$a_{m+n} + a_{m-n} = a+(m+n-1)d + a+(m-n-1)d$$

$$\Rightarrow a_{m+n} + a_{m-n} = a+md+nd-d+a+md-nd-d$$

$$\Rightarrow a_{m+n} + a_{m-n} = 2a+2md+-2d$$

$$\Rightarrow a_{m+n} + a_{m-n} = 2(a+md-d)$$

$$\Rightarrow a_{m+n} + a_{m-n} = 2[a+(m-1)d]$$

$$\Rightarrow a_{m+n} + a_{m-n} = 2a_m$$

2. If the sum of three numbers in A.P., is 24 and their product is 440. Find the numbers.

Ans. Let $(a-d), a, (a+d)$ be three numbers in A.P.

According to question, $(a-d)+a+(a+d)=24$

$$\Rightarrow 3a = 24$$

$\Rightarrow\ a=8$

And $(a-d)(a)(a+d)=440$

$\Rightarrow (a^2-d^2)a=440$

$\Rightarrow\ (64-d^2)8=440$

$\Rightarrow\ 64-d^2=55$

$\Rightarrow\ d^2=64-55$

$\Rightarrow\ d^2=9$

$\Rightarrow\ d=\pm 3$

Taking $d=3$, A.P. is (8 – 3), 8, (8 + 3)

$\Rightarrow$ 5, 8, 11

Taking $d=-3$, A.P. is (8 + 3), 8, (8 – 3)

$\Rightarrow$ 11, 8, 5

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3. Let sum of $n, 2n, 3n$ terms of an A.P. be S_1, S_2 and S_3 respectively, show that $S_3 = 3\ (S_2 - S_1)$.

Ans. Given: $S_1=\frac{n}{2}\left[2a+(n-1)d\right]$ .....(i)

$S_2=\frac{2n}{2}\left[2a+(2n-1)d\right]$.....(ii)

And $S_3=\frac{3n}{2}\left[2a+(3n-1)d\right]$

Now, $S_2-S_1=\frac{2n}{2}\left[2a+(2n-1)d\right]-\frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow S_2-S_1=(n-\frac{n}{2})2a+\left[n(2n-1)-\frac{n}{2}(n-1)\right]d$

$$=na+\frac{1}{2}\left[4n^2-2n-n^2+n\right]d$$

$$= \frac{n}{2}\left[2a + (3n-1)d\right] = \frac{1}{3}\left\{\frac{3n}{2}\left[2a + (3n-1)d\right]\right\} = \frac{1}{3}S_3$$

$\Rightarrow 3(S_2 - S_1) = S_3$

Hence proved.

4. Find the sum of all numbers between 200 and 400 which are divisible by 7.

Ans. Given: A.P. 203, 210, 217, ………, 399

Here $a = 203, d = 210 - 203 = 7$ and $a_n = 399$

Now, $a_n = a + (n-1)d$

$\Rightarrow 399 = 203 + (n-1) \times 7$

$\Rightarrow 399 - 203 = (n-1) \times 7$

$\Rightarrow 196 = (n-1) \times 7$

$\Rightarrow n - 1 = \frac{196}{7} = 28$

$\Rightarrow n = 29$

$\therefore S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow S_{29} = \frac{29}{2}(203 + 399)$

$\Rightarrow S_{29} = \frac{29}{2} \times 602 = 8729$

5. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Ans. Given: A.P. which is divisible by 2

2, 4, 6, ..., 100

Here $a = 2, d = 4 - 2 = 2$ and $a_n = 100$

Now, $a_n = a + (n-1)d$

$\Rightarrow 100 = 2 + (n-1) \times 2$

$\Rightarrow 100 - 2 = (n-1) \times 2$

$\Rightarrow 98 = (n-1) \times 2$

$\Rightarrow n - 1 = \frac{98}{2} = 49$

$\Rightarrow n = 50$

$\therefore S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow S_1 = \frac{50}{2}(2 + 100)$

