CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 3 Trigonometric Functions

Exercise 3.3

Prove that:

1. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Ans. Taking L.H.S.

$= \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2=\frac{1}{4}+\frac{1}{4}-1$

$= \frac{1+1-4}{4}=\frac{-2}{4}=\frac{-1}{2}$ R.H.S.

2. $2\sin^2\frac{\pi}{6}+\cos ec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Ans. Taking L.H.S

$= 2\sin^2\frac{\pi}{6}+\cos ec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\sin^2\frac{\pi}{6}+\cos ec^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$= 2\sin^2\frac{\pi}{6}-\cos ec^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

= $2\times\left(\frac{1}{2}\right)^2+(-2)^2\times\left(\frac{1}{2}\right)^2$

= $2\times\frac{1}{4}+4\times\frac{1}{4}=\frac{6}{4}=\frac{3}{2}$

3. $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Ans. Taking L.H.S

= $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

= $\cot^2\frac{\pi}{6}+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\tan^2\frac{\pi}{6}$

= $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{\pi}{6}+3\tan^2\frac{\pi}{6}$

= $(\sqrt{3})^2+2+3\times(\frac{1}{\sqrt{3}})^2$

= $3+2+3\times\frac{1}{3}=5+1=6$ = R.H.S.

4. $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Ans. L.H.S. = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

= $2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

= $2\sin^2\frac{\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$= 2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times(2)^2$

$= 2\times\frac{1}{2}+2\times\frac{1}{2}+2\times4=1+1+8=10$ = R.H.S.

5. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Ans. (i) $\sin 75^\circ=\sin\left(45^\circ+30^\circ\right)=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ=\tan\left(45^\circ-30^\circ\right)=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$

$\left[\because \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$

$= \frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{3+1-2\sqrt{3}}{3-1}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$

Prove the following:

6. $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Ans. Taking L.H.S

$= \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$= \cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right]$

$\left[\because \cos(x+y)=\cos x\cos y-\sin x\sin y\right]$

$= \cos\left[\frac{\pi}{2}-(x+y)\right]=\sin(x+y)$ = R.H.S.

7. $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left[\frac{1+\tan x}{1-\tan x}\right]^2$

Ans. Taking L.H.S

$= \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

[Using $\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$, $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$]

$= \frac{\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}} = \frac{\frac{1+\tan x}{1-\tan x}}{\frac{1-\tan x}{1+\tan x}}$

= $\frac{(1+\tan x)^2}{(1-\tan x)^2}$ = R.H.S.

8. $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Ans. Taking L.H.S

= $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$

= $\frac{-\cos x.\cos x}{\sin x(-\sin x)}$ = $\frac{-\cos^2 x}{-\sin^2 x}=\cot^2 x$ = R.H.S.

9. $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

Ans. Taking L.H.S

= $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

= $\sin x.\cos x(\tan x+\cot x)$

= $\sin x.\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

= $\sin x.\cos x\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$

= 1 = R.H.S.

10. $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Ans. Taking L.H.S.

= $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

= $\cos\left[(n+1)x-(n+2)x\right]$

= $\cos\left[nx+x-nx-2x\right]$

= $\cos(-x)=\cos x$ = R.H.S.

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11. $cos\left(\frac{3\pi}{4}+x\right)-cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}sinx$

Ans. Taking L.H.S

= $cos\left(\frac{3\pi}{4}+x\right)-cos\left(\frac{3\pi}{4}-x\right)$

= $-2\sin\frac{3\pi}{4}\sin x$ = $-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$

= $-2\sin\frac{\pi}{4}\sin x$ = $-2\times\frac{1}{\sqrt{2}}\sin x$

= $-\sqrt{2}\sin x$ = R.H.S.

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12. $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

Ans. L.H.S. = $\sin^2 6x-\sin^2 4x$

= $\sin(6x+4x).\sin(6x-4x)$

$\left[\because \sin^2 x-\sin^2 y=\sin(x+y)\sin(x-y)\right]$

= $\sin 10x\sin 4x$ = R.H.S.

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13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Ans. L.H.S. = $\cos^2 2x - \cos^2 6x$

= $\sin(2x+6x).\sin(6x-2x)$

$\left[\because \cos^2 y - \cos^2 x = \sin(x+y)\sin(x-y)\right]$

= $\sin 8x \sin 4x$ = R.H.S.

