CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 14 Mathematical Reasoning

Exercise 14.5

1. Show that the statement

p: **"If x is a real number such that $x^3+4x=0$ then x is 0" is true by (i) direct method (ii) method of contradiction (iii) method of contrapositive.**

Ans. The given compound statement is of the form "if p then q".

$p: x \in$ R such that $x^3+4x=0$

$q: x=0$

(i) Direct method: If we assume that p is true then

$x \in$ R such that $x^3+4x=0$

$\Rightarrow x \in$ R such that $x\left(x^2+4\right)=0$

$\Rightarrow x \in$ R such that $x=0$ or $x^2+4=0$

$\Rightarrow x=0$ as $x^2+4 \neq 0$

$\Rightarrow q$ is true, i.e., when p is true then q is true.

Therefore, the given compound statement is true.

(ii) Method of contradiction: If we assume that p is true and q is false, which means say $x \neq 0$

then if

$x \in$ R such that $x^3+4x=0$

$\Rightarrow x \in$ R such that $x\left(x^2+4\right)=0$

$\Rightarrow x \in$ R such that $x = 0$ or $x^2 + 4 = 0$

$\Rightarrow x = 0$ as $x^2 + 4 \neq 0$

which is contradiction to our assumption that $x \neq 0$.

Therefore, the given compound statement is true.

(iii) Method of contrapositive: If we assume that q is false, then

$x \neq 0$ $x \in$ R such that $x^3 + 4x \neq 0$

$\Rightarrow p$ is false, i.e., if q is false, then p is false.

Therefore, the given compound statement is true.

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2. Show that the statement "For any real numbers a and b, $a^2 = b^2$ implies that $a = b$" is not true by giving a counter example.

Ans. The given compound statement is of the form "if p then q".

If we assume that p is true then, $a, b \in$ R such that $a^2 = b^2$

Let $a = -3$ and $b = 3$

Now $a^2 = b^2$ but $a \neq b$ therefore, when p is true, then q is false.

Therefore, the given compound statement is not true.

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3. Show that the following statement is true by the method of contrapositive.

p : **"If x is an integer and x^2 is even, then x is also even"**

Ans. The given compound statement is of the form "if p then q".

$p : x \in$ Z and x^2 is even.

$q : x$ is an even integer.

If we assume that q is false then x is not an even integer.

$\Rightarrow$ x is an odd integer.

$\Rightarrow$ x^2 is an odd integer.

$\Rightarrow$ p is false, i.e., when q is false, then p is false.

Therefore, the given compound statement is true.

Alternative Solution:

Ans. The given compound statement is of the form "if p then q".

$p: x \in$ Z and x^2 is even.

$q: x$ is an even integer.

If we assume that q is false then x is not an even integer.

Let x = 2n+1 then;

$x^2 = (2n+1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1$

As $4(n^2 + n)$ is an even number therefore $4(n^2 + n) + 1$ is an odd number.

i.e " If q is not true then p is not true"

Therefore, the given compound statement is true.

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4. By giving a counter example, show that the following statements are not true:

(i) p: "If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle".

(ii) q: "The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2".

Ans. (i) Since the triangle is obtuse angled triangle means $\theta > 90^\circ$

Let $\theta = 100^\circ$

Also all the angles of the triangle are equal.

$\therefore$ Sum of all angles of a triangle = $300^0 > 180^0$ which is not possible.

Therefore, the given compound statement is not true.

(ii) We see that $x = 1$ is a root of the equation then $x^2 - 1 = 0$,

which lies between 0 and 2. Therefore, the given compound statement is not true.

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5. Which of the following statements are true and which are false? In each case give a valid reason for saying so.

(i) p : **Each radius of a circle is a chord of the circle.**

(ii) q : **The centre of a circle bisects each chord of the circle.**

(iii) r : **Circle is a particular case of an ellipse.**

(iv) s : **If** x **and** y **are integers such that** $x > y$ **then** $-x < -y$.

(v) t : $\sqrt{11}$ **is a rational number.**

Ans. (i) A chord of a circle is a line whose two end points lie on the circles and all the points on the line lie inside the circle.

So the radius of the circle is not a chord of the circle.

Therefore, the given statement is false.

(ii) The centre of a circle bisects chord of circle when the chord is diameter of the circle.

When the chord is other than diameter then centre of the circle does not lie on the chord.

Therefore, the given statement is false.

(iii) In the equation of an ellipse if we put, $a = b$ then we get an equation of circle.

i.e when $a = b$

The equation of ellipse becomes $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1\Rightarrow x^2+y^2=a^2$which is the equation of a circle

Therefore, the given statement is true.

(iv) It is given that $x, y \in$ Z such that $x > y$

Multiplying both sides by negative sign, then $x, y \in$ Z such that $-x < -y$ (by the rule of inequality)

Therefore, the given statement is true.

(v) Since 11 is a prime number and $\sqrt{11}$ cannot be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Therefore, the given statement is false.