CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 9 Sequences and Series

Exercise 9.3

1. Find the 20th and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Ans. Here $a = \frac{5}{2}$ and $r = \frac{5}{4} \div \frac{5}{2} = \frac{1}{2}$

$\therefore\ a_n = ar^{n-1}$

$\Rightarrow\ a_{20} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{20-1}$

$\Rightarrow\ a_{20} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{19} = \frac{5}{2^{20}}$

And $a_n = \frac{5}{2} \times \left(\frac{1}{2}\right)^{n-1} = \frac{5}{2 \times 2^{n-1}} = \frac{5}{2^n}$

2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Ans. Let a be the first term of given G.P. Here $r = 2$ and $a_8 = 192$

$\therefore\ a_n = ar^{n-1}$

$\Rightarrow\ a_8 = a \times (2)^{8-1} = 192$

$\Rightarrow\ a \times (2)^7 = 192$

$\Rightarrow\ a \times 128 = 192$

$\Rightarrow a = \frac{192}{128} = \frac{3}{2}$

$\therefore a_{12} = ar^{12-1}$

$\Rightarrow a_{12} = \frac{3}{2} \times 2^{11}$ = 3×2^{10}

= 3×1024 = 3072

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3. The 5th, 8th and 11th terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$.

Ans. Let a be the first term and r be the common ratio of given G.P.

$\therefore a_5 = p \Rightarrow ar^4 = p$ ....(i)

$a_8 = q \Rightarrow ar^7 = q$ ....(ii)

$a_{11} = s \Rightarrow ar^{10} = s$ ....(iii)

Squaring both sides of eq. (ii), we get $q^2 = \left(ar^7\right)^2$

$\Rightarrow q^2 = a^2 r^{14}$

$\Rightarrow q^2 = \left(ar^4\right)\left(ar^{10}\right)$

$\Rightarrow q^2 = ps$ [From eq. (i) and (iii)]

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4. The 4th term of a G.P. is square of its second term and the first term is -3. Determine its 7th term.

Ans. Let a be the first term and r be the common ratio of given G.P.

Here $a = -3$ and $a_4 = \left(a_2\right)^2$

Now, $a_4 = \left(a_2\right)^2$

$\Rightarrow ar^3 = (ar)^2$

$\Rightarrow ar^3 = a^2r^2$

$\Rightarrow r = a$

$\Rightarrow r = -3 \ [\because a = -3]$

$\therefore \ a_7 = ar^{7-1} = (-3)\times(-3)^6$

$= -3\times 729 = -2187$

5. Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \dots\dots$ **is 128?**

(b) $\sqrt{3}, 3, 3\sqrt{3}, \dots\dots\dots$ **is 729?**

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots\dots\dots$ **is** $\frac{1}{19683}$?

Ans. (a) Here $a = 2$, $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$ and $a_n = 128$

$\therefore \ a_n = ar^{n-1}$

$\Rightarrow 128 = 2\times\left(\sqrt{2}\right)^{n-1}$

$\Rightarrow 64 = \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow \left(\sqrt{2}\right)^{12} = \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

Therefore, 13^{th} term of the given G.P. is 128.

(b) Here $a=\sqrt{3}$, $r=\frac{3}{\sqrt{3}}=\sqrt{3}$ and $a_n=729$

$\therefore\ a_n=ar^{n-1}$

$\Rightarrow\ 729=\sqrt{3}\times\left(\sqrt{3}\right)^{n-1}$

$\Rightarrow\left(\sqrt{3}\right)^{12}=\left(\sqrt{3}\right)^{n}$

$\Rightarrow n=12$

Therefore, 12^{th} term of the given G.P. is 729.

(c) Here $a=\frac{1}{3}$, $r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$ and $a_n=\frac{1}{19683}$

$\therefore\ a_n=ar^{n-1}$

$\Rightarrow\ \frac{1}{19683}=\frac{1}{3}\times\left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow\left(\frac{1}{3}\right)^{9}=\left(\frac{1}{3}\right)^{n}$

$\Rightarrow n=9$

Therefore, 9^{th} term of the given G.P. is $\frac{1}{19683}$.