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Ans. L.H.S. = $\sin 2x + 2\sin 4x + \sin 6x$

= $[\sin 4x + \sin 2x] + [\sin 6x + \sin 4x]$

= $2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + 2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)$

= 2sin3x.cosx + 2sin5x cos x

= $2\cos x[\sin 3x + \sin 5x]$

= $2\cos x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $2\cos x[2\sin 4x.\cos x]$

= $4\cos^2 x \sin 4x$ = R.H.S.

15. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Ans. L.H.S. = $\cot 4x(\sin 5x + \sin 3x)$

= $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $\frac{\cos 4x}{\sin 4x}[2\sin 4x\cos x]$ = $2\cos 4x\cos x$

R.H.S. = cot x(sin5x - sin3x)

= $\frac{cosx}{sinx}\left[2cos(\frac{5x+3x}{2})sin(\frac{5x-3x}{2})\right]$

= $\frac{cosx}{sinx}[2cos4xsinx]$

= $2\cos 4x\cos x$

$\therefore$ L.H.S. = R.H.S.

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16. $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$

Ans. L.H.S. = $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$

$$=\frac{-2sin(\frac{9x+5x}{2})sin(\frac{9x-5x}{2})}{2cos(\frac{17x+3x}{2})sin(\frac{17x-3x}{2})}$$

= $\frac{-2\sin 7x\sin 2x}{2\cos 10x\sin 7x}=-\frac{\sin 2x}{\cos 10x}$ = R.H.S.

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17. $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Ans. L.H.S. = $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$

$$=\frac{2sin(\frac{5x+3x}{2})cos(\frac{5x-3x}{2})}{2cos(\frac{5x+3x}{2})cos(\frac{5x-3x}{2})}$$

= $\frac{2\sin 4x}{2\cos 4x}=\tan 4x$ = R.H.S.

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18. $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\dfrac{x-y}{2}\right)$

Ans. L.H.S. = $\dfrac{\sin x-\sin y}{\cos x+\cos y}$

= $\dfrac{2\cos\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}$

= $\dfrac{\sin\left(\dfrac{x-y}{2}\right)}{\cos\left(\dfrac{x-y}{2}\right)}=\tan\left(\dfrac{x-y}{2}\right)$ = R.H.S.

19. $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Ans. L.H.S. = $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}$

= $\dfrac{2\sin\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}{2\cos\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}$

= $\dfrac{\sin 2x}{\cos 2x}=\tan 2x$ = R.H.S.

20. $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Ans. L.H.S. = $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

= $\dfrac{-(\sin 3x-\sin x)}{-(\cos^2 x-\sin^2 x)}$

= $\dfrac{2\cos\left(\dfrac{3x+x}{2}\right)\sin\left(\dfrac{3x-x}{2}\right)}{\cos 2x}$

= $\dfrac{2\cos 2x\sin x}{\cos 2x}=2\sin x$ = R.H.S.

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21. $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Ans. L.H.S. = $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$

= $\dfrac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$

= $\dfrac{2\cos\left(\dfrac{4x+2x}{2}\right)\cos\left(\dfrac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\dfrac{4x+2x}{2}\right)\cos\left(\dfrac{4x-2x}{2}\right)+\sin 3x}$

= $\dfrac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$

= $\dfrac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$ = $\cot 3x$ = R.H.S.

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22. $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Ans. We know that $\cot 3x=\cot(2x+x)$

$$\Rightarrow \cot 3x = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}$$

$$\Rightarrow \cot 3x(\cot 2x + \cot x) = \cot 2x \cot x - 1$$

$$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x = \cot 2x \cot x - 1$$

$$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x - \cot 2x \cot x + 1 = 0$$

$$\Rightarrow \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

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23. $\tan 4x = \dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x + \tan^4 x}$

Ans. L.H.S. = $\tan 4x = \dfrac{2\tan 2x}{1-\tan^2 2x}$

$$= \frac{2.\dfrac{2\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}{\left(1-\tan^2 x\right)^2}}$$

$$= \frac{4\tan x}{1-\tan^2 x} \times \frac{\left(1-\tan^2 x\right)^2}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$$ = R.H.S.

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24. $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

Ans. L.H.S. = $\cos 4x = 1 - 2\sin^2 2x$

= $1 - 2(2\sin x\cos x)^2$

= $1 - 2(4\sin^2 x\cos^2 x)$

= $1 - 8\sin^2 x\cos^2 x$ = R.H.S.

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25. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Ans. L.H.S. = $\cos 6x = 2\cos^2 3x - 1$

= $2\left[4\cos^3 x - 3\cos x\right]^2 - 1$

= $2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$

= $32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$

= $32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ = R.H.S.