CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 11 Conic Sections

Exercise 11.4

In each of the Exercises 1 to 6, find the coordinates of the foci, and the vertices, the eccentricity and the length of the latus rectum of the following hyperbolas.

1. $\frac{x^2}{16}-\frac{y^2}{9}=1$

Ans. Given: Equation of hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ in the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Since, the foci and vertices of the hyperbola lies on $x-$ axis.

$\therefore a^2=16$

$\Rightarrow a=4$ and $b^2=9$

$\Rightarrow b=3$

$\therefore c^2=a^2+b^2$

$\Rightarrow c^2=16+9=25$

$\Rightarrow c=5$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow (\pm 5, 0)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 4, 0)$

Eccentricity $(e)=\frac{c}{a}=\frac{5}{4}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$ units

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2. $\frac{y^2}{9}-\frac{x^2}{27}=1$

Ans. Given: Equation of hyperbola $\frac{y^2}{9}-\frac{x^2}{27}=1$ is in the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Since, the foci and vertices of the hyperbola lies on y- axis.

$\therefore a^2=9$

$\Rightarrow a=3$ and $b^2=27$

$\Rightarrow b=3\sqrt{3}$

$\therefore c^2=a^2+b^2$

$\Rightarrow c^2=9+27=36$

$\Rightarrow c=6$

$\therefore$ Coordinates of foci are $(0,\pm c)$

$\Rightarrow (0,\pm 6)$

Coordinates of vertices are $(0,\pm a)$

$\Rightarrow (0,\pm 3)$

Eccentricity $(e)=\frac{c}{a}=\frac{6}{3}=2$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 27}{3} = 18$ units

3. $9y^2 - 4x^2 = 36$

Ans. Given: Equation of hyperbola $9y^2 - 4x^2 = 36$

$\Rightarrow \frac{9y^2}{36} - \frac{4x^2}{36} = 1$ (divide both sides of equation by 36)

$\Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$ in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since, the foci and vertices of the hyperbola lies on $y-$axis.

$\therefore a^2 = 4$

$\Rightarrow a = 2$ and $b^2 = 9$

$\Rightarrow b = 3$

$\therefore c^2 = a^2 + b^2$

$\Rightarrow c^2 = 4 + 9 = 13$

$\Rightarrow c = \sqrt{13}$

$\therefore$ Coordinates of foci are $(0, \pm c)$

$\Rightarrow \left(0, \pm\sqrt{13}\right)$

Coordinates of vertices are $(0, \pm a)$

$\Rightarrow (0, \pm 2)$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 9}{2}=9$ units

4. $16x^2-9y^2=576$

Ans. Given: Equation of hyperbola $16x^2-9y^2=576$

$\Rightarrow \frac{16x^2}{576}-\frac{9y^2}{576}=1$ (divide both sides of equation by 576)

$\Rightarrow \frac{x^2}{36}-\frac{y^2}{64}=1$ in the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Since, the foci and vertices of the hyperbola lies on $x-$axis.

$\therefore a^2=36$

$\Rightarrow a=6$ and $b^2=64$

$\Rightarrow b=8$

$\therefore c^2=a^2+b^2$

$\Rightarrow c^2=36+64=100$

$\Rightarrow c=10$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow (\pm 10, 0)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 6, 0)$

Eccentricity $(e)=\frac{c}{a}=\frac{10}{6}=\frac{5}{3}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 64}{6}=\frac{64}{3}$ units

5. $5y^2-9x^2=36$

Ans. Given: Equation of hyperbola $5y^2-9x^2=36$

$\Rightarrow \frac{5y^2}{36}-\frac{9x^2}{36}=1$ (divide both sides of eqaution by 36)

$\Rightarrow \frac{y^2}{36/5}-\frac{x^2}{4}=1$ in the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Since, the foci and vertices of the hyperbola lies on $y-$axis.

$\therefore a^2=\frac{36}{5}$

$\Rightarrow a=\frac{6}{\sqrt{5}}$ and $b^2=4$

$\Rightarrow b=2$

$\therefore c^2=a^2+b^2$

$\Rightarrow c^2=\frac{36}{5}+4=\frac{56}{5}$

$\Rightarrow c=\sqrt{\frac{56}{5}}$

$\therefore$ Coordinates of foci are $(0,\pm c)$

$\Rightarrow \left(0,\pm\sqrt{\frac{56}{5}}\right)$

Coordinates of vertices are $(0, \pm a)$

$$\Rightarrow \left(0, \pm\frac{6}{\sqrt{5}}\right)$$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{\frac{56}{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{56}}{6} = \frac{2\sqrt{14}}{6} = \frac{\sqrt{14}}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 4}{6/\sqrt{5}} = \frac{2\times 4\times\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$ units

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6. $49y^2 - 16x^2 = 784$

Ans. Given: Equation of hyperbola $49y^2 - 16x^2 = 784$

$\Rightarrow \frac{49y^2}{784} - \frac{16x^2}{784} = 1$ (divide both sides of eqaution by 784)

$\Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$ in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since, the foci and vertices of the hyperbola lies on $y-$axis.

