CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 11 Conic Sections

Exercise 11.1

In each of the following Exercises 1 to 5, find the equation of the circle with:

1. Centre (0, 2) and radius 2.

Ans. Given: $h=0, k=2$ and $r=2$

Equation of the circle;

$(x-h)^2+(y-k)^2=r^2$

$\therefore (x-0)^2+(y-2)^2=(2)^2$

$\Rightarrow x^2+y^2+4-4y=4$

$\Rightarrow x^2+y^2-4y=0$

2. Centre $(-2, 3)$ and radius 4.

Ans. Given: $h=-2, k=3$ and $r=4$

Equation of the circle ;

$(x-h)^2+(y-k)^2=r^2$

$\therefore (x+2)^2+(y-3)^2=(4)^2$

$\Rightarrow x^2+4+4x+y^2+9-6y=16$

$\Rightarrow x^2+y^2+4x-6y-3=0$

3. Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$.

Ans. Given: $h = \frac{1}{2}, k = \frac{1}{4}$ and $r = \frac{1}{12}$

Equation of the circle ;

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\therefore \left(x-\frac{1}{2}\right)^2 + \left(y-\frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$\Rightarrow x^2 + \frac{1}{4} - x + y^2 + \frac{1}{16} - \frac{1}{2}y = \frac{1}{144}$$

$$\Rightarrow 144x^2 + 36 - 144x + 144y^2 + 9 - 72y = 1$$

$$\Rightarrow 144x^2 + 144y^2 - 144x - 72y + 44 = 0$$

$$\Rightarrow 4\left(36x^2 + 36y^2 - 36x - 18y + 11\right) = 0$$

$$\Rightarrow 36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

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4. Centre $(1,1)$ and radius $\sqrt{2}$.

Ans. Given: $h = 1, k = 1$ and $r = \sqrt{2}$

Equation of the circle;

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\therefore (x-1)^2 + (y-1)^2 = \left(\sqrt{2}\right)^2$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 - 2y = 2$$

$\Rightarrow x^2+y^2-2x-2y=0$

5. Centre $(-a,-b)$ and radius $\sqrt{a^2-b^2}$.

Ans. Given: $h=-a, k=-b$ and $r=\sqrt{a^2-b^2}$

Equation of the circle ;

$(x-h)^2+(y-k)^2=r^2$

$\therefore (x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$

$\Rightarrow x^2+a^2+2ax+y^2+b^2+2by=a^2-b^2$

$\Rightarrow x^2+y^2+2ax+2by+2b^2=0$

In each of the following Exercises 6 to 9, find the centre and radius of the circles.

6. $(x+5)^2+(y-3)^2=36$

Ans. Given: Equation of the circle;

$(x+5)^2+(y-3)^2=36$

$\Rightarrow (x+5)^2+(y-3)^2=(6)^2$ ....(i)

On comparing eq. (i) with $(x-h)^2+(y-k)^2=r^2$

$h=-5, k=3$ and $r=6$

7. $x^2+y^2-4x-8y-45=0$

Ans. Given: Equation of the circle: $x^2+y^2-4x-8y-45=0$

$\Rightarrow \left(x^2-4x\right)+\left(y^2-8y\right)=45$

$\Rightarrow \left(x^2-4x+2^2\right)+\left(y^2-8y+4^2\right)=45+2^2+4^2$

$=> (x-2)^2+(y-4)^2=(\sqrt{65})^2 \ldots\ldots.(i)$

On comparing eq. (i) with $(x-h)^2+(y-k)^2=r^2$

$h=2, k=4$ and $r=\sqrt{65}$

8. $x^2+y^2-8x-10y-12=0$

Ans. Given: Equation of the circle;

$=> (x^2-8x)+(y^2-10y)=12$

$=> (x^2-8x+4^2)+(y^2-10y+5^2)=12+4^2+5^2$

$=> (x-4)^2+(y-5)^2=(\sqrt{53})^2$ .... (i)

On comparing eq. (i) with $(x-h)^2+(y-k)^2=r^2$

We get, h = 4 , k = 5 and $r=\sqrt{53}$

9. $2x^2+2y^2-x=0$

Ans. Given: Equation of the circle: $2x^2+2y^2-x=0$

$\Rightarrow x^2+y^2-\frac{x}{2}=0$ (divide the equation by 2)

