CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 15 Statistics

Miscellaneous Exercise

1. The mean and variance of eight observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Ans. Let two required observations be x and y. Then,

According to question, $\dfrac{6+7+10+12+12+13+x+y}{8}=9$

$\Rightarrow 60+x+y=72$

$\Rightarrow x+y=12$ ....(i)

Also $\frac{1}{8}\left[6^2+7^2+10^2+12^2+12^2+13^2+x^2+y^2\right]-9^2=9.25$

$\Rightarrow \frac{1}{8}\left[36+49+100+144+144+169+x^2+y^2\right]-9^2=9.25$

$\Rightarrow 642+x^2+y^2=722$

$\Rightarrow x^2+y^2=80$ ....(ii)

$\because (x+y)^2+(x-y)^2=2\left(x^2+y^2\right)$

$\Rightarrow (12)^2+(x-y)^2=2\times 80$

$\Rightarrow (x-y)^2=16$

$\Rightarrow x-y=\pm 4$

When $x-y=4$, then on solving $x+y=12$ and $x-y=4$, we get $x=8$ and $y=4$

When $x-y=-4$, then on solving $x+y=12$ and $x-y=-4$, we get $x=4$ and $y=8$

2. The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

Ans. Let two required observations be x and y. Then,

According to question, $\frac{2+4+10+12+14+x+y}{7}=8$

$\Rightarrow 42+x+y=56$

$\Rightarrow x+y=14$ ....(i)

Also $\frac{1}{7}\left[2^2+4^2+10^2+12^2+14^2+x^2+y^2\right]-8^2=16$

$\Rightarrow \frac{1}{7}\left[4+16+100+144+196+x^2+y^2\right]-64=16$

$\Rightarrow 460+x^2+y^2=560$

$\Rightarrow x^2+y^2=100$ ....(ii)

$\because (x+y)^2+(x-y)^2=2\left(x^2+y^2\right)$

$\Rightarrow (14)^2+(x-y)^2=2\times 100$

$\Rightarrow (x-y)^2=4$

$\Rightarrow x-y=\pm 2$

When $x-y=2$, then on solving $x+y=14$ and $x-y=2$, we get $x=8$ and $y=6$

When $x-y=-2$, then on solving $x+y=14$ and $x-y=-2$, we get $x=6$ and $y=8$

3. The mean and standard deviation of six observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Ans. Let six observations be $x_1, x_2, x_3, x_4, x_5, x_6$, then

According to questions, $\frac{x_1+x_2+x_3+x_4+x_5+x_6}{6}=8$

$\Rightarrow x_1+x_2+x_3+x_4+x_5+x_6=48$

Now, if each observation is multiplied by 3, then

new observations are $3x_1$, $3x_2$, $3x_3$, $3x_4$, $3x_5$ and $3x_6$

New mean = $\frac{3(x_1+x_2+x_3+x_4+x_5+x_6)}{6}=\frac{1}{2}\times 48$ = 24

Also $\frac{1}{6}\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2\right)-(8)^2=16$

$\Rightarrow x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2=480$

Now, if each observation is multiplied by 3, then

New Variance = $\frac{1}{6}\times(3)^2\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2\right)-(24)^2=\frac{9}{6}\times 480-576$

= 720 – 576 = 144

Therefore, New S.D. = $\sqrt{144}$ = 12

4. Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, \ldots\ldots\ldots, x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, \ldots\ldots\ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$ respectively $(a \neq 0)$.

Ans. Given: $\bar{x}=\frac{x_1+x_2+x_3+x_4+\ldots\ldots\ldots\ldots+x_n}{n}=\frac{\sum x}{n}$

Also $\frac{x_1^2+x_2^2+x_3^2+x_4^2+\ldots\ldots\ldots\ldots+x_n^2}{n}=\frac{\sum x^2}{n}$

New Mean =

$$\bar{x} = \frac{ax_1 + ax_2 + ax_3 + ax_4 + \ldots\ldots\ldots + ax_n}{n} = \frac{a(x_1 + x_2 + x_3 + x_4 + \ldots\ldots\ldots + x_n)}{n} = a\bar{x}$$

Also $\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$

New Variance = $\sigma'^2 = \frac{\sum_{i=1}^{n} (ax_i)^2}{n} - \left(\frac{\sum_{i=1}^{n} ax_i}{n}\right)^2$

$$\sigma'^2 = a^2\left[\frac{\sum_{i=1}^{n} {x_i}^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2\right]$$

= $a^2\sigma^2$

5. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) if wrong item is omitted

(ii) if it is replaced by 12.