$\Rightarrow S_1 = \frac{50}{2} \times 102$ = 2550

Again A.P. which is divisible by 5

5, 10, 15, ..., 100

Here $a = 5, d = 10 - 5 = 5$ and $a_n = 100$

Now, $a_n = a + (n-1)d$

$\Rightarrow 100 = 5 + (n-1) \times 5$

$\Rightarrow 100 - 5 = (n-1) \times 5$

$\Rightarrow 95 = (n-1)\times 5$

$\Rightarrow n-1 = \frac{95}{5} = 19$

$\Rightarrow n = 20$

$\therefore S_n = \frac{n}{2}(a+a_n)$

$\Rightarrow S_2 = \frac{20}{2}(5+100)$

$\Rightarrow S_2 = \frac{20}{2}\times 105$ = 1050

Again A.P. which is divisible by both 2 and 5

10, 20, 30, ..., 100

Here $a = 10, d = 20-10 = 10$ and $a_n = 100$

Now, $a_n = a+(n-1)d$

$\Rightarrow 100 = 10+(n-1)\times 10$

$\Rightarrow 100-10 = (n-1)\times 10$

$\Rightarrow 90 = (n-1)\times 10$

$\Rightarrow n-1 = \frac{90}{10} = 9$

$\Rightarrow n = 10$

$\therefore S_3 = \frac{n}{2}(a+a_n)$

$\Rightarrow\ S_3 = \frac{10}{2}(10+100)$

$\Rightarrow\ S_3 = \frac{10}{2}\times 110$ = 550

Now sum of integers divisible by 2 or 5=sum of integers divisible by 2+sum of integers divisible by 5-sum of integers divisible by 2 and 5

$\therefore$ required sum = (2550 + 1050) – 550 = 3050

6. Find the sum of all two digit numbers which when divided by 4, yield 1 as remainder.

Ans. Given: A.P. 13, 17, 21, ………, 97

Here $a = 13, d = 17 - 13 = 4$ and $a_n = 97$

Now, $a_n = a + (n-1)d$

$\Rightarrow 97 = 13 + (n-1)\times 4$

$\Rightarrow 97 - 13 = (n-1)\times 4$

$\Rightarrow 84 = (n-1)\times 4$

$\Rightarrow\ n - 1 = \frac{84}{4} = 21$

$\Rightarrow\ n = 22$

$\therefore\ S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow\ S_{22} = \frac{22}{2}(13+97)$

$\Rightarrow\ S_{22} = \frac{22}{2}\times 110$ = 1210

7. If f is a function satisfying $f(x+y)=f(x)f(y)$ for all $x, y \in$ N such that $f(1)$ =3 and $\sum_{x=1}^{n} f(x)$ = 120, find the value of n.

Ans. Given $f(1)=3$ and $f(x+y)=f(x)f(y)$ for all $x, y \in$ N ....(i)

Putting $x=1, y=1$ in eq. (i), $f(1+1)=f(1)f(1)$

$\Rightarrow f(2)= 3\times 3 = 9$

Putting $x=1, y=2$ in eq. (i), $f(1+2)=f(1)f(2)$

$\Rightarrow f(3)= 3\times 9 = 27$

Putting $x=1, y=3$ in eq. (i), $f(1+3)=f(1)f(3)$

$\Rightarrow f(4)= 3\times 27 = 81$

Now, $\sum_{x=1}^{n} f(x)=120$

$\Rightarrow f(1)+f(2)+f(3)+..........f(n)=120$

$\Rightarrow 3+9+27+81+.........$ up to n term $= 120$

$\Rightarrow \dfrac{3(3^n-1)}{3-1}=120$

$\Rightarrow 3(3^n-1)=240$

$\Rightarrow 3^n-1=80$

$\Rightarrow 3^n=81$

$\Rightarrow 3^n=3^4$

$\Rightarrow n = 4$

8. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and the number of terms.

Ans. Given: $a = 15, r = 2$ and $S_n = 315$

$\therefore \quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$

$\Rightarrow 315 = \frac{5\left(2^n - 1\right)}{2 - 1}$

$\Rightarrow \frac{315}{5} = 2^n - 1$

$\Rightarrow 2^n - 1 = 63$

$\Rightarrow 2^n = 64 = 2^6$

$\Rightarrow n = 6$

$\therefore \quad a_6 = ar^{6-1} = 5 \times 2^5 = 5 \times 32 = 160$

Hence the number of terms=6 and the last term =160

9. The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find common ratio of G.P.

Ans. Given: $a = 1$ and $a_3 + a_5 = 90$

$\Rightarrow ar^2 + ar^4 = 90$

$\Rightarrow a\left(r^2 + r^4\right) = 90$

$\Rightarrow 1 \times \left(r^2 + r^4\right) = 90$

$\Rightarrow r^2 + r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow r^2 = \dfrac{-1 \pm \sqrt{(1)^2 - 4 \times (-90) \times 1}}{2 \times 1}$

$= \dfrac{-1 \pm \sqrt{1+360}}{2} = \dfrac{-1 \pm \sqrt{361}}{2}$

$= \dfrac{-1 \pm 19}{2}$

$\Rightarrow r^2 = \dfrac{-1+19}{2} = \dfrac{18}{2} = 9$ or $r^2 = \dfrac{-1-19}{2} = \dfrac{-20}{2} = -10$ which is not possible

Therefore, common ratio is $r = \pm 3$

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10. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Ans. Let a, ar, ar^2 be three numbers in G.P., therefore, $a + ar + ar^2 = 56$

$\Rightarrow a(1 + r + r^2) = 56$ ..(i)

According to question, $a-1, ar-7, ar^2-21$ are in A.P.