6. For what values of x, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.?

Ans. Given: $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$\therefore \frac{x}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{x}$

$\Rightarrow x^2 = \frac{-2}{7} \times \frac{-7}{2}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

Therefore for $x = \pm 1$ the given numbers are in G.P

Find the sum to indicated number of terms in each of the geometric progression in Exercises 7 to 10:

7. 0.15, 0.015, 0.0015, ... 20 terms

Ans. Here, $a = 0.15$ and $r = \frac{0.015}{0.15} = \frac{1}{10}$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_{20} = \frac{0.15\left[1-\left(\frac{1}{10}\right)^{20}\right]}{1-\frac{1}{10}}$

$\Rightarrow S_{20} = \frac{15}{100} \times \frac{10}{9}\left[1-(0.1)^{20}\right]$

$\Rightarrow S_{20} = \frac{1}{6}\left[1-(0.1)^{20}\right]$

8. $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots\ldots\ldots n$ **terms**

Ans. Here, $a=\sqrt{7}$ and $r=\frac{\sqrt{21}}{\sqrt{7}}=\sqrt{3}$

$$\therefore \ S_n=\frac{a\left(r^n-1\right)}{r-1} \text{ when } r>1$$

$$\Rightarrow S_n=\frac{\sqrt{7}\left[\left(\sqrt{3}\right)^n-1\right]}{\sqrt{3}-1}$$

$$\Rightarrow S_n=\frac{\sqrt{7}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}\left[(3)^{\frac{n}{2}}-1\right]$$

$$\Rightarrow S_n=\frac{\sqrt{7}\left(\sqrt{3}+1\right)}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

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9. $1,-a,a^2,-a^3,$.................................... n**terms (if** $a\neq-1$**)**

Ans. Here, $a=1$ and $r=\frac{-\alpha}{1}=-\alpha$

$$\therefore \ S_n=\frac{a\left(1-r^n\right)}{1-r} \text{ when } r<1$$

$$\Rightarrow S_n=\frac{1\left[1-(-\alpha)^n\right]}{1-(-\alpha)}$$

$$\Rightarrow S_n=\frac{1}{1+\alpha}\left[1-(-\alpha)^n\right]$$

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10. $x^3,x^5x^7,$.................. n **terms (if** $x\neq\pm1$**)**

Ans. Here, $a=x^3$ and $r=\frac{x^5}{x^3}=x^2$

$\therefore\ S_n = \frac{a\left(1-r^n\right)}{1-r}$ when $r<1$

$\Rightarrow S_n = \frac{x^3\left[1-\left(x^2\right)^n\right]}{1-x^2}$

$\Rightarrow S_n = \frac{x^3}{1-x^2}\left[1-x^{2n}\right]$

11. Evaluate: $\sum_{k=1}^{11}\left(2+3^k\right)$

Ans. Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

= $\left(2+3^1\right)+\left(2+3^2\right)+\left(2+3^3\right)+........+\left(2+3^{11}\right)$

= $\left(2+2+2+......11 \text{ times}\right)+\left(3+3^2+3^3+.......+3^{11}\right)$

= $22+\left(3+3^2+3^3+.......+3^{11}\right)$ ....(i)

Here $3, 3^2, 3^3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, 3^{11}$ is in G.P.

$\therefore\ a=3$ and $r=\frac{3^2}{3}=3$

$\therefore\ S_n = \frac{3\left(3^{11}-1\right)}{3-1} = \frac{3}{2}\left(3^{11}-1\right)$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}\left(2+3^k\right) = 22+\frac{3}{2}\left(3^{11}-1\right)$

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common

ratio and the terms.

Ans. Let $\frac{a}{r}, a, r$ be first three terms of the given G.P.

