CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 7 Permutations and Combinations

Exercise 7.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digits is repeated?

Ans.

Given digits from 1 to 9 and repitition of digits are not allowed. We need to a three digit number.

9 different digits taken 3 at a time =$^9P_3 = \frac{9!}{3!\times 6!}$

$= \frac{9\times 8\times 7\times 6\,!}{6\,!} = 9\times 8\times 7 = 504.$

2. How many 4-digit numbers are there with no digit repeated?

Ans.

Thousand	Hundred	Tenth	Unit

Thousandth place can be arranged any of the digits from 1 to 9 because 0 cannot be included. So the number of ways to arrange thousandth place is 9.

The hundred, Tenths and Unit places can be arranged by any of the digits from 0 to 9 but digits cannot be repeated.

Thus the hundred,tenths and unit places can be arranged in 9 ways.

$\therefore$ there will be as many such 3 digit numbers as there are permutations 9 different digits taken 3 at a time.

$\therefore$ Number of such three digits $=>$ $^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\times 8\times 7\times 6!}{6!} = 9\times 8\times 7 = 504$

Thus by multiplication principle,

Total number of 4 digit numbers $= 9 \times 504 = 4536.$

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7 if no digit is repeated?

Ans. Total number of digits = 6 and unit place can be filled with any one of the digits 2, 4, 6.

$\therefore$ Number of ways to arrange unit place $= {}^3P_1 = \frac{3!}{2!} = \frac{3\times2\times1}{2\times1} = 3$

Now, the tens and hundreds places can be filled by remaining 5 digits.

$\therefore$ Number of ways to arrange the hundredth and tenth places $= {}^5P_2 = \frac{5!}{3!} = \frac{5\times4\times3\times2\times1}{3\times2\times1} = 20.$

Therefore, total number of 3 - digit even numbers $= 3 \times 20 = 60.$

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Ans.

Thousand	Hundred	Tenth	Unit

Even numbers always end with a digit of 0, 2, 4, 6 or 8

Given digits are 1, 2, 3, 4, 5, Here even numbers end with 2 and 4.

The number of ways in which unit place can be arranged is **2**

Since repetition of digits are not allowed, unit place is already arranged with one of the even numbers. So remaining places can be arranged with 4 digits.

Therefore, the number of ways to arrange remainig places is ${}^4P_3 = \frac{4!}{(4-3)!} = \frac{4\,!}{1\,!} = 4! = 24.$

Therefore, the required number of even numbers$= 2 \times 24 = 48.$

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Ans. We need to form a committee of 8 people with a chairman and a vice - chairman. It is evident that no committee contains two or more chairman and two or more vice - chairman.

So repetition is not allowed.

Therefore, the number of ways to choose a chairman and a vice - chairman is the permutation of 8 things taken 2 at a time.

Required number of ways $= ^8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8\times7\times6!}{6!} = 8\times7 = 56.$

6. Find n if $^{(n-1)}P_3 : ^nP_4 = 1:9$

Ans. Given: $^{(n-1)}P_3 : ^nP_4 = 1:9$

$\Rightarrow \frac{^{(n-1)}P_3}{^nP_4} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9} \Rightarrow n = 9.$

7. Find r if:

(i) $^5\mathrm{P}_r = 2\,^6\mathrm{P}_{r-1}$

(ii) $^5\mathrm{P}_r = ^6\mathrm{P}_{r-1}$

Ans. (i) Given: $^5\mathrm{P}_r = 2\,^6\mathrm{P}_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = 2\times\frac{6!}{(7-r)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{2\times6\times5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = \frac{2\times6}{(7-r)(6-r)}$

$\Rightarrow (7-r)(6-r) = 12$

$\Rightarrow 42 - 7r - 6r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 3r - 10r + 30 = 0$

$\Rightarrow (r-3)(r-10) = 0$

$\Rightarrow r = 3, 10$

$r = 10$,is not possible because $r > n$, therefore, $r = 3$

(ii) Given: ${}^5\mathrm{P}_r = {}^6\mathrm{P}_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6\times5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = \frac{6}{(7-r)(6-r)}$

$\Rightarrow (7-r)(6-r) = 6$

$\Rightarrow 42 - 7r - 6r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow (r-9)(r-4) = 0$

$\Rightarrow r = 9 \text{ or } r = 4$

$r = 9$ is not possible because $r > n$, therefore,$r = 4$.

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8. How many words, with or without meaning can be formed using all the letters of the word EQUATION, using each letter exactly once?