$\therefore (ar-7)-(a-1) = (ar^2-21)-(ar-7)$

$\Rightarrow ar - 7 - a + 1 = ar^2 - 21 - ar + 7$

$\Rightarrow ar - a - 6 = ar^2 - ar - 14$

$\Rightarrow ar^2 - 2ar + a = 8$

$\Rightarrow a(r^2 - 2r + 1) = 8$ .....(ii)

Dividing eq. (i) by eq. (ii), $\frac{a(1+r+r^2)}{a(r^2-2r+1)}=\frac{56}{8}$

$\Rightarrow 1+r+r^2=7r^2-14r+7$

$\Rightarrow 6r^2-15r+6=0$

$\Rightarrow 2r^2-5r+2=0$

$\Rightarrow r=\frac{-(-5)\pm\sqrt{(-5)^2-4\times2\times2}}{2\times2}$

$=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm\sqrt{9}}{4}=\frac{5\pm3}{4}$

$\Rightarrow r=\frac{5+3}{4}=\frac{8}{4}=2$ or $r=\frac{5-3}{4}=\frac{2}{4}=\frac{1}{2}$

Putting $r=2$ in eq. (i), $a(1+2+2^2)=56$

$\Rightarrow a=\frac{56}{7}=8$

Then the required numbers are 8, 16, 32.

Putting $r=\frac{1}{2}$ in eq. (i), $a\left(1+\frac{1}{2}+\frac{1}{4}\right)=56$

$\Rightarrow a\times\frac{7}{4}=56$

$\Rightarrow a=32$

Then the required numbers are 32, 16, 8.

11. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Ans. Let the number of terms be $2n$ then we have the number of odd terms is n

Let the G.P be $a, ar, ar^2, \ldots\ldots\ldots ar^{2n-1}$

Then the odd terms $a, ar^2, ar^4, ar^6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ form a G.P

$\therefore \ S_{2n} = \dfrac{a\left(r^{2n}-1\right)}{r-1}$ and $S_n = a\left[\frac{(r^2)^n - 1}{r^2-1}\right]$

According to question, $S_{2n} = 5S_n$

$\Rightarrow a\left[\frac{r^{2n}-1}{r-1}\right] = 5a\left[\frac{(r^2)^n-1}{r^2-1}\right]$

$\Rightarrow \frac{1}{r-1} = \frac{5}{r^2-1}$

$\Rightarrow r+1=5$

$\Rightarrow r=4$

12. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Ans. Given: $a=11$ and $S_4 = 56$

$\Rightarrow S_4 = \frac{4}{2}\left[2\times 11 + (4-1)d\right]$

$\Rightarrow 2(22+3d) = 56$

$\Rightarrow 22+3d = 28$

$\Rightarrow 3d = 6$

$\Rightarrow d = 2$

Also, $l + (l-d) + (l-2d) + (l-3d) = 112$

$\Rightarrow 4l - 6d = 112$

$\Rightarrow 4l = 112 + 6d$

$\Rightarrow 4l = 112 + 6 \times 2$

$\Rightarrow 4l = 112 + 12$

$\Rightarrow 4l = 124$

$\Rightarrow l = 31$

$\therefore \ a_n = a + (n-1)d$

$\Rightarrow 31 = 11 + (n-1) \times 2$

$\Rightarrow 2(n-1) = 20$

$\Rightarrow n - 1 = 10$

$\Rightarrow n = 11$

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13. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx} (x \neq 0)$, then show that a, b, c and d are in G.P.

Ans. Taking $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$

$\Rightarrow (a+bx)(b-cx) = (b+cx)(a-bx)$

$\Rightarrow ab - acx + b^2x - bcx^2 = ab - b^2x + acx - bcx^2$

$\Rightarrow 2b^2x = 2acx$

$\Rightarrow b^2 = ac$

$\Rightarrow \frac{b}{a} = \frac{c}{b}$ ....(i)

Taking $\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

$\Rightarrow \ (b+cx)(c-dx)=(c+dx)(b-cx)$

$\Rightarrow \ 2c^2x=2bdx$

$\Rightarrow \ c^2=bd$

$\Rightarrow \ \frac{c}{b}=\frac{d}{c}$ ....(ii)

From eq. (i) and (ii), $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

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14. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n=S^n$.