$\therefore a^2 = 16$

$\Rightarrow a = 4$ and $b^2 = 49$

$\Rightarrow b = 7$

$\therefore c^2 = a^2 + b^2$

$\Rightarrow c^2 = 16 + 49 = 65$

$\Rightarrow c = \sqrt{65}$

$\therefore$ Coordinates of foci are $(0, \pm c)$

$\Rightarrow (0, \pm\sqrt{65})$

Coordinates of vertices are $(0, \pm a)$

$\Rightarrow (0, \pm 4)$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 49}{4} = \frac{49}{2}$ units

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In each of the Exercises 7 to 15, find the equation of the hyperbola satisfying the given conditions.

7. Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

Ans. Given: The vertices $(\pm 2, 0)$ lie on $x-$ axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore (\pm a, 0)$ = $(\pm 2, 0)$

$\Rightarrow a = 2$

Foci $(\pm ae, 0)$ = $(\pm 3, 0)$

$\Rightarrow ae = 3$

$\Rightarrow e = \frac{3}{a} = \frac{3}{2}$

Also $b = a\sqrt{e^2 - 1}$ = $2\sqrt{\frac{9}{4} - 1} = 2 \times \frac{\sqrt{5}}{2} = \sqrt{5}$

Therefore, the required equation of hyperbola is $\frac{x^2}{(2)^2} - \frac{y^2}{\left(\sqrt{5}\right)^2} = 1$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1.$

8. Vertices $(0, \pm 5)$, **foci** $(0, \pm 8)$

Ans. Given: The vertices $(0, \pm 5)$ lie on $y-$axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore (0, \pm a) = (0, \pm 5)$

$\Rightarrow a = 5$

Foci $(0, \pm ae)$ = $(0, \pm 8)$

$\Rightarrow ae = 8$ and a = 5

$\Rightarrow \frac{8}{5}$

Also $b = a\sqrt{e^2 - 1}$ = $5\sqrt{\frac{64}{25} - 1} = 5 \times \frac{\sqrt{39}}{5} = \sqrt{39}$

Therefore, the required equation of hyperbola is $\frac{y^2}{(5)^2} - \frac{x^2}{\left(\sqrt{39}\right)^2} = 1$

$\Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1.$

9. Vertices $(0, \pm 3)$, foci $(0, \pm 5)$

Ans. Given: The vertices $(0, \pm 3)$ lie on $y-$axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore$ $(0, \pm a)$ = $(0, \pm 3)$

$\Rightarrow a = 3$

Foci $(0, \pm ae)$ = $(0, \pm 5)$

$\Rightarrow ae = 5$

$\Rightarrow e = \frac{5}{a} = \frac{5}{3}$

Also $b = a\sqrt{e^2 - 1}$ = $3\sqrt{\frac{25}{9} - 1} = 3 \times \frac{4}{3} = 4$

Therefore, the required equation of hyperbola is $\frac{y^2}{(3)^2} - \frac{x^2}{(4)^2} = 1$

$\Rightarrow \frac{y^2}{9} - \frac{x^2}{16} = 1$.

10. Foci $(\pm 5, 0)$, the transverse axis is of length 8.

Ans. Given: The foci $(\pm 5, 0)$ lie on $x-$ axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Length of transverse axis: $2a = 8$

$\Rightarrow a = 4$

Foci $\left(\pm ae, 0\right)$ = $\left(\pm 5, 0\right)$

$\Rightarrow \ ae = 5$

$\Rightarrow \ e = \frac{5}{a} = \frac{5}{4}$

Also $b = a\sqrt{e^2 - 1}$ = $4\sqrt{\frac{25}{16} - 1} = 4 \times \frac{3}{4} = 3$

Therefore, the required equation of hyperbola is $\frac{x^2}{(4)^2} - \frac{y^2}{(3)^2} = 1$

$\Rightarrow \ \frac{x^2}{16} - \frac{y^2}{9} = 1$.

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11. Foci $\left(0, \pm 13\right)$, the conjugate axis is of length 24.

