CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 8 Binomial Theorem

Exercise 8.1

Expand each of the expression in Exercises 1 to 5.

1. $(1-2x)^5$

Ans. Using Binomial Theorem,

$$(1-2x)^5 = {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + {}^5C_4(-2x)^4 + {}^5C_5(-2x)^5$$

$$= 1+5(-2x)+10(-2x)^2+10(-2x)^3+5(-2x)^4+(-2x)^5$$

$$= 1+5(-2x)+10(4x^2)+10(-8x^3)+5(16x^4)+(-32x^5)$$

$= 1-10x+40x^2-80x^3+80x^4-32x^5$ Ans.

2. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Ans. Using Binomial Theorem,

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 + {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right)^1 + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2 + {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)^1$$

$$= \frac{32}{x^5} + 5\left(\frac{16}{x^4}\right)\left(\frac{-x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x}{4}^2\right) + 10\left(\frac{4}{x^2}\right)\left(\frac{-x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x}{16}^4\right) + \left(\frac{-x^5}{32}\right)$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

3. $(2x-3)^6$

Ans. Using Binomial Theorem,

$(2x-3)^6 = {}^6C_0(2x)^6 + {}^6C_1(2x)^5(-3) + {}^6C_2(2x)^4(-3)^2 + {}^6C_3(2x)^3(-3)^3 + {}^6C_4(2x)^2(-3)^4$

$+{}^6C_5(2x)(-3)^5 + {}^6C_6(-3)^6$

=

$64x^6 + 6(32x^5)(-3) + 15(16x^4)(9) + 20(8x^3)(-27) + 15(4x^2)(81) + 6(2x)(-243) + 7$

= $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

---

4. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Ans. Using Binomial Theorem,

$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5$

= $\frac{x^5}{243} + 5.\frac{x^4}{81}.\frac{1}{x} + 10.\frac{x^3}{27}.\frac{1}{x^2} + 10.\frac{x^2}{9}.\frac{1}{x^3} + 5.\frac{x}{3}.\frac{1}{x^4} + \frac{1}{x^5}$

= $\frac{x^5}{243} + \frac{5}{81}x^3 + \frac{10}{27}x + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$

---

5. $\left(x+\frac{1}{x}\right)^6$

Ans. Using Binomial Theorem,

$\left(x+\frac{1}{x}\right)^6 = {}^6C_0.x^6 + {}^6C_1.x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 + {}^6C_3.x^3\left(\frac{1}{x}\right)^3 + {}^6C_4.x^2\left(\frac{1}{x}\right)^4 + {}^6C_5.x\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$

= $x^6 + 6x^5 \cdot \frac{1}{x} + 15x^4 \cdot \frac{1}{x^2} + 20x^3 \cdot \frac{1}{x^3} + 15x^2 \cdot \frac{1}{x^4} + 6x \cdot \frac{1}{x^5} + \frac{1}{x^6}$

= $x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

Using binomial theorem evaluate each of the following:

6. $(96)^3$

Ans. First we have to express 96 as the sum or difference of two numbers whose powers are easier to calculate and then use Binomial Theorem

We can write $96 = 100 - 4$

Therefore $(96)^3 = (100-4)^3$

Using Binomial Theorem,

$$(100-4)^3 = {}^3C_0(100)^3 + {}^3C_1(100)^2(-4) + {}^3C_2(100)(-4)^2 + {}^3C_3(-4)^3$$

$= (100)^3 + 3.10000(-4) + 3.100.16 + (-64)$

= 1000000 – 120000 + 4800 – 64

= 1004800 – 120064 = 884736

---

7. $(102)^5$

Ans. First we have to express 102 as the sum or difference of two numbers whose powers are easier to calculate and then use Binomial Theorem

We can write 102=100+2

Therefore $(102)^5 = (100+2)^5$

Using Binomial Theorem,

$$(100+2)^5 = {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$$

$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5.100(2)^4 + (2)^5$

= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32

= 11040808032

8. $(101)^4$

Ans. First we have to express 101 as the sum or difference of two numbers whose powers are easier to calculate and then use Binomial Theorem

We can write 101=100+1

Therefore $(101)^4 = (100+1)^4$

Using Binomial Theorem,

$$(100+1)^4 = {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)^1(1)^3 + {}^4C_4(1)^4$$

= $(100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + 1$

= 100000000 + 4000000 + 60000 + 400 + 1

= 104060401

9. $(99)^5$

Ans. First we have to express 99 as the sum or difference of two numbers whose powers are easier to calculate and then use Binomial Theorem

We can write $99 = 100 - 1$

Therefore $(99)^5 = (100-1)^5$

Using Binomial Theorem,

$$(100-1)^5 = {}^5C_0(100)^5 + {}^5C_1(100)^4(-1) + {}^5C_2(100)^3(-1)^2 + {}^5C_3(100)^2(-1)^3 + {}^5C_4(100)(-1)^4 + {}^5C_5(-1)^5$$