Ans. Let the G.P be $a, ar, ar^2, ar^3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, ar^{n-1}$

Here $S=\frac{a(r^n-1)}{r-1}$

P = $a.ar.ar^2\ldots\ldots\ldots ar^{n-1}=a^n.r^{1+2+3+\ldots\ldots+(n-1)}=a^n.r^{\frac{n(n-1)}{2}}$

and R = $\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\ldots\ldots\ldots\frac{1}{ar^{n-1}}=\frac{r^{n-1}+r^{n-2}+r^{n-3}+\ldots\ldots\ldots+1}{ar^{n-1}}$

$=\frac{1(r^n-1)}{r-1}.\frac{1}{ar^{n-1}}=\frac{r^n-1}{ar^{n-1}(r-1)}$

Now $p^2R^n=\frac{a^{2n}.r^{n(n-1)}(r^n-1)^n}{a^nr^{n(n-1)}(r-1)^n}=\frac{a^n(r^n-1)^n}{(r-1)^n}=a^n\left(\frac{r^n-1}{r-1}\right)^n=S^n$

Hence proved.

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15. The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively. Show that

$(q-r)a+(r-p)b+(p-q)c=0.$

Ans. According to question, $a_p=a+(p-1)d=a$, $a_q=a+(q-1)d=b$ and

$a_r = a + (r-1)d = c$

Now $(q-r)a + (r-p)b + (p-q)c = 0$

Putting values of a, b and $c,$ we get

$(q-r)[a+(p-1)d] + (r-p)[a+(q-1)d] + (p-q)[a+(r-1)d] = 0$

$\Rightarrow (q-r)[a+pd-d] + (r-p)[a+qd-d] + (p-q)[a+rd-d] = 0$

$\Rightarrow aq + pqd - qd - ra - rpd + rd + ar + qrd - dr - pa - pqd + pd + pa + prd - pd -$

$\Rightarrow$ 0 = 0 Proved.

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16. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., prove that a, b, c are in A.P.

Ans. Given: $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

$\Rightarrow a\left(\frac{b+c}{bc}\right), b\left(\frac{c+a}{ca}\right), c\left(\frac{a+b}{ab}\right)$ are in A.P.

$\Rightarrow \frac{ab+ac}{bc}, \frac{bc+ab}{ca}, \frac{ac+bc}{ab}$ are in A.P.

$\Rightarrow \frac{ab+ac}{bc}+1, \frac{bc+ab}{ca}+1, \frac{ac+bc}{ab}+1$ are in A.P. [Adding 1 to each term in the sequence]

$\Rightarrow \frac{ab+ac+bc}{bc}, \frac{bc+ab+ca}{ca}, \frac{ac+bc+ab}{ab}$ are in A.P.

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.[Dividing each fraction by $ab+bc+ca$]

$\Rightarrow$ $\frac{abc}{bc}, \frac{abc}{ca}, \frac{abc}{ab}$ are in A.P.[Multiplying each fraction by abc]

$\Rightarrow$ a, b, c are in A.P.

17. If a, b, c, d are in G.P., prove that $\left(a^n+b^n\right), \left(b^n+c^n\right), \left(c^n+d^n\right)$ are in G.P.

Ans. Given: a, b, c, d are in G.P.

To prove: $\left(a^n+b^n\right), \left(b^n+c^n\right), \left(c^n+d^n\right)$ are in G.P.

$\Rightarrow$ $\frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore$ $\frac{b}{a} = k$

$\Rightarrow$ $b = ak$

And $\frac{c}{b} = k$

$\Rightarrow$ $c = bk = (ak)k = ak^2$

Also $\frac{d}{c} = k$

$\Rightarrow$ $d = ck = \left(ak^2\right)k = ak^3$

Now, $\frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

$$\Rightarrow \frac{(ak)^n + \left(ak^2\right)^n}{a^n + (ak)^n} = \frac{\left(ak^2\right)^n + \left(ak^3\right)^n}{(ak)^n + \left(ak^2\right)^n}$$

$$\Rightarrow \frac{a^nk^n + a^nk^{2n}}{a^n + a^nk^n} = \frac{a^nk^{2n} + a^nk^{3n}}{a^nk^n + a^nk^{2n}}$$

$$\Rightarrow \frac{a^nk^n\left(1+k^n\right)}{a^n\left(1+k^n\right)} = \frac{a^nk^{2n}\left(1+k^n\right)}{a^nk^n\left(1+k^n\right)}$$

$$\Rightarrow k^n = k^n$$

Therefore, $\left(a^n + b^n\right), \left(b^n + c^n\right), \left(c^n + d^n\right)$ are in G.P.