According to question, $\frac{a}{r} + a + ar = \frac{39}{10}$ ....(i)

And $\frac{a}{r} \times a \times r = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Putting value of a in eq. (i), $\frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow 10 + 10r + 10r^2 = 39r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$$\Rightarrow r = \frac{-(-29) \pm \sqrt{(-29)^2 - 4 \times 10 \times 10}}{2 \times 10}$$

$$\Rightarrow r = \frac{29 \pm \sqrt{841 - 400}}{20}$$

$$\Rightarrow r = \frac{29 \pm 21}{20}$$

Taking $r = \frac{29+21}{20} = \frac{50}{20} = \frac{5}{2}$ and Taking $r = \frac{29-21}{20} = \frac{8}{20} = \frac{2}{5}$

When $r = \frac{5}{2}$, then first three terms are $\frac{1}{5/2}, 1, 1 \times \frac{5}{2}$

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$

When $r=\frac{2}{5}$, then first three terms are $\frac{1}{2/5}, 1, 1\times\frac{2}{5}$

$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$

13. How many terms of G.P. $3, 3^2, 3^3, \ldots\ldots\ldots$ are needed to give the sum 120 ?

Ans. Here, $\therefore\ a=3$ and $r=\frac{3^2}{3}=3$

$\therefore S_n=\frac{a(r^n-1)}{r-1}$ when $r>1$

$\Rightarrow 120=\frac{3(3^n-1)}{3-1}$

$\Rightarrow 120=\frac{3}{2}(3^n-1)$

$\Rightarrow 120\times\frac{2}{3}=3^n-1$

$\Rightarrow 3^n=81$

$\Rightarrow 3^n=(3)^4$

$\Rightarrow n=4$

Therefore, the sum of 4 terms of the given G.P. is 120.

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum of n terms of the G.P.

NA. Let a be the first term and r be the common ratio of given G.P.

$\therefore\ a+ar+ar^2=16$

$\Rightarrow\ a\left(1+r+r^2\right)=16$ ....(i)

And $ar^3+ar^4+ar^5=128$

$\Rightarrow\ ar^3\left(1+r+r^2\right)=128$ ....(ii)

Putting the value from eq. (i) into eq. (ii), we get

$16r^3=128$

$\Rightarrow r^3=8$

$\Rightarrow r=2$

Putting value of r in eq. (i), we get $a\left(1+2+2^2\right)=16$

$\Rightarrow a=\frac{16}{7}$

$\therefore\ S_n=\frac{a\left(r^n-1\right)}{r-1}$ when $r>1$

$\Rightarrow\ S_n=\frac{\frac{16}{7}\left(2^n-1\right)}{2-1}=\frac{16}{7}\left(2^n-1\right)$

15. Given a G.P. with a = 729 and 7$^{\text{th}}$ term 64, determine S_7.

Ans. Given: a = 729 and $a_7=64$

$\Rightarrow\ ar^6=64$

$\Rightarrow 729r^6=64$

$\Rightarrow r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6$

$\Rightarrow r = \frac{2}{3}$

$\therefore\ S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}} = \frac{729\left[1-\frac{128}{2187}\right]}{\frac{3-2}{3}}$

$\Rightarrow S_7 = 729 \times 3\left(\frac{2187-128}{2187}\right)$

$\Rightarrow S_7 = \frac{729 \times 3 \times 2059}{2187} = 2059$

16. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

Ans. Let a be the first term and r be the common ratio of given G.P.

Given: $a + ar = -4$

$\Rightarrow a(1+r) = -4$ ....(i)

And $a_5 = 4a_3$

$\Rightarrow ar^4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

Putting $r = 2$ in eq. (i), we get $a(1+2) = -4$

$\Rightarrow a = \frac{-4}{3}$

Therefore, required G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$

Putting $r = -2$ in eq. (i), we get $a(1-2) = -4$

$\Rightarrow a = 4$

Therefore, required G.P. is $4, -8, 16, -32, \ldots\ldots\ldots\ldots$

17. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P.

Ans. Let a be the first term and r be the common ratio of given G.P.

$\therefore\ a_4 = x$

$\Rightarrow ar^3 = x$ ....(i)

$a_{10} = y$

$\Rightarrow ar^9 = y$ ....(ii)

$a_{16} = z$

$\Rightarrow ar^{15} = z$ ....(iii)

From eq. (ii), $ar^9 = y$

$\Rightarrow \left(ar^9\right)^2 = y^2$

$\Rightarrow y^2 = \left(ar^3\right)\left(ar^{15}\right)$

$\Rightarrow y^2 = xz$ [From eq. (i) and (iii)]

$\therefore\ x, y, z$ are in G.P.