= $100^5 + 5(100)^4(-1) + 10(100)^3(1) + 10(100)^2(-1) + 5(100)(1) + (-1)$

= 10000000000 – 500000000 + 10000000 – 100000 + 500 – 1

= 9509900499

10. Using binomial theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans. We have $1.1 = 1 + 0.1$

$\therefore (1.1)^{10000} = (1+0.1)^{10000}$

Using Binomial Theorem,

$(1+0.1)^{10000} = 1 + {}^{10000}C_1(.1) + {}^{10000}C_2(0.1)^2 + {}^{10000}C_3(0.1)^3 + \ldots\ldots$

= 1 + 10000 (0.1) + other positive numbers

= 1 + 1000 + other positive numbers

which is greater than 1000

$\therefore (1.1)^{10000} > 1000$

11. Find $(a+b)^4 - (a-b)^4$. **Hence evaluate:** $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$.

Ans. Given: $(a+b)^4 - (a-b)^4$

Using Binomial Theorem,

$(a+b)^4 - (a-b)^4 = \left[{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4\right]$

$-\left[{}^4C_0a^4 + {}^4C_1a^3(-b) + {}^4C_2a^2(-b)^2 + {}^4C_3a(-b)^3 + {}^4C_4(-b)^4\right]$

= $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 - \left[a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\right]$

= $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 - a^4 + 4a^3b - 6a^2b^2 + 4ab^3 - b^4$

= $8a^3b + 8ab^3 = 8ab\left(a^2 + b^2\right)$

Putting $a=\sqrt{3}$ and $b=\sqrt{2}$

$$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=8\sqrt{3}.\sqrt{2}\left[\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2\right]$$

= $8\sqrt{6}[3+2]=40\sqrt{6}$ Ans.

---

12. Find $(x+1)^6+(x-1)^6$. Hence or otherwise evaluate $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$.

Ans. Given: $(x+1)^6+(x-1)^6$

Using Binomial Theorem,

$$(x+1)^6+(x-1)^6=\left[{}^6C_0x^6+{}^6C_1x^5(1)+{}^6C_2x^4(1)^2+{}^6C_3x^3(1)^3+{}^6C_4x^2(1)^4+{}^6C_5x(1)^5+{}^6C_6(1)^6\right]$$

$$+\left[{}^6C_0x^6+{}^6C_1x^5(-1)+{}^6C_2x^4(-1)^2+{}^6C_3x^3(-1)^3+{}^6C_4x^2(-1)^4+{}^6C_5x(-1)^5+{}^6C_6(-1)^6\right]$$

= $x^6+6x^5+15x^4+20x^3+15x^2+6x+1+x^6-6x^5+15x^4-20x^3+15x^2-6x+1$

= $2x^6+30x^4+30x^2+2$ = $2\left[x^6+15x^4+15x^2+1\right]$

Putting $x=\sqrt{2}$

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6=2\left[\left(\sqrt{2}\right)^6+15\left(\sqrt{2}\right)^4+15\left(\sqrt{2}\right)^2+1\right]$$

= $2\left[8+15\times4+15\times2+1\right]$

= 2 [8 + 60 + 30 + 1] = 2×99 = 198

---

13. Show that $9^{n+1}-8n-9$ is divisible by 64 whenever n is a positive integer.

Ans. We know that b is divisible by a(or a divides b) $\Rightarrow b=ak$, k is an integer

Here we have to show that 64 divides $9^{n+1}-8n-9$

$\Rightarrow 9^{n+1}-8n-9=64k$,k is an integer

We have $9^{n+1}=(1+8)^{n+1}$

Using Binomial Theorem,we have

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^nC_nx^n$

$\therefore 9^{n+1}=(1+8)^{n+1}={}^{n+1}C_0+{}^{n+1}C_1(8)+{}^{n+1}C_2(8)^2+{}^{n+1}C_3(8)^3+\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n+1}$

$=1+(n+1)\times 8+{}^{n+1}C_2(8)^2+{}^{n+1}C_3(8)^3+\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n+1}$

$=1+8n+8+{}^{n+1}C_2(8)^2+{}^{n+1}C_3(8)^3+\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n+1}$

$\Rightarrow 9^{n+1}-8n-9={}^{n+1}C_2(8)^2+{}^{n+1}C_3(8)^3+\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n+1}$

$\Rightarrow$
$9^{n+1}-8n-9=64\left[{}^{n+1}C_2+{}^{n+1}C_3(8)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n-1}\right]$

$\Rightarrow 9^{n+1}-8n-9=64k$,where $k={}^{n+1}C_2+{}^{n+1}C_3(8)+\ldots\ldots\ldots+{}^{n+1}C_{n+1}(8)^{n-1}$ is an integer

which shows that $9^{n+1}-8n-9$ is divisible by 64.

14. Prove that $\sum_{r=0}^{n} 3^r\ {}^nC_r=4^n$

Ans. L.H.S. = $\sum_{r=0}^{n} 3^r\ {}^nC_r=3^0\ {}^nC_0+3^1\ {}^nC_1+3^2\ {}^nC_2+\ldots\ldots\ldots+3^n\ {}^nC_n$

But we have $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^nC_nx^n$

$\therefore \sum_{r=0}^{n} 3^r\,{}^nC_r=(1+3)^n=4^n$

Hence proved