Ans. Given: The foci $\left(0, \pm 13\right)$ lie on $y-$axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore$ Length of conjugate axis: $2b = 24$

$\Rightarrow b = 12$

Foci : $(\pm c, 0)$

= $\left(\pm 5, 0\right)$ $c = 5$

Also $c^2 = a^2 + b^2$

$\Rightarrow \ (13)^2 = a^2 + (12)^2$

$\Rightarrow a^2 = 169 - 144 = 25$

Therefore, the required equation of hyperbola is $\frac{y^2}{25} - \frac{x^2}{(12)^2} = 1$

$\Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1$.

12. Foci $\left(\pm 3\sqrt{5}, 0\right)$, the latus rectum is of length 8.

Ans. Given: The foci $\left(\pm 3\sqrt{5}, 0\right)$ lie on $x-$ axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Foci $\left(\pm c, 0\right)$ = $\left(\pm 3\sqrt{5}, 0\right)$

$\Rightarrow c = 3\sqrt{5}$

Length of latus rectum: $\frac{2b^2}{a} = 8$

$\Rightarrow b^2 = 4a$

Also $c^2 = a^2 + b^2$

$\Rightarrow \left(3\sqrt{5}\right)^2 = a^2 + 4a$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow \left(a+9\right)\left(a-5\right) = 0$

$\Rightarrow a = -9, a = 5$ [$a = -9$ not possible]

$\Rightarrow a = 5$

$\Rightarrow a^2 = 25$

Length of latus rectum: $b^2 = 4a$

$\Rightarrow b^2 = 4 \times 5 = 20$

Therefore, the required equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$.

13. Foci $(\pm 4, 0)$, the latus rectum is of length 12.

Ans. Given: The foci $(\pm 4, 0)$ lie on $x-$ axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Foci $(\pm c, 0) = (\pm 4, 0)$

$\Rightarrow c = 4$

Length of latus rectum: $\frac{2b^2}{a} = 12$

$\Rightarrow b^2 = 6a$

Also $c^2 = a^2 + b^2$

$\Rightarrow (4)^2 = a^2 + 6a$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow (a+8)(a-2) = 0$

$\Rightarrow a = -8, a = 2$ [$a = -8$ not possible]

$\Rightarrow a = 2$

$\Rightarrow a^2 = 4$

Length of latus rectum: $b^2 = 6a$

$\Rightarrow b^2 = 6 \times 2 = 12$

Therefore, the required equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$.

14. Vertices $(\pm 7, 0), e = \frac{4}{3}$

Ans. Given: The vertices $(\pm 7, 0)$ lie on $x-$ axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Vertices $(\pm a, 0)$ = $(\pm 7, 0)$

$\Rightarrow a = 7$

$e = \frac{4}{3}$

$\Rightarrow \frac{c}{a} = \frac{4}{3}$

$\Rightarrow \frac{c}{7} = \frac{4}{3}$

$\Rightarrow c = \frac{28}{3}$

Also $c^2 = a^2 + b^2$

$\Rightarrow \left(\frac{28}{3}\right)^2 = (7)^2 + b^2$

$\Rightarrow b^2 = \frac{784}{9} - 49 = \frac{343}{9}$

Therefore, the required equation of hyperbola is $\frac{x^2}{(7)^2} - \frac{y^2}{343/9} = 1$

$\Rightarrow \frac{x^2}{49} - \frac{9y^2}{343} = 1$.

15. Foci $\left(0, \pm\sqrt{10}\right)$, passing through (2, 3).

Ans. Given: The foci $\left(0, \pm\sqrt{10}\right)$ lie on $y-$axis.

$\therefore$ The equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore$ Foci $(0, \pm c)$ = $\left(0, \pm\sqrt{10}\right)$

$\Rightarrow c = \sqrt{10}$

Also $c^2 = a^2 + b^2$

$\Rightarrow \left(\sqrt{10}\right)^2 = a^2 + b^2$

$\Rightarrow b^2 = 10 - a^2$

Since given hyperbola passes through (2, 3)

$\therefore \frac{9}{a^2} - \frac{4}{b^2} = 1$

$\Rightarrow \frac{9}{a^2} - \frac{4}{10 - a^2} = 1$

$\Rightarrow \frac{9\left(10 - a^2\right) - 4a^2}{a^2\left(10 - a^2\right)} = 1$

$\Rightarrow\ 9\left(10-a^2\right)-4a^2=a^2\left(10-a^2\right)$

$\Rightarrow\ a^4-23a^2+90=0$

$\Rightarrow\ a^4-18a^2-5a^2+90=0$

$\Rightarrow\ \left(a^2-18\right)\left(a^2-5\right)=0$

$\Rightarrow\ a^2=18, a^2=5$

$\therefore\ b^2=10-a^2=10-18=-8$ [which is not possible]

And $b^2=10-a^2=10-5=5$

Therefore, the required equation of hyperbola is $\frac{y^2}{5}-\frac{x^2}{5}=1$.