$\Rightarrow \left(x^2-\frac{x}{2}\right)+y^2=0$

$\Rightarrow \left[x^2-\frac{x}{2}+\left(\frac{1}{4}\right)^2\right]+y^2=0+\left(\frac{1}{4}\right)^2$

$\Rightarrow (x - \frac{1}{4})^2 + (y - 0)^2 = (\frac{1}{4})^2$.......(i)

On comparing eq. (i) with $(x-h)^2 + (y-k)^2 = r^2$

$h = \frac{1}{4}, k = 0$ and $r = \frac{1}{4}$

10. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre lies on the line $4x + y = 16$.

Ans. The equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$ ....(i)

$\because$ Circle passes through point (4, 1)

$\therefore (4-h)^2 + (1-k)^2 = r^2$

$\Rightarrow 16 + h^2 - 8h + 1 + k^2 - 2k = r^2$

$\Rightarrow h^2 + k^2 - 8h - 2k + 17 = r^2$ ....(ii)

Again Circle passes through point (6, 5)

$\therefore (6-h)^2 + (5-k)^2 = r^2$

$\Rightarrow 36 + h^2 - 12h + 25 + k^2 - 10k = r^2$

$\Rightarrow h^2 + k^2 - 12h - 10k + 61 = r^2$ ....(iii)

From eq. (ii) and (iii), we have

$h^2 + k^2 - 8h - 2k + 17 = h^2 + k^2 - 12h - 10k + 61$

$\Rightarrow 4h + 8k = 44$

$\Rightarrow h + 2k = 11$ ....(iv)

Since the centre (h, k) of the circle lies on the line $4x + y = 16$

$\therefore 4h + k = 16$ ....(v)

On solving eq. (iv) and (v), we have $h = 3, k = 4$

Putting the values of h and k in eq. (ii), we have

$3^2 + 4^2 - 8 \times 3 - 2 \times 4 + 17 = r^2$

$\Rightarrow r^2 = 9 + 16 - 24 - 8 + 17 = 10$

Therefore, the equation of the required circle is

$(x-3)^2 + (y-4)^2 = 10$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y = 10$

$\Rightarrow x^2 + y^2 - 6x - 8y + 15 = 0$

11. Find the equation of the circle passing through the points (2, 3) and $(-1, 1)$ and whose centre lies on the line $x - 3y - 11 = 0$.

Ans. The equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$ ....(i)

$\because$ Circle passes through point (2, 3)

$\therefore (2-h)^2 + (3-k)^2 = r^2$

$\Rightarrow 4 + h^2 - 4h + 9 + k^2 - 6k = r^2$

$\Rightarrow h^2 + k^2 - 4h - 6k + 13 = r^2$ ....(ii)

Again Circle passes through point (–1, 1)

$\therefore (-1-h)^2 + (1-k)^2 = r^2$

$\Rightarrow 1 + h^2 + 2h + 1 + k^2 - 2k = r^2$

$\Rightarrow h^2 + k^2 + 2h - 2k + 2 = r^2$ ....(iii)

From eq. (ii) and (iii), we have

$=> h^2+k^2-4h-6k+13=h^2+k^2+2h-2k+2$

$\Rightarrow -6h-4k=-11$

$\Rightarrow 6h+4k=11$ ....(iv)

Since the centre (h,k) of the circle lies on the line $x-3y-11=0$

$\therefore h-3k=11$..........(v)

On solving eq. (iv) and (v), we have $h=\frac{7}{2}, k=\frac{-5}{2}$

Putting the values of h and k in eq. (ii), we have

$\left(\frac{7}{2}\right)^2+\left(\frac{-5}{2}\right)^2-\frac{4\times7}{2}-6\times\frac{-5}{2}+13=r^2$

$\Rightarrow \frac{49}{4}+\frac{25}{4}-14+15+13=r^2$

$\Rightarrow r^2=\frac{65}{2}$

Therefore, the equation of the required circle is

$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}$

$\Rightarrow x^2+\frac{49}{4}-7x+y^2+\frac{25}{4}+5y=\frac{65}{2}$

$\Rightarrow 4x^2+49-28x+4y^2+25+20y=130$

$\Rightarrow 4x^2+4y^2-28x+20y-56=0$

$\Rightarrow 4\left(x^2+y^2-7x+5y-14\right)=0$

$\Rightarrow x^2+y^2-7x+5y-14=0$

12. Find the equation of the circle with radius 5 whose centre lies on $x-$ axis and passes through the point (2, 3).

Ans. Since the centre of circle lies on $x-$ axis, therefore the coordinates of centre is $(h,0)$.