18. If a and b are the roots $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that $(q+p):(q-p) = 17:15$.

Ans. Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore \frac{b}{a} = k$

$\Rightarrow b = ak$

And $\frac{c}{b} = k$

$\Rightarrow c = bk = (ak)k = ak^2$

Also $\frac{d}{c} = k$

$\Rightarrow d = ck = \left(ak^2\right)k = ak^3$

$\because$ a and b are the roots $x^2 - 3x + p = 0$

$\therefore a + b = \frac{-(-3)}{1} = 3$

$\Rightarrow a+ak=3$

$\Rightarrow a(1+k)=3$ ....(i)

And $ab=\frac{p}{1}$

$\Rightarrow a(ak)=p$

$\Rightarrow a^2k=p$ ....(ii)

Also c, d are roots of $x^2-12x+q=0$

$\therefore c+d=\frac{-(-12)}{1}=12$

$\Rightarrow ak^2+ak^3=12$

$\Rightarrow ak^2(1+k)=12$ ....(iii)

And $cd=\frac{q}{1}$

$\Rightarrow ak^2(ak^3)=q$

$\Rightarrow a^2k^5=q$ ....(iv)

Dividing eq. (iii) by eq. (i), $\frac{ak^2(1+k)}{a(1+k)}=\frac{12}{3}$

$\Rightarrow k^2=4$

$\Rightarrow k=\pm 2$

Now $\frac{q+p}{q-p}=\frac{a^2k^5+a^2k}{a^2k^5-a^2k}=\frac{a^2k(k^4+1)}{a^2k(k^4-1)}$

= $\dfrac{(\pm 2)^4+1}{(\pm 2)^4-1}=\dfrac{16+1}{16-1}=\dfrac{17}{15}$

Therefore, $(q+p):(q-p)=17:15$

19. The ratio of the A.M. and G.M. of two positive numbers a and b, is $m:n$. Show that $a:b$ = $\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$.

Ans. Given: $\dfrac{a+b}{2}:\sqrt{ab}=m:n$

$$\Rightarrow\ \frac{a+b}{2\sqrt{ab}}=\frac{m}{n}$$

By componendo and dividendo,

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{m+n}{m-n}$$

$$\Rightarrow\ \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{m+n}{m-n}$$

$$\Rightarrow\ \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{m+n}}{\sqrt{m-n}}$$

Again by componendo and dividendo,

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\Rightarrow\ \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\Rightarrow \frac{a}{b}=\frac{\left(\sqrt{m+n}+\sqrt{m-n}\right)^2}{\left(\sqrt{m+n}-\sqrt{m-n}\right)^2}$$

$$\Rightarrow \frac{a}{b}=\frac{m+n+m-n+2\sqrt{(m+n)(m-n)}}{m+n+m-n-2\sqrt{(m+n)(m-n)}}$$

$$\Rightarrow \frac{a}{b}=\frac{2m+2\sqrt{(m+n)(m-n)}}{2m-2\sqrt{(m+n)(m-n)}}$$

$$\Rightarrow \frac{a}{b}=\frac{m+\sqrt{(m+n)(m-n)}}{m-\sqrt{(m+n)(m-n)}}$$

Therefore, $a:b$ = $\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$

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20. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

Ans. Since, a, b, c are in A.P.

$\therefore\ b-a=c-b$

$\Rightarrow\ 2b=a+c$

$\Rightarrow\ b=\frac{a+c}{2}$

Since, b, c, d are in G.P. $\therefore\ \frac{c}{b}=\frac{d}{c}$

$\Rightarrow\ c^2=bd$ ....(i)

Also $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

$$\therefore \quad \frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d}$$

$$\Rightarrow \frac{2}{d}=\frac{1}{c}+\frac{1}{e}$$

$$\Rightarrow \frac{2}{d}=\frac{c+e}{ce}$$

$$\Rightarrow d=\frac{2ce}{c+e}$$

Putting values of b and d in eq. (i), $c^2=\left(\frac{c+a}{2}\right)\left(\frac{2ce}{c+e}\right)$

$$\Rightarrow c^2=\frac{ce(c+a)}{c+e}$$

$$\Rightarrow c^2(c+e)=ec(c+a)$$

$$\Rightarrow c^2+ce=ce+ae$$

$\Rightarrow c^2=ae$ which shows that a, c, e are in G.P.

