CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 8 Binomial Theorem

Miscellaneous Exercise

1. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively.

Ans. We have ,

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + {}^nC_3a^{n-3}b^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^n$

Given: $T_1 = {}^nC_0a^nb^0 = 729 \Rightarrow a^n = 729$ .....(i)

$T_2 = {}^nC_1a^{n-1}b = 7290$

$\Rightarrow na^{n-1}b = 7290$ .....(ii)

And $T_3 = {}^nC_2a^{n-2}b^2 = 30375$

$\Rightarrow \frac{n(n-1)}{2}a^{n-2}b^2 = 30375$ .....(iii)

Multiplying eq. (i) and eq. (iii),

$\frac{n(n-1)}{2}a^{2n-2}b^2 = 729 \times 30375$ .....(iv)

Squaring both sides of eq. (ii),

$n^2a^{2n-2}b^2 = (7290)^2$ .....(v)

Dividing eq. (iv) by eq. (v),

$$\frac{n(n-1)a^{2n-2}b^2}{2n^2a^{2n-2}b^2} = \frac{729 \times 30375}{7290 \times 7290}$$

$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{72900}$

$\Rightarrow \frac{(n-1)}{2n} = \frac{5}{12}$

$\Rightarrow 12n - 12 = 10n$

$\Rightarrow 2n = 12$

$\Rightarrow n = 6$

$\therefore$ From eq. (i), $a^6 = 729$

$\Rightarrow a^6 = 3^6$

$\Rightarrow a = 3$

And From eq. (ii), $6 \times 3^5 \times b = 7290$

$\Rightarrow b = 5$

Therefore, $n = 6$, $a = 3$ and $b = 5$

2. Find a if the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.

Ans. We have ,

$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^n$

$\therefore$

$(3+ax)^9 = {}^9C_0 3^9 + {}^9C_1 3^8 (ax) + {}^9C_2 3^7 (ax)^2 + {}^9C_3 3^6 (ax)^3 + \ldots\ldots\ldots\ldots + {}^9C_9 (ax)^9$

$= {}^9C_0 3^9 + {}^9C_1 3^8 ax + {}^9C_2 3^7 a^2 x^2 + {}^9C_3 3^6 a^3 x^3 + \ldots\ldots\ldots\ldots + {}^9C_9 a^9 x^9$

$\therefore$ Coefficient of x^2 = ${}^9C_2 (3)^7 a^2$ and Coefficient of x^3 = ${}^9C_3 (3)^6 a^3$

According to question, ${}^9C_2 (3)^7 a^2 = {}^9C_3 (3)^6 a^3$

$\Rightarrow 36\times 3^7 a^2 = 84\times 3^6 a^3$

$\Rightarrow a = \dfrac{36\times 3^7}{84\times 3^6} = \dfrac{108}{84} = \dfrac{9}{7}$

3. Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

Ans. Using Binomial Theorem,

$(1+2x)^6(1-x)^7 = \left[{}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6\right]$

$\left[{}^7C_0 - {}^7C_1x + {}^7C_2x^2 - {}^7C_3x^3 + {}^7C_4x^4 - {}^7C_5x^5 + {}^7C_6x^6 - {}^7C_7x^7\right]$

$= \left[1+12x+60x^2+160x^3+240x^4+192x^5+64x^6\right]$

$= \left[1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7\right]$

It is enough to consider only those terms in the product which involve x^5 and they are

$1(-21x^5) + (12x)(35x^4) + (60x^2)(-35x^3) + (160x^3)(21x^2) + (240x^4)(-7x) + (192x^5$

$\therefore$ Coefficient of x^5 in the product

$= \{1\times(-21)\} + \{12\times 35\} + \{60\times(-35)\} + \{160\times 21\} + \{240\times(-7)\} + \{192\times 1\}$

$= -21+420-2100+3360-1680+192 = 171$

4. If a and b are distinct integers, prove that $a-b$ is a factor of a^n-b^n, whenever n is a positive integer.

Ans. We have, $a^n = \left[(a-b)+b\right]^n$

$\Rightarrow a^n = {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}.b + {}^nC_2(a-b)^{n-2}.b^2 + \ldots\ldots\ldots + {}^nC_n.b^n$