Ans. Total number of letters in word " EQUATION " = 8

Number of letters to arranged = 8 (All distinct)

So it is a permutation of 8 distinct letters taken 8 at a time.

$\therefore$ Number of permutations $={}^{8}P_{8}=\frac{8!}{(8-8)!}=\frac{8!}{0!}=8!$

$=8\times7\times6\times5\times4\times3\times2\times1=40320.$

9. How many words, with or without meaning can be formed using all the letters of the word MONDAY, assuming that no letter is repeated if:

(i) 4 letters are used at a time

(ii) all letters are used at a time

(iii) all letters are used by first letter is a vowel?

Ans. (ii) We can arrange 6 - letter words from the letters on " MONDAY " without repetition.

It is the permuation of 6 distinct letters taken 6 at a time.

Therefore, required number of words$={}^{6}P_{6}=\frac{6!}{(6-6)!}=\frac{6!}{0!}=6!$

$=6\times5\times4\times3\times2\times1=720.$

(i) Total number of letters in word " MONDAY " = 6

Number of letters to arranged = 4 (All distinct)

So it is a permutation of 6 distinct letters taken 4 at a time.

$\therefore$ Number of permutations $={}^{6}P_{4}=\frac{6!}{(6-2)!}=\frac{6!}{2!}$

=$6\times5\times4\times3=360$

(iii) Total number of letters in word MONDAY = 6 , four consonants M, N, D, Y and two vowels A and O.

There are two different ways to arrange the first letter. This can be done in ${}^{2}\mathrm{P}_{1}$ ways. Since the letters cannot be repeated and the first letter is already arranged with one of the vowels, so the remaining letters can be arranged with 5 distinct letters. This can be done in ${}^{5}\mathrm{P}_{5}$ ways.

Therefore total number of words formed $={}^{5}\mathrm{P}_{5}\times{}^{2}\mathrm{P}_{1}=5!\times2!$

$= 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 240.$

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Ans. In the word " MISSISSIPPI " ,letter " I " appears 4 times, letter " S " appears 4 times, letter " P " appears 2 times, letter " M " appears once.

Therefore , the number of distinct permutations of the given word " MISSISSIPPI " is

$= \frac{11\,!}{4\,!\times4\,!\times2\,!} = \frac{11\times10\times9\times8\times7\times6\times5\times4\,!}{4\,!\times4\times3\times2\times1\times2\times1}$

$= \frac{11\times10\times9\times8\times7\times6\times5}{4\times3\times2\times1\times2\times1} = 34650.$

Now, there are 4 I's in the word " MISSISSIPPI " . When they occur together , treat them as a single letter. This single letter and the remaining 7 letters together we get total 8 letters.

In this 8 letters there are 4 " S" and 2 " P " , So it can be arranged in $\frac{8!}{4!\times2!} = \frac{8\times7\times6\times5\times4!}{4!\times2\times1} = 840$

Number of distinct permutations = 34650 - 840 = 33810

11. In how many ways the letters of word PERMUTATIONS be arranged if the

(i) words start with P and end with S.

(ii) vowels are together.

(iii) there are always 4 letters between P and S

Ans. (i) In PERMUTATIONS number of letters = 12 and there are 2 T's in it.

No. of ways letters of PERMUTATIONS be arranged so that words always start from P and end with S = $\frac{10!}{2!}$ = 1814400

(ii) There are 5 vowels in the word PERMUTATIONS and if they are together are considered them as 1, so no. of letters then is 8

No. of ways vowels are together = $\frac{8!}{2!} \times 5!$ = $\frac{8\times7\times6\times5\times4\times3\times2!\times5\times4\times3\times2\times1}{2!}$=2419200

(iii) For 4 letters between P and S , P and S can be arranged in ways as shown:

P_ _ _ _S_ _ _ _ _ _   _ _P _ _ _ _S _ _ _ _   _P_ _ _ _S_ _ _ _ _   _ _ _P _ _ _ _S _ _ _

_ _ _ _P _ _ _ _S _ _   _ _ _ _ _P_ _ _ _S _   _ _ _ _ _ _P _ _ _ _S

As P can be filled in places 1,2,3,4,5,6 and 7 , consequently S can be filled in places 6,7,8,9,10,11 and 12 leaving 4 places between them. So P and S or S and P can be filled in

7 +7=14ways. Now the remaining 10 places can be filled in $\frac{10!}{2!}$ = 1814400

Therefore No. of ways in which 4 letters between P and S = 1814400 $\times$ 14= 2540100