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21. Find the sum of the following series up to n terms:

(i) 5 + 55 + 555 + ...

(ii) .6 + .66 + .666 + ...

Ans. (i) S_n = 5 + 55 + 555 + ... up to n terms

= 5 [1 + 11 + 111 + ... up to n terms]

= $\frac{5}{9}\left[9+99+999+\text{..........up uo } n \text{ terms}\right]$

$= \frac{5}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\text{.......... up to } n \text{ terms}\right]$

$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$

$= \frac{5}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

$= \frac{50}{81}(10^n-1)-\frac{5}{9}n$

(ii) S_n = .6 + .66 + .666 + ... up to n terms

= 6 [.1 + .11 + .111 + ... up to n terms]

$= \frac{6}{9}[.9+.99+.999+\text{..........up uo } n \text{ terms}]$

$= \frac{6}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\text{.......... up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)\text{.......... up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[n-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\text{.......... up to } n \text{ terms}\right)\right]$

$= \frac{2}{3}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{2}{3}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

= $\frac{2n}{3}-\frac{2}{27}\left(1-\frac{1}{10^n}\right)$

22. Find the 20th term of the series $2\times4+4\times6+6\times8+\dots\dots\dots\dots+$ terms.

Ans. Given: $2\times4+4\times6+6\times8+\dots\dots\dots\dots+n$ terms

$\therefore\ a_n=\left(n^{th}\text{ term of }2,4,6,\dots\dots\right)\left(n^{th}\text{ term of }4,6,8,\dots\dots\right)$

$\Rightarrow\ a_n=\left[2+(n-1)2\right]\left[4+(n-1)2\right]$ = $2n(2n+2)$

$\therefore\ a_{20}=2\times20(2\times20+2)=40\times42=1680$

23. Find the sum of the first n terms of the series: 3 + 7 + 13 + 21 + 21 + ……….

Ans. Given: $S_n=3+7+13+21+31+\dots\dots\dots\dots+a_{n-1}+a_n$ ……….(i)

Also $S_n=3+7+13+21+31+\dots\dots\dots\dots+a_{n-2}+a_{n-1}+a_n$ ……….(ii)

Subtracting eq. (i) from eq. (ii), $0=3+\left(4+6+8+10+\dots\dots\text{ up to }(n-1)\text{ terms}\right)-a_n$

$\Rightarrow\ a_n=3+\frac{n-1}{2}\left[2\times4+(n-2)\times2\right]$

$\Rightarrow\ a_n=3+\frac{n-1}{2}\left[8+2n-4\right]$

$\Rightarrow\ a_n=3+(n-1)(n+2)$

$\Rightarrow\ a_n=3+n^2+n-2$

$\Rightarrow\ a_n=n^2+n+1$

$\therefore\ S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}(k^2+k+1)$

= $\left(1^2+1+1\right)+\left(2^2+2+1\right)+\left(3^2+3+1\right)+..........+\left(n^2+n+1\right)$

= $\left(1^2+2^2+3^2+..........+n^2\right)+\left(1+2+3+..........+n\right)+n$

= $\dfrac{n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}+n$

= $n\left[\dfrac{2n^2+3n+1+3n+3+6}{6}\right]$

= $n\left[\dfrac{2n^2+6n+10}{6}\right]$

= $\dfrac{n}{3}\left(n^2+3n+5\right)$

24. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes respectively, show that $9S_2^2=S_3\left(1+8S_1\right)$.

Ans. Here $S_1=\dfrac{n(n+1)}{2}$, $S_2=\dfrac{n(n+1)(2n+1)}{6}$ and $S_3=\dfrac{n^2(n+1)^2}{4}$

Now, $9S_2^2=S_3\left(1+8S_1\right)$

$\Rightarrow 9\left[\dfrac{n(n+1)(2n+1)}{6}\right]^2=\left[\dfrac{n^2(n+1)^2}{4}\right]\left[1+8\left\{\dfrac{n(n+1)}{2}\right\}\right]$

$\Rightarrow \frac{9}{36}n^2(n+1)^2(2n+1)^2=\frac{n^2(n+1)^2}{4}+n^3(n+1)^3$

$\Rightarrow \dfrac{9}{36}n^2(n+1)^2(2n+1)^2=\dfrac{n^2(n+1)^2}{4}\left[4n^2+4n+1\right]$

$\Rightarrow \dfrac{1}{4}n^2(n+1)^2(2n+1)^2=\dfrac{1}{4}n^2(n+1)^2(2n+1)^2$ Proved.