18. Find the sum to n terms of the sequences 8, 88, 888, 8888, ……

Ans. Here $S_n = 8 + 88 + 888 + 8888 + \ldots\ldots$ up to n terms

$$\Rightarrow S_n = 8(1 + 11 + 111 + 1111 + \ldots\ldots \text{ up to } n \text{ terms})$$

$$\Rightarrow S_n = \frac{8}{9}(9 + 99 + 999 + 9999 + \ldots\ldots \text{ up to } n \text{ terms})$$

$$\Rightarrow S_n = \frac{8}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1) + \ldots\ldots \text{ up to } n \text{ terms}\right]$$

$$\Rightarrow S_n = \frac{8}{9}\left[(10 + 10^2 + 10^3 + \ldots\ldots \text{ up to } n \text{ terms}) - (1 + 1 + 1 + \ldots\ldots \text{ up to } n \text{ terms})\right]$$

$$\Rightarrow S_n = \frac{8}{9}\left[\frac{10 \times (10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

$$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$$

19. Find the sum of the product of the corresponding terms of the sequences 2, 4, 8,16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Ans. Multiplying the corresponding terms of the given sequences 2, 4, 8,16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

$$(2 \times 128), (4 \times 32), (8 \times 8), (16 \times 2), \left(32 \times \frac{1}{2}\right)$$

$\Rightarrow 256, 128, 64, 32, 16$ are in G.P.

Here $a = 256$, $r = \frac{128}{256} = \frac{1}{2}$ and $n = 5$

$$\therefore \ S_n = \frac{a\left(1-r^n\right)}{1-r} \text{ when } r < 1$$

$$\Rightarrow S_5 = \frac{256\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = 256 \times 2\left(1-\frac{1}{32}\right)$$

$$\Rightarrow S_5 = 256 \times 2 \times \frac{31}{32} = 496$$

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20. Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots\ldots ar^{n-1}$ and $A, AR, AR^2, \ldots\ldots AR^{n-1}$ form a G.P. and find the common ratio.

Ans. Multiplying the corresponding terms of the given sequences, we have

$$(a \times A), (ar \times AR), (ar^2 \times AR^2), \ldots\ldots\ldots\ldots, (ar^{n-1} \times AR^{n-1})$$

$$\Rightarrow (aA), (aArR), (aAr^2R^2), \ldots\ldots\ldots\ldots, (aAr^{n-1}R^{n-1}) \text{ are in G.P.}$$

Now $\frac{a_2}{a_1} = \frac{aArR}{aA} = rR and \frac{a_3}{a_2} = \frac{aAr^2R^2}{aArR} = rR$

Since the ratio of the two succeeding terms are same, the resulting sequence is also in G.P

and common ratio = $\frac{aArR}{aA} = rR$

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21. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9 and the second term is greater than by 4^{th} by 18.

Ans. Let the four numbers in G.P. be a, ar, ar^2, ar^3

$\therefore\ ar^2 = a + 9$ and $ar = ar^3 + 18$

Now, $ar^2 - a = 9$

$\Rightarrow\ a\left(r^2 - 1\right) = 9$ ....(i)

And $ar - ar^3 = 18$

$\Rightarrow ar\left(1 - r^2\right) = 18$

$\Rightarrow -ar\left(r^2 - 1\right) = 18$ ....(ii)

Dividing eq. (ii) by eq. (i), we have

$$\frac{-ar\left(r^2-1\right)}{a\left(r^2-1\right)} = \frac{18}{9}$$

$\Rightarrow r = -2$

Putting value of r in eq. (i), we get

$a\left(4 - 1\right) = 9$

$\Rightarrow a = 3$

$\therefore\ ar = 3 \times \left(2\right) = -6$

$ar^2 = 3 \times \left(-2\right)^2 = 12$

$ar^3 = 3 \times \left(-2\right)^3 = -24$

Therefore, the required numbers are $3, -6, 12, -24$.

22. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c respectively. Prove that $a^{q-r} b^{r-p} c^{p-q} = 1$.

