CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 12 Introduction to Three Dimensional Geometry

Exercise 12.2

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) $(-3, 7, 2)$ **and** $(2, 4, -1)$

(iii) $(-1, 3, -4)$ **and** $(1, -3, 4)$

(iv) $(2, -1, 3)$ **and** $(-2, 1, 3)$

Ans. (i) Let A (2, 3, 5) and B (4, 3, 1) be two points, then

AB = $\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$

= $\sqrt{4+0+16}$ = $\sqrt{20}=2\sqrt{5}$ units

(ii) Let A (–3, 7, 2) and B (2, 4, –1) be two points, then

AB = $\sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$

= $\sqrt{25+9+9}$ = $\sqrt{43}$ units

(iii) Let A (–1, 3, –4) and B (1, –3, 4) be two points, then

AB = $\sqrt{\{1-(-1)\}^2+(-3-3)^2+\{4-(-4)\}^2}$

= $\sqrt{4+36+64}$ = $\sqrt{104}=2\sqrt{26}$ units

(iv) Let A (2, –1, 3) and B (–2, 1, 3) be two points, then

AB = $\sqrt{(-2-2)^2+\{1-(-1)\}^2+(3-3)^2}$

= $\sqrt{16+4+0}$ = $\sqrt{20}=2\sqrt{5}$ units

2. Show that the points $(-2,3,5),(1,2,3)$ and $(7,0,-1)$ are collinear.

Ans. Let A (–2, 3, 5), B (1, 2, 3) and C (7, 0, –1) be three points, then

AB = $\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

= $\sqrt{9+1+4}$ = $\sqrt{14}$ units

BC = $\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$

= $\sqrt{36+4+16}$ = $\sqrt{56}=2\sqrt{14}$ units

AC = = $\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$

= $\sqrt{81+9+36}$ = $\sqrt{126}=3\sqrt{14}$ units

Here, AC = AB + BC

Therefore A, B and C are collinear.

3. Verify the following:

(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of right angled triangle.

(iii) $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Ans. (i) Let A (0, 7, –10) B (1, 6, –6) and C (4, 9, –6) be three vertices of ΔABC, then

AB = $\sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}$

= $\sqrt{1+1+16}$ = $\sqrt{18}=3\sqrt{2}$ units

BC = $\sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}$

= $\sqrt{9+9+0}$ = $\sqrt{18}=3\sqrt{2}$ units

AC = = $\sqrt{(4-0)^2+(9-7)^2+(-6+10)^2}$

= $\sqrt{16+4+16}$ = $\sqrt{36}=6$ units

Here, AB = BC

Therefore ΔABC is an isosceles triangle.

(ii) Let A (0,7,10), B (–1, 6, 6) and C (–4, 9, 6) be three vertices of ΔABC, then

AB = $\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$

= $\sqrt{1+1+16}$ = $\sqrt{18}=3\sqrt{2}$ units

BC = $\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

= $\sqrt{9+9+0}$ = $\sqrt{18}=3\sqrt{2}$ units

AC = = $\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$

= $\sqrt{16+4+16}$ = $\sqrt{36}=6$ units

Here, $AC^2 = AB^2 + BC^2$

Therefore ΔABC is a right angled triangle.

(iii) Let A (-1, 2, 1), B (1, –2, 5), C (4, –7, 8) and D (2, –3, 4) be four vertices of a quadrilateral ABCD, then

AB = $\sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$

= $\sqrt{4+16+16}$ = $\sqrt{36}=6$ units

BC = $\sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}$

= $\sqrt{9+25+9}$ = $\sqrt{43}$ units

CD = $\sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}$

= $\sqrt{4+16+16}$ = $\sqrt{36}=6$ units

AD = $\sqrt{(2+1)^2+(-3-2)^2+(4-1)^2}$

= $\sqrt{9+25+9}$ = $\sqrt{43}$ units

AC = $\sqrt{(4+1)^2+(-7-2)^2+(8-1)^2}$

= $\sqrt{25+81+49}$ = $\sqrt{155}$ units

BD = $\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}$

= $\sqrt{1+1+1}$ = $\sqrt{3}$ units

Here, AB = CD, BC = AD and AC $\neq$ BD

Therefore A, B, C and are the vertices of a parallelogram ABCD.

4. Find the equation of the set of points which are equidistant from the point (1, 2, 3) and $(3, 2, -1)$.

Ans. Let A(x, y, z) be any point which is equidistant from points B (1, 2, 3) and C$(3, 2, -1)$. Then

According to question, AB = AC

$$\Rightarrow \sqrt{(x-1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$$

Squaring both sides, we get

$\Rightarrow (x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$

$\Rightarrow$

$x^2+1-2x+y^2+4-4y+z^2+9-6z=x^2+9-6x+y^2+4-4y+z^2+1+2z$

$\Rightarrow -2x-4y-6z=-6x-4y+2z$

$\Rightarrow -2x-6z+6x-2z=0$

$\Rightarrow 4x-8z=0$

$\Rightarrow x-2z=0$

5. Find the equation of the set of points P, the sum of whose distance from A (4, 0, 0) and B$(-4,0,0)$ is equal to 10.

Ans. Let P(x,y,z) be any point, then

According to question, PA + PB = 10

$\Rightarrow \sqrt{(x-4)^2+(y-0)^2+(z-0)^2}+\sqrt{(x+4)^2+(y-0)^2+(z-0)^2}=10$

$\Rightarrow \sqrt{x^2-8x+16+y^2+z^2}+\sqrt{x^2+8x+16+y^2+z^2}=10$

$\Rightarrow \sqrt{x^2-8x+16+y^2+z^2}=10-\sqrt{x^2+8x+16+y^2+z^2}$

Squaring both sides, we get

$\Rightarrow$

$x^2-8x+16+y^2+z^2=100+x^2+8x+16+y^2+z^2-20\sqrt{x^2+8x+16+y^2+z^2}$

$\Rightarrow 16x+100=20\sqrt{x^2+8x+16+y^2+z^2}$

$\Rightarrow 4x+25=5\sqrt{x^2+8x+16+y^2+z^2}$

Again squaring both sides, we get

$\Rightarrow 25\left(x^2+16+8x+y^2+z^2\right)=16x^2+625+200x$

$\Rightarrow 25x^2+400+200x+25y^2+25z^2-16x^2-625-200x=0$

$\Rightarrow 9x^2+25y^2+25z^2-225=0$

This is the required equation.