25. Find the sum of the following series up to n terms:

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots\ldots\ldots$$

Ans. Given: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots\ldots\ldots$ up to n terms

$$\therefore\quad a_n=\frac{1^3+2^3+3^3+\ldots\ldots\ldots+n^3}{1+3+5+\ldots\ldots\ldots(2n-1)}$$

$$=\frac{\sum n^3}{\frac{n}{2}[2+(n-1)2]}=\frac{\sum n^3}{\frac{n}{2}(2n)}=\frac{\sum n^3}{n^2}=\frac{n^2(n+1)^2}{4n^2}$$

$$=\frac{1}{4}\left(n^2+2n+1\right)$$

$$\therefore\quad S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}\frac{k^2+2k+1}{4}$$

$$=\frac{1}{4}\left[\left(1^2+2.1+1\right)+\left(2^2+2.2+1\right)+\left(3^2+2.3+1\right)+\ldots\ldots\ldots\ldots+\left(n^2+2n+1\right)\right]$$

$$=\frac{1}{4}\left[\sum n^2+2\sum n+n\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}+n\right]$$

$$=\frac{n}{4}\left[\frac{2n^2+3n+1+6n+6+6}{6}\right]$$

$$=\frac{n}{24}\left(2n^2+9n+13\right)$$

26. Show that $\frac{1\times2^2+2\times3^2+\ldots\ldots\ldots+n\times(n+1)^2}{1^2\times2+2^2\times3+\ldots\ldots\ldots+n^2(n+1)}=\frac{3n+5}{3n+1}$

Ans. Given: $\dfrac{1\times 2^2+2\times 3^2+.......+n\times(n+1)^2}{1^2\times 2+2^2\times 3+.......+n^2(n+1)}=\dfrac{3n+5}{3n+1}$

$= \dfrac{\sum n(n+1)^2}{\sum n^2(n+1)} = \dfrac{\sum n(n^2+2n+1)}{\sum(n^3+n^2)}$

$= \dfrac{\sum n^3+2\sum n^2+\sum n}{\sum n^3+\sum n^2}$

$= \dfrac{\dfrac{n^2(n+1)^2}{4}+\dfrac{2n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}}{\dfrac{n^2(n+1)^2}{4}+\dfrac{n(n+1)(2n+1)}{6}}$

$= \dfrac{\dfrac{n(n+1)}{2}\left[\dfrac{n(n+1)}{2}+\dfrac{2(2n+1)}{3}+1\right]}{\dfrac{n(n+1)}{2}\left[\dfrac{n(n+1)}{2}+\dfrac{(2n+1)}{3}\right]}$

$= \dfrac{3n^2+11n+10}{3n^2+7n+2} = \dfrac{(n+2)(3n+5)}{(n+2)(3n+1)} = \dfrac{3n+5}{3n+1}$

27. A farmer buys a used tractor for Rs. 12000. He pays Rs. 6000 cash and agrees to pay the balance in annual installments of Rs. 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

Ans. Total cost of the tractor = Rs. 12000, Cash paid = Rs. 6000

Balance to be paid = 12000 – 6000 = Rs. 6000

Annual installment = Rs. 500

$\therefore$ Number of installment $= \frac{6000}{500} = 12$

Interest of 1st installment = $\frac{6000 \times 12 \times 1}{100}$ = Rs. 720

Amount of 1st installment = 500 + 720 = Rs. 1220

Interest of 2nd installment = $\frac{5500 \times 12 \times 1}{100}$ = Rs. 660

Amount of 2nd installment = 500 + 660 = Rs. 1160

Interest of 3rd installment = $\frac{5000 \times 12 \times 1}{100}$ = Rs. 600

Amount of 3rd installment = 500 + 600 = Rs. 1100

$\therefore$ Sequence of installments is 1220, 1160, 1100, ... which is in A.P

Here, $a = 1220, d = 1160 - 1220 = -60$ and $n = 12$

$\therefore \ S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

= $\frac{12}{2}\left[2 \times 1220 + (12-1) \times (-60)\right]$

= 6 [2440 – 660] = Rs. 10680

Therefore, the total cost of tractor is (10680 + 6000) = Rs. 16680.