$\therefore$

$a^n - b^n = {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots\ldots\ldots\ldots\ldots + {}^nC_{n-1}(a-b)b^{n-1} + b^n - b^n$

$\Rightarrow a^n - b^n = (a-b)^n + {}^nC_1(a-b)^{n-1}.b + {}^nC_2(a-b)^{n-2}.b^2 + \dots\dots\dots + {}^nC_{n-1}(a-b)b^{n-1}$

$\Rightarrow$

$a^n - b^n = (a-b)\left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}.b + {}^nC_2(a-b)^{n-3}.b^2 + \dots\dots\dots + {}^nC_{n-1}b^{n-1}\right]$

$\Rightarrow a^n - b^n = k(a-b)$,where

$k = \left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \dots\dots\dots + {}^nC_{n-1}b^{n-1}\right]$ is an integer

which shows that $(a-b)$ is a factor of $a^n - b^n$.

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5. Evaluate: $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

Ans. First we will consider $(a+b)^6 - (a-b)^6$

Using Binomial Theorem, we have

$(a+b)^6 - (a-b)^6 =$
$\left[{}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6\right]$

$-\left[{}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5ab^5 + {}^6C_6b^6\right]$

$= \left[a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6\right]$
$-\left[a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6\right]$

$= 12a^5b + 40a^3b^3 + 12ab^5 = 4ab\left(3a^4 + 10a^2b^2 + 3b^4\right)$

$\therefore (a+b)^6 - (a-b)^6 = 4ab\left(3a^4 + 10a^2b^2 + 3b^4\right)$ ....(i)

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$ in equation(i),we get

$(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6 = 4\sqrt{3}\sqrt{2}\left[3(\sqrt{3})^4 + 10(\sqrt{3})^2(\sqrt{2})^2 + 3(\sqrt{2})^4\right]$

$= 4\sqrt{6}\,(27 + 60 + 12) = 4\sqrt{6} \times 99 = 396\sqrt{6}$

6. Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$.

Ans. Putting $a^2=x$ and $\sqrt{a^2-1}=y$, we have

$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4=(x+y)^4+(x-y)^4$

$=\left[{}^4C_0x^4+{}^4C_1x^3y+{}^4C_2x^2y^2+{}^4C_3x^1y^3+{}^4C_4y^4\right]+\left[{}^4C_0x^4-{}^4C_1x^3y+{}^4C_2x^2y^2-\right.$

$=2\left[{}^4C_0x^4+{}^4C_2x^2y^2+{}^4C_4y^4\right]=2\left[x^4+6x^2y^2+y^4\right]$

$=2\left[\left(a^2\right)^4+6\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2+\left(\sqrt{a^2-1}\right)^4\right]$

$=2\left[a^8+6a^4\left(a^2-1\right)+\left(a^2-1\right)^2\right]$

$=2\left[a^8+6a^6-6a^4+a^4-2a^2+1\right]$

$=2\left[a^8+6a^6-5a^4-2a^2+1\right]$

7. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Ans. We have ,

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^nC_nx^n$

Here $(0.99)^5=(1-0.01)^5=\left[1+(-0.01)^5\right]={}^5C_0-{}^5C_1(0.01)-{}^5C_2(0.01)^2\ldots\ldots\ldots\ldots$

$=1-5\times(0.01)-10\times(0.01)^2\ldots\ldots\ldots\ldots=1-0.05+0.001\ldots\ldots\ldots\ldots=1.001-0.05=0.951$

8. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

Ans. We have the general term of the binomial expansion

$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \ldots\ldots\ldots\ldots\ldots\ldots + {}^nC_n b^n$ is

$T_{r+1} = {}^nC_r a^{n-r} b^r$

Also r^th term from the end=(n-r+2)^th term from the beginning

$\therefore$ 5^th term from the beginning = $T_5 = T_{4+1} = {}^nC_4 a^{n-4} b^4$ and

5^th term from the end = $T_{n-5+2} = T_{n-3} = T_{(n-4)+1} = {}^nC_{n-4} a^4 b^{n-4}$

Hence in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$

5^th term from the beginning = ${}^nC_4 \left(\sqrt[4]{2}\right)^{n-4} \left(\frac{1}{\sqrt[4]{3}}\right)^4$ and