Ans. Let A be the first term and R be the common ratio of given G.P.

$\therefore\ a_p = a$

$\Rightarrow\ AR^{p-1} = a$ ....(i)

$a_q = b$

$\Rightarrow\ AR^{q-1} = b$ ....(ii)

$a_r = c$

$\Rightarrow\ AR^{r-1} = c$ ....(iii)

Now, L.H.S. = $a^{q-r}b^{r-p}c^{p-q} = \left(AR^{p-1}\right)^{q-r}.\left(AR^{q-1}\right)^{r-p}.\left(AR^{r-1}\right)^{p-q}$

= $A^{q-r}R^{(p-1)(q-r)}.A^{r-p}R^{(q-1)(r-p)}.A^{p-q}R^{(r-1)(p-q)}$

= $A^{q-r+r-p+p-q}R^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$

= $A^0R^0 = 1\times1$ = 1 = R.H.S.

23. If the first and the n^{th} term of a G.P. are a and b respectively and if P is the product of n terms, prove that $\mathbf{P}^2 = (ab)^n$.

Ans. Let r be the common ratio of the given G.P

Here, first term of G.P. is a

and $a_n = b$

$\Rightarrow ar^{n-1} = b$ ....(i)

Given: P = $a.ar.ar^2.ar^3..........ar^{n-1}$

$\Rightarrow$ P = $a^n.r^{1+2+3+......+n-1}$

$\Rightarrow p = a^n r^{\frac{n(n-1)}{2}}$

$\Rightarrow p^2 = a^{2n} r^{n(n-1)} = \left[aar^{n-1}\right]^n$ [Squaring both sides]

$\Rightarrow P^2 = (ab)^n$ [From eq. (i)]

Hence proved

24. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Ans. Let a be the first term and r be the common ratio of given G.P.

Then $\dfrac{\text{Sum of first } n \text{ terms}}{\text{Sum of terms from } (n+1)^{th} \text{ to } (2n)^{th}}$

$= \frac{a+ar+ar^2+................+ar^{n-1}}{ar^n+ar^{n+1}+................+ar^{2n-1}}$

$= \dfrac{a+ar+ar^2+..........+ar^{n-1}}{r^n\left[a+ar+ar^2+..........+ar^{n-1}\right]} = \dfrac{1}{r^n}$

25. If a, b, c and d are in G.P., show that $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right) = (ab+bc+cd)^2$.

Ans. Given a,b,c,d are in G.P

Let r be the common ratio of given G.P.

Then $b = ar, c = ar^2$ and $d = ar^3$

Now, L.H.S. = $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$

= $\left(a^2+a^2r^2+a^2r^4\right)\left(a^2r^2+a^2r^4+a^2r^6\right)$

= $a^2\left(1+r^2+r^4\right)a^2r^2\left(1+r^2+r^4\right)$ = $a^4r^2\left(1+r^2+r^4\right)^2$

R.H.S. = $\left(ab+bc+cd\right)^2$

= $\left(a.ar+ar.ar^2+ar^2.ar^3\right)^2$

= $\left(a^2r+a^2r^3+a^2r^5\right)^2$

= $\left(a^2r\right)^2\left(1+r^2+r^4\right)^2$ = $a^4r^2\left(1+r^2+r^4\right)^2$

Therefore, L.H.S. = R.H.S.

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26. Insert two numbers between 3 and 81 so that the resulting sequence us G.P.

Ans. Let $G_1 and\ G_2$ be two numbers between 3 and 81 such that $3,\ G_1,\ G_2,\ 81$ are in G.P.

Let r be the common ratio

Here $a=3$ and $a_4=81$

$\Rightarrow\ ar^3=81$

$\Rightarrow 3\times r^3=81$

$\Rightarrow r^3=27$

$\Rightarrow r=3$

$\therefore\ G_1=ar=3\times 3=9$

And $G_2=ar^2=3\times(3)^2=27$

Therefore, the required numbers are 9 and 27.

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27. Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Ans. Since, G.M. between two numbers a and b is $\sqrt{ab}$.