Ans. Given: $n = 20, \bar{x} = 10$ and $\sigma = 2$

Now, Mean = $\frac{\sum x_i}{20} = 10$

$\Rightarrow \sum x_i = 20 \times 10$

$\Rightarrow \sum x_i = 200$

$\therefore$ Incorrect $\sum x_i = 200$

Now $\frac{1}{n}\sum x_i^2 - (\bar{x})^2 = 4$

$\Rightarrow \frac{1}{20}\sum x_i^2 - (10)^2 = 4$

$\Rightarrow \sum x_i^2 = 2080$

(i) If wrong item is omitted: then we have 19 observations.

$\therefore$ Correct $\sum x_i$ = Incorrect $\sum x_i - 8$

$\Rightarrow$ Correct $\sum x_i$ = 200 – 8 = 192

$\therefore$ Correct Mean = $\frac{192}{19}$ = 10.11

Also Correct $\sum x_i^2$ = Incorrect $\sum x_i^2 - (8)^2 = 2080 - 64 = 2016$

$\therefore$ Correct Variance = $\frac{1}{19}$ (Correct $\sum x_i^2$) – (Correct mean)^2

= $\frac{1}{19} \times 2016 - \left(\frac{192}{19}\right)^2$ = $\frac{2016}{19} - \frac{36864}{361}$ = $\frac{1440}{361}$

Correct S.D. = $\sqrt{\frac{1440}{361}}$ = 1.997

(ii) If it is replaced by 12: then

$\therefore$ Correct $\sum x_i$ = Incorrect $\sum x_i - 8 + 12$

$\Rightarrow$ Correct $\sum x_i$ = 200 – 8 + 12 = 204

$\therefore$ Correct Mean = $\frac{204}{20}$ = 10.2

Also Correct $\sum x_i^2$ = Incorrect $\sum x_i^2 - (8)^2 + (12)^2 = 2080 - 64 + 144 = 2160$

$\therefore$ Correct Variance = $\frac{1}{20}$ (Correct $\sum x_i^2$) $-\left(Correct\ mean\right)^2$

$= \frac{2160}{20} - (10.2)^2$ =108-104.04=3.96

Correct S.D. = $\sqrt{3.96}$ = 1.989

6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Ans. For Mathematics, $\bar{x}$ = 42 and σ = 12

$\therefore$ C.V. of Mathematics = $\frac{12}{42} \times 100$ = 28.57%

For Physics, $\bar{x}$ = 32 and σ = 15

$\therefore$ C.V. of Physics = $\frac{15}{32} \times 100$ = 46.88%

For Chemistry, $\bar{x}$ = 40.9 and σ = 20

$\therefore$ C.V. of Chemistry = $\frac{20}{40.9} \times 100$ = 48.9%

Therefore, Chemistry with highest C.V. shows highest variability and Mathematics with lowest C.V. shows lowest variability.

7. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observations were incorrect, which

were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Ans. Given: $n = 100, \bar{x} = 20$ and $\sigma = 3$

Now, $\bar{x} = \frac{1}{n}\sum x_i = 20$

$\Rightarrow \frac{1}{100}\sum x_i = 20$

$\Rightarrow \sum x_i = 2000$

$\therefore$ Incorrect $\sum x_i = 2000$

Now $\frac{1}{n}\sum x_i^2 - (\bar{x})^2 = 9$

$\Rightarrow \frac{1}{100}\sum x_i^2 - (20)^2 = 9$

$\Rightarrow \sum x_i^2 = 40900$

When wrong items 21, 21 and 18 are omitted from the data, then there will be 97observations.

$\therefore$ Correct $\sum x_i$ = Incorrect $\sum x_i - 21 - 21 - 18$

$\Rightarrow$ Correct $\sum x_i$ = 2000 – 21 – 21 – 18 = 1940

$\therefore$ Correct Mean = $\frac{1940}{97}$ = 20

Also Correct $\sum x_i^2$ = Incorrect $\sum x_i^2 - (21)^2 - (21)^2 - (18)^2$

$= 40900 - 441 - 441 - 324 = 39694$

$\therefore$ Correct Variance = $\frac{1}{97}$ (Correct $\sum x_i^2$) – (Correct mean)^2

= $\frac{1}{97} \times 39694 - (20)^2$ = $409.22 - 400$ = 9.22

Correct S.D. = $\sqrt{9.22}$ = 3.036