5^th term from the end = ${}^nC_{n-4} \left(\sqrt[4]{2}\right)^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$

Given that

$$\frac{{}^nC_4 \left(\sqrt[4]{2}\right)^{n-4} \left(\frac{1}{\sqrt[4]{3}}\right)^4}{{}^nC_{n-4} \left(\sqrt[4]{2}\right)^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}} = \frac{\sqrt{6}}{1}$$

$\Rightarrow \left(\sqrt[4]{2}\right)^{n-4-4} \left(\frac{1}{\sqrt[4]{3}}\right)^{4-n+4} = \frac{\sqrt{6}}{1}$, since ${}^nC_r = {}^nC_{n-r}$

$(2)^{\frac{n-8}{4}} . (3)^{\frac{n-8}{4}} = 2^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$\Rightarrow \frac{n-8}{4} = \frac{1}{2}$

$\Rightarrow n-8=2$

$\Rightarrow n=10$

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9. Expand using binomial theorem $\left[1+\frac{x}{2}-\frac{2}{x}\right]^4, x \neq 0.$

Ans. We have, $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots\ldots\ldots\ldots\ldots\ldots + {}^nC_nx^n$

Now, $\left[1+\frac{x}{2}-\frac{2}{x}\right]^4 = \left[1+\left(\frac{x}{2}-\frac{2}{x}\right)\right]^4$

$= {}^4C_0 + {}^4C_1\left(\frac{x}{2}-\frac{2}{x}\right) + {}^4C_2\left(\frac{x}{2}-\frac{2}{x}\right)^2 + {}^4C_3\left(\frac{x}{2}-\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{x}{2}-\frac{2}{x}\right)^4$

$= 1+4\left(\frac{x}{2}-\frac{2}{x}\right)+6\left(\frac{x^2}{4}+\frac{4}{x^2}-2\right)+4\left(\frac{x^3}{8}-\frac{8}{x^3}-\frac{3x}{2}+\frac{6}{x}\right)$

$+\left[{}^4C_0\left(\frac{x}{2}\right)^4 - {}^4C_1\left(\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + {}^4C_2\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - {}^4C_3\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{2}{x}\right)^4\right]$

$= 1+\left(2x-\frac{8}{x}\right)+\left(\frac{3}{2}x^2+\frac{24}{x^2}-12\right)+\left(\frac{x^3}{2}-\frac{32}{x^3}-6x+\frac{24}{x}\right)+\left(\frac{x^4}{16}-x^2+6-\frac{16}{x^2}+\frac{16}{x^4}\right)$

$= \left(\frac{24}{x}-\frac{8}{x}\right)+\left(\frac{24}{x^2}-\frac{16}{x^2}\right)-\frac{32}{x^3}+\frac{16}{x^4}+(2x-6x)+\left(\frac{3}{2}x^2-x^2\right)+\left(\frac{x^3}{2}\right)+\left(\frac{x^4}{16}\right)+($

$= \frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5$

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10. Find the expansion of $\left(3x^2-2ax+3a^2\right)^3$ **using binomial theorem.**

Ans. Here $\left(3x^2-2ax+3a^2\right)^3 = \left[\left(3x^2-2ax\right)+3a^2\right]^3$

$= {}^3C_0\left(3x^2-2ax\right)^3 + {}^3C_1\left(3x^2-2ax\right)^2\left(3a^2\right) + {}^3C_2\left(3x^2-2ax\right)\left(3a^2\right)^2 + {}^3C_3\left(3a^2\right)^3$

= $\left(3x^2-2ax\right)^3+3\times 3a^2\left(3x^2-2ax\right)^2+3\times 9a^4\left(3x^2-2ax\right)+27a^6$

=
$27x^6-8a^3x^3-54ax^5+36a^2x^4+9a^2\left(9x^4+4a^2x^2-12ax^3\right)+27a^4\left(3x^2-2ax\right)+27a^6$

=
$27x^6-8a^3x^3-54ax^5+36a^2x^4+81a^2x^4+36a^4x^2-108a^3x^3+81a^4x^2-54a^5x+27a^6$

= $27x^6-54ax^5+117a^2x^4-116a^3x^3+117a^4x^2-54a^5x+27a^6$