28. Shams had Ali buys a scooter for Rs. 22000. He pays Rs. 4000 cash and agrees to pay the balance in annual installment of Rs. 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Ans. Total cost of the scooter = Rs. 22000, Cash paid = Rs. 4000

Balance to be paid = 22000 – 4000 = Rs. 18000

Annual installment = Rs. 1000

$\therefore$ Number of installment $= \frac{18000}{1000} = 18$

Interest of 1st installment = $\dfrac{18000 \times 10 \times 1}{100}$ = Rs. 1800

Amount of 1st installment = 1000 + 1800 = Rs. 2800

Interest of 2nd installment = $\dfrac{17000 \times 10 \times 1}{100}$ = Rs. 1700

Amount of 2nd installment = 1000 + 1700 = Rs. 2700

Interest of 3rd installment = $\dfrac{16000 \times 10 \times 1}{100}$ = Rs. 1600

Amount of 3rd installment = 1000 + 1600 = Rs. 2600

$\therefore$ Sequence of installments is 2800, 2700, 2600, ... in A.P

Here, $a = 2800, d = 2700 - 2800 = -100$ and $n = 18$

$\therefore$ $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ = $\dfrac{18}{2}\left[2 \times 2800 + (18-1) \times (-100)\right]$

= 9 [5600 – 1700] = Rs. 35100

Therefore, the total cost of tractor is (35100 + 4000) = Rs. 39100.

29. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed.

Ans. Total letters in the first set = 4, Total letters in the second set = 4^2 = 16

Total letters in the third set = 4^3 = 64

$\therefore$ Sequence of letters is 4, 16, 64, ... in G.P.

Here $a = 4, r = \frac{16}{4} = 4$ and $n = 8$

$\therefore \quad S_n = \frac{a(r^n - 1)}{r - 1}$

= $\frac{4(4^8 - 1)}{8 - 1}$

= $\frac{4}{3}(65536 - 1)$

= $\frac{4}{3} \times 65535$ = 87380

Hence,total number of letters mailed = 87380

The amount of postage on each letter = 50 paise

Therefore total amount spent on postage = 87380×0.50 = Rs. 43690.

30. A man deposited Rs. 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.

Ans. Total amount deposited = Rs. 10000, Rate of interest = 5% per annum

Interest of first year = $\frac{10000 \times 5 \times 1}{100}$ = Rs. 500

Here $a = 10000, d = 500$

$\therefore$ Amount in 15th year = $a_{15} = 10000 + (15 - 1) \times 500$ = 10000 + 7000 = Rs. 17000

Total amount after 20 years = Amount in the 21st year = $a_{21} = 10000 + (21 - 1)500$

=10000 + 10000 = Rs.20000

31. A manufacturer reckons that the value of a machine, which cost him Rs. 15625 will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Ans. Present value of the machine = Rs. 15625

Rate of depreciation = 20%

After 1 year value of machine = $15625 - 15625 \times \frac{20}{100}$ = 15625 – 3125 = Rs. 12500

After 2 year value of machine = $12500 - 12500 \times \frac{20}{100}$ = 12500 – 2500 = Rs. 10000

After 3 year value of machine = $10000 - 10000 \times \frac{20}{100}$ = 10000 – 2000 = Rs. 8000

$\therefore$ Sequence of values of machine after depreciation is 12500, 10000, 8000, ... is a G.P.

Here $a = 12500, r = \frac{10000}{12500} = \frac{4}{5}$

$\therefore \quad a_5 = ar^4 = 12500 \times \left(\frac{4}{5}\right)^4 = 12500 \times \frac{256}{625}$ = Rs. 5120

Therefore, the value of machine at the end of 5 years is Rs. 5120.

32. 150 workers were engaged to finish a job in a certain number of boys. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work find the number of days in which the work was completed.

Ans. Number of workers on the first day = 150

Number of workers on the second day = 150 – 4 = 146

Number of workers on the third day = 146 – 4 = 142

$\therefore$ Sequence of number of workers is 150, 146, 142, ... in A.P.

Here $a = 150$, $d = 146 - 150 = -4$

$\therefore$ Total number of workers required to finish the work in n days

= $\frac{n}{2}[2 \times 150 + (n-1)(-4)]$

$= \frac{n}{2}(300 - 4n + 4)$

= $n(152 - 2n)$ ....(i)

If no worker had dropped out, then the work should have finished in $(n-8)$ days with 150 workers on each day.

$\therefore$ Total number of workers required to finish the work in $(n-8)$ days = $150(n-8)$ ....(ii)

From eq. (i) and (ii), $n(152 - 2n) = 150(n-8)$

$\Rightarrow$ $152n - 2n^2 = 150n - 1200$

$\Rightarrow$ $2n^2 - 2n - 1200 = 0$

$\Rightarrow$ $n^2 - n - 600 = 0$

$\Rightarrow$ $(n-25)(n+24) = 0$

$\Rightarrow$ $n = 25$ and $n = -24$ which is not possible

Therefore, the work was completed in 25 days.