CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 2 Relations and Functions

Miscellaneous Exercise

1. The relation f is defined by $f(x)=\begin{cases} x^2 & 0\le x\le 3 \\ 3x & 3\le x\le 10 \end{cases}$. The relation g is defined by $g(x)=\begin{cases} x^2 & 0\le x\le 2 \\ 3x & 2\le x\le 10 \end{cases}$. Show that f is a function and g is not a function.

Ans. Given: $f(x)=x^2 \quad 0\le x\le 3$ and

$f(x)=3x \quad 3\le x\le 10$

At $x=3$, $f(3)=(3)^2=9$ and

$f(3)=3\times 3=9$

It is observed that $f(x)$ takes unique value at each point in its domain [0, 10]. Therefore, f is a function.

Now, $g(x)=x^2 \quad 0\le x\le 2$ and

$g(x)=3x \quad 2\le x\le 10$

At $x=2$, $g(2)=(2)^2=4$ and

$g(2)=3\times 2=6$

Therefore, $g(x)$ does not have unique value at $x=2$.

Hence, $g(x)$ is not a function.

2. If $f(x)=x^2$, **find** $\frac{f(1.1)-f(1)}{(1.1-1)}$.

Ans. Given: $f(x)=x^2$

At $x=1.1$ $f(1.1)=(1.1)^2=1.21$

and $f(1)=(1)^2=1$

$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$

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3. Find the domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$.

Ans. Given: $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

$f(x)$ is a rational function of x.

$f(x)$ assumes real values of all x except for those values of x for which

$x^2-8x+12=0$

$\Rightarrow (x-6)(x-2)=0$

$\Rightarrow x=2,6$

$\therefore$ Domain of function = R – {2, 6}

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4. Find the domain and range of the real function f **defined by** $f(x)=\sqrt{x-1}$.

Ans. Given: $f(x)=\sqrt{x-1}$, $f(x)$ assumes real values if $x-1\geq 0 \Rightarrow x\geq 1$

$\Rightarrow x\in[1,\infty)$

$\therefore$ Domain of $f(x) = [1, \infty)$

For $x \geq 1, f(x) \geq 0$

$\therefore$ Range of $f(x)$ = all real numbers $\geq$ 0 = $[0, \infty)$

5. Find the domain and range of the real function f defined by $f(x) = |x-1|$.

Ans. Given: $f(x) = |x-1|$

The function $f(x)$ is defined for all values of x.

$\therefore$ Domain of $f(x)$ = R

When $x > 1$, $|x-1| = x-1 > 0$

When $x = 1$, $|x-1| = 0$

When $x < 1$, $|x-1| = -x+1 > 0$

6. Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in R \right\}$ be a function from R into R. Determine the range of f.

Ans. Here $f(x) = \frac{x^2}{1+x^2}$

Putting $y = \frac{x^2}{1+x^2}$

$\Rightarrow y + yx^2 = x^2$

$\Rightarrow x^2(1-y) = y$

$\Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$

Now, x will be real if $\frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y}{y-1} \leq 0$

$\Rightarrow 0 \leq y < 1$

$\Rightarrow y \in [0,1)$

$\therefore$ Range $f(x) = [0,1)$

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7. Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined respectively by $f(x) = x+1, g(x) = 2x-3.$ Find $f+g, f-g$ and $\frac{f}{g}.$

Ans. Given : $f(x) = x+1$ and $g(x) = 2x-3$

Now, $(f+g)(x) = f(x) + g(x) = x+1+2x-3 = 3x-2$

And $(f-g)(x) = f(x) - g(x) = x+1-2x+3 = -x+4$

And $\frac{(f)}{(g)}(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

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8. Let $f = \{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function from Z to Z defined by $f(x) = ax+b$ for some integers $a, b.$ Determine $a, b.$

Ans. Given: $f(x) = ax+b$ and

$f = \{(1,1),(2,3),(0,-1),(1,-3)\}$

$\Rightarrow f(1) = 1, f(2) = 3, f(0) = -1, f(-1) = -3$

Now $f(1) = 1 \Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$ ....(i)

And $f(2)=3 \Rightarrow a\times 2+b=3$

$\Rightarrow 2a+b=3$ ....(ii)

Solving eq. (i) and (ii), we get $a=2$ and $b=-1$

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9. Let R be a relation from N to N defined by R = $\{(a,b): a,b\in \text{N and } a=b^2\}$. Are the following true:

(i) $(a,a)\in$ **R for all** $a\in$**N**

(ii) $(a,b)\in$ **R implies** $(b,a)\in$ **R**

(iii) $(a,b)\in$R, $(b,c)\in$ **R implies** $(a,c)\in$ **R**

Ans. Given: R = $\{(a,b): a,b\in \text{N and } a=b^2\}$

(i) No, (3, 3) $\notin$ R because $3\neq 3^2$

(ii) No, (9, 3) $\in$ R but (3, 9) $\notin$ R

(iii) No, (81, 9) $\in$ R and (9,3) $\in R$ but (81, 3) $\notin$ R

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10. Let A = {1, 2, 3, 4}, B = {1, 5 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}.-Are the following true:

(i) f is a relation from A to B.

(ii) f is a function from A to B.

Justify your answer in each case.

Ans. (i) Here A = {1, 2, 3,4} and B = {1, 5, 9, 11, 15, 16}

$\therefore A \times B$ = {(1, 1), (1, 5), (1,9), (1, 11), (1, 15), (1, 16) ,(2, 1), (2, 5), (2, 9), (2, 11),

(2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5),

(4, 9), (4, 11), (4, 15), (4, 16)}

f = {(1, 5),(2, 9), (3, 1), (4, 5), (2, 11)}

Now, (1, 5),(2, 9), (3, 1), (4, 5), (2, 11) $\in A \times B$

$\therefore f$ is a relation from A to B.

(ii) f is not a function because (2,9) $\in f$ and $(2, 11) \in f$

11. Let f be a subset of $Z \times Z$ defined by $f = \{(ab, a+b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

Ans.-We observed that 1×4 = 4 and 2×2 = 4

$\Rightarrow (1 \times 4,\ 1 + 4) \in f$ and $(2 \times 2,\ 2 + 2) \in f$

$\Rightarrow$ (4, 5) $\in f$ and (4, 4) $\in f$

It shows that f is not a function from Z to Z.

12. Let A = {9, 10, 11, 12 13} and let $f : A \to N$ be defined by $f(n)$ = the highest prime factor of n. Find the range of f.

Ans. Here A = {9, 10, 11, 12, 13}

For $n = 9$, $f(9) = 3$

[$\because 9 = 3 \times 3$ and 3 is highest prime factor of 9]

For $n = 10$, $f(10) = 5$

[$\because 10 = 2 \times 5$ and 5 is highest prime factor of 10]

For $n = 11$, $f(11) = 11$

[$\because 11 = 1 \times 11$ and 11 is highest prime factor of 11]

For $n = 12$, $f(12) = 3$

[$\because 12 = 3 \times 3 \times 2$ and 3 is highest prime factor of 12]

For $n = 13$, $f(13) = 13$

[$\because 13 = 1 \times 13$ and 13 is highest prime factor of 13]

$\therefore$ Range of f = {5, 11, 3, 13}

= {3, 5, 11, 13}