

CBSE Class-11 Mathematics
NCERT Solutions
Chapter - 7 Permutations and Combinations
Exercise 7.4

1. If ${}^nC_8 = {}^nC_2$, find nC_2 .

Ans. Given: ${}^nC_8 = {}^nC_2$

$$\Rightarrow {}^nC_8 = {}^nC_{n-2}$$

$$\Rightarrow 8 = n - 2$$

$$\Rightarrow n = 10$$

$$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!8!} = 45$$

2. Determine if:

(i) ${}^{2n}C_3 : {}^nC_2 = 12 : 1$

(ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

Ans. (i) Given: ${}^{2n}C_3 : {}^nC_2 = 12 : 1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3 \times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{12}{1}$$

$$\Rightarrow \frac{4(2n-1)}{3} = \frac{12}{1}$$

$$\Rightarrow 8n - 4 = 36$$

$$\Rightarrow n = 5$$

(ii) Given: ${}^{2n}C_3 : {}^nC_3 = 11:1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-3)!}{n!} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4(2n-1)}{(n-2)} = \frac{11}{1}$$

$$\Rightarrow 8n - 4 = 11n - 22$$

$$\Rightarrow 3n = 18$$

$$\Rightarrow n = 6$$

3. How many chords can be drawn through 21 points on a circle?

Ans. There are 21 points on the circumference of a circle. Since one and only one chord can be drawn by joining 2 distinct points, therefore the required number of chords given by

$${}^{21}C_2 = \frac{21!}{2!19!} = \frac{21 \times 20 \times 19!}{2!19!} = 210$$

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4

girls?

Ans. There are 5 boys and 4 girls and 3 out of 5 boys and 3 out of 4 girls have to be selected.

$$\therefore \text{Number of ways of selection} = {}^5C_3 \times {}^4C_3$$

$$= \frac{5!}{3!2!} \times \frac{4!}{3!1!} = 10 \times 4 = 40$$

5. Find the number ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans. There are 6 red balls, 5 white balls and 5 blue balls. 3 balls of each colour have to be found.

$$\therefore \text{Number of ways of selection} = {}^6C_3 \times {}^5C_3 \times {}^5C_3$$

$$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$

$$= 20 \times 10 \times 10 = 2000$$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans. There are 4 aces and 48 other cards in a deck of 52 cards. We have to select 1 ace out of 4 aces and 4 other cards out of 48 other cards.

$$\therefore \text{Number of ways of selection} = {}^4C_1 \times {}^{48}C_4$$

$$= \frac{4!}{1!3!} \times \frac{48!}{44!4!}$$

$$= \frac{4 \times 3!}{3!} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4 \times 3 \times 2 \times 44!} = 778320$$

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans. There are 5 bowlers and 12 other players in a team of 17 players we have to select 4 bowlers out of 5 bowlers and 7 other players out of 12 other players.

$$\therefore \text{Number of ways of selection} = {}^5C_4 \times {}^{12}C_7$$

$$= \frac{5!}{1!4!} \times \frac{12!}{7!5!}$$

$$= \frac{5 \times 4!}{4!} \times \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{5 \times 4 \times 3 \times 2 \times 1 \times 7!}$$

$$= 5 \times 792 = 3960$$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans. There are 5 black and 6 red balls. We have to select 2 black balls out of 5 black balls and 3 red balls out of 6 red balls.

$$\therefore \text{Number of ways of selection} = {}^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!}$$

$$= \frac{5 \times 4 \times 3!}{2!3!} \times \frac{6 \times 5 \times 4 \times 3!}{3!3!} = 200$$

9. In how many ways can a student choose a program of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Ans. There are 9 courses and number of courses to be selected are 5 in which 2 specific courses are compulsory. 3 courses out of remaining 7 courses have to be selected.

$$\therefore \text{Number of ways of selection} = {}^7C_3 = \frac{7!}{4!3!}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$