According to question, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$

$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n}=a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1}+b^{n+1}=(a^n+b^n)a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1}+b^{n+1}=a^{n+\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$

$$\Rightarrow a^{n+1}-a^{n+\frac{1}{2}}b^{\frac{1}{2}}=a^{\frac{1}{2}}b^{n+\frac{1}{2}}-b^{n+1}$$

$$\Rightarrow a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)=b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)$$

$$\Rightarrow a^{n+\frac{1}{2}}=b^{n+\frac{1}{2}}$$

$$\Rightarrow \frac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}}=1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}}=\left(\frac{a}{b}\right)^{0}$$

$$\Rightarrow n+\frac{1}{2}=0$$

$$\Rightarrow n=-\frac{1}{2}$$

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Ans. Let the numbers be a and b

Given: $a+b=6\sqrt{ab} \Rightarrow \frac{a+b}{2\sqrt{ab}}=\frac{3}{1}$

Applying componendo and dividendo, we get

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{3+1}{3-1}$$

$$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{4}{2}$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{2}}{1}$$

Again applying componendo and dividendo, we get

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Squaring both sides, $\frac{a}{b}=\frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$$\Rightarrow \frac{a}{b}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Therefore, the numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

29. If A and G be A.M. and G.M. respectively between two positive numbers, prove that the numbers are $A \pm\sqrt{(A+G)(A-G)}$.

Ans. Let the two positive numbers be a and b

Therefore $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$

Now, $A \pm \sqrt{(A+G)(A-G)} = A \pm \sqrt{A^2 - G^2}$

$= \frac{a+b}{2} \pm \sqrt{\left(\frac{a+b}{2}\right)^2 - \left(\sqrt{ab}\right)^2}$

$= \frac{a+b}{2} \pm \sqrt{\frac{a^2+b^2+2ab}{4} - ab}$

$= \frac{a+b}{2} \pm \sqrt{\frac{a^2+b^2+2ab-4ab}{4}}$

$= \frac{a+b}{2} \pm \sqrt{\frac{(a-b)^2}{4}} = \frac{a+b}{2} \pm \frac{a-b}{2}$

$= \frac{a+b}{2} + \frac{a-b}{2}$ and $\frac{a+b}{2} - \frac{a-b}{2}$

$= \frac{a+b+a-b}{2}$ and $\frac{a+b-a+b}{2}$

$= \frac{2a}{2} = a$ and $\frac{2b}{2} = b$

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and n^{th} hour?

Ans. Bacteria present in the culture originally = 30

Since the bacteria doubles itself after each hour, then the sequence of bacteria after each hour is a G.P.

Here $a = 30$ and $r = 2$

$\therefore$ Bacteria at the end of 2nd hour = $30 \times 2^{3-1} = 30 \times 2^2 = 120$

And Bacteria at the end of 4th hour = $30 \times 2^{5-1} = 30 \times 2^4 = 480$

And Bacteria at the end of n^{th} hour = $a_{n+1} = 30(2^{(n+1)-1}) = 30(2^n)$

31. What will Rs. 500 amount to 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Ans. Original amount = Rs. 500, Rate of interest = 10% compounded annually

$\therefore$ Interest of one year = $\frac{500 \times 10 \times 1}{100}$ = Rs. 50

And Amount after one year = 500 + 50 = Rs. 550

Here $a = 500$ and $r = \frac{550}{500} = 1.1$

Therefore, amount after 10 years =Amount in the 11th year=

$500 \times (1.1)^{11-1} = \text{Rs. } 500(1.1)^{10}$

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5 respectively then obtain the quadratic equation.

Ans. Let a and b be the roots of required quadratic equation.

Then A.M. = $\frac{a+b}{2} = 8$

$\Rightarrow a + b = 16$

And G.M. = $\sqrt{ab} = 5$

$\Rightarrow ab = 25$

Now, Quadratic equation $x^2 - (\text{Sum of roots})\,x + (\text{Product of roots}) = 0$

$\Rightarrow x^2 - (a+b)x + ab = 0$

$\Rightarrow x^2 - 16x + 25 = 0$

Therefore, required equation is $x^2 - 16x + 25 = 0$.