Now the circle passes through the point (2, 3). According to the question,

$\sqrt{(h-2)^2+(0-3)^2}=5$

$\Rightarrow \sqrt{h^2+4-4h+9}=5$

$\Rightarrow h^2+4-4h+9=25$

$\Rightarrow h^2-4h-12=0$

$\Rightarrow (h-6)(h+2)=0$

$\Rightarrow h=6$ or $h=-2$

Taking $h=6$, Equation of the circle is $(x-6)^2+(y-0)^2=(5)^2$

$\Rightarrow x^2+36-12x+y^2=25$

$\Rightarrow x^2+y^2-12x+11=0$

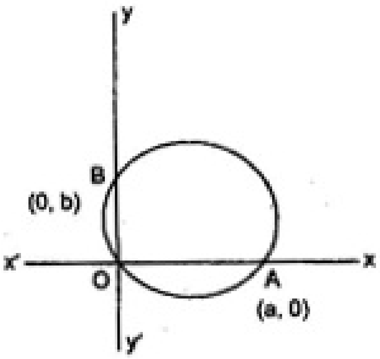
Taking $h=-2$, Equation of the circle is $(x+2)^2+(y-0)^2=(5)^2$

$\Rightarrow x^2+4+4x+y^2=25$

$\Rightarrow x^2+y^2+4x-21=0$

13. Find the equation of the circle passing through (0, 0) and making intercept a and b on the coordinate axes.

Ans. The circle makes intercepts a with $x-$axis and b with $y-$axis.



$\therefore$ OA = a and OB = b

$\therefore$ Coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.

Now the circle passes through the points O (0, 0), A $(a, 0)$ and B $(0, b)$.

Putting these coordinates of three points in the equation of the circle,

$x^2+y^2+2gx+2fy+c=0$ ....(i)

Circle passing through (0,0)

$\Rightarrow c=0$

the circle also passes through (a,0) and (0,b)

$a^2+2ga=0$

$\Rightarrow a(a+2g)=0$

$\Rightarrow g=\frac{-1}{2}a$

And $b^2+2fb=0$

$\Rightarrow b(b+2f)=0$

$\Rightarrow f = \frac{-1}{2}b$

Putting the values of g, f and c in eq. (i), we have

$x^2 + y^2 + 2 \times \frac{-1}{2}ax + 2 \times \frac{-1}{2}by + 0 = 0$

$\Rightarrow x^2 + y^2 - ax - by = 0$

14. Find the equation of the circle with centre (2, 2) and passes through the point (4, 5).

Ans. The equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$..........(i)

Since the circle passes through point (4, 5) and coordinates of centre are (2, 2).

$\therefore$ Radius of circle = $\sqrt{(4-2)^2 + (5-2)^2}$ = $\sqrt{4+9}$ = $\sqrt{13}$

Therefore, the equation of the required circle is

$(x-2)^2 + (y-2)^2 = \left(\sqrt{13}\right)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 4 - 4y = 13$

$\Rightarrow x^2 + y^2 - 4x - 4y - 5 = 0$

15. Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

Ans. Given: Equation of the circle $x^2 + y^2 = 25$

$\Rightarrow (x-0)^2 + (y-0)^2 = (5)^2$

On comparing with $(x-h)^2 + (y-k)^2 = r^2$, we have $h = 0, k = 0$ and $r = 5$

Now distance of the point $(-2.5, 3.5)$ from the centre (0, 0)

= $\sqrt{(0+2.5)^2 + (0-3.5)^2}$ = $\sqrt{6.25 + 12.25}$ = $\sqrt{18.5}$ = 4.3 <

Therefore, the point $(-2.5, 3.5)$ lies inside the circle.