CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 5 Complex Numbers and Quadratic Equations

Exercise 5.2

Find the modulus and the argument of each of the complex numbers in exercises 1 to 2.

1. $z=-1-i\sqrt{3}$

Ans. Given: $z=r(\cos\theta+i\sin\theta)=-1-i\sqrt{3}$

$\therefore\ r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta+\sin^2\theta)=1+3$

$\Rightarrow r^2=4$

$\Rightarrow r=2$

$\therefore\ 2\cos\theta=-1$ and $2\sin\theta=-\sqrt{3}$

$\Rightarrow \cos\theta=\dfrac{-1}{2}$ and $\sin\theta=\dfrac{-\sqrt{3}}{2}$

[θ lies in third quadrant]

$\therefore\ \theta=\left(-\pi+\dfrac{\pi}{3}\right)=\dfrac{-2\pi}{3}$

Therefore, $|z|=2$ and $\arg(z)=\dfrac{-2\pi}{3}$

2. $z=-\sqrt{3}+i$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = -\sqrt{3} + i$

$\therefore r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = 2$

$\therefore 2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

[θ lies in second quadrant]

$\therefore \theta = \left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{6}$

Therefore, $|z| = 2$ and $\arg(z) = \frac{5\pi}{6}$

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form.

3. $1 - i$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = 1 - i$

$\therefore r\cos\theta = 1$ and $r\sin\theta = -1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{-1}{\sqrt{2}}$

[θ lies in fourth quadrant]

$\therefore \theta = \frac{-\pi}{4}$

Therefore, Polar form of z is $\sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right]$.

4. $-1+i$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = -1+i$

$\therefore r\cos\theta = -1$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

[θ lies in second quadrant]

$\therefore \theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Therefore, Polar form of z is $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$.

5. $-1-i$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = -1-i$

$\therefore r\cos\theta = -1$ and $r\sin\theta = -1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{-1}{\sqrt{2}}$

[θ lies in third quadrant]

$\therefore \theta = \left(-\pi + \frac{\pi}{4}\right) = \frac{-3\pi}{4}$

Therefore, Polar form of z is $\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right) + i\sin\left(\frac{-3\pi}{4}\right)\right]$.

6. -3

Ans. Given: $z = r(\cos\theta + i\sin\theta) = -3$

$\therefore r\cos\theta = -3$ and $r\sin\theta = 0$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 9+0$

$\Rightarrow r^2 = 9$

$\Rightarrow r = 3$

$\therefore 3\cos\theta = -3$ and $3\sin\theta = 0$

$\Rightarrow \cos\theta = -1$ and $\sin\theta = 0$

[θ lies in second quadrant]

$\therefore \theta = (\pi - 0) = \pi$

Therefore, Polar form of z is $3[\cos\pi + i\sin\pi]$.

---

7. $\sqrt{3} + i$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = \sqrt{3} + i$

$\therefore r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = 2$

$\therefore 2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

[θ lies in first quadrant]

$\therefore \theta = \frac{\pi}{6}$

Therefore, Polar form of z is $3\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$.

---

8. i

Ans. Given: $z = r\left(\cos\theta + i\sin\theta\right) = i$

$\therefore\ r\cos\theta = 0$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 0 + 1$

$\Rightarrow r^2 = 1$

$\Rightarrow r = 1$

$\therefore\ 1\cos\theta = 0$ and $1\sin\theta = 1$

$\Rightarrow \cos\theta = 0$ and $\sin\theta = 1$

[θ lies in first quadrant]

$\therefore\ \theta = \frac{\pi}{2}$

Therefore, Polar form of z is $cos\frac{\pi}{2} + isin\frac{\pi}{2}$