CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 10 Straight Lines

Exercise 10.3

1. Reduce the following equations into slope-intercept form and find their slopes and the y- intercepts.

(i) $x+7y=0$

(ii) $6x+3y-5=0$

(iii) $y=0$

Ans. (i) Given: $x+7y=0$

$\Rightarrow 7y=-x$

$\Rightarrow y=\frac{-1}{7}x+0$ ....(i)

Comparing with $y=mx+c$, we have $m=\frac{-1}{7}$ and $c=0$

(ii) Given: $6x+3y-5=0$

$\Rightarrow 3y=-6x+5$

$\Rightarrow y=-2x+\frac{5}{3}$ ....(i)

Comparing with $y=mx+c$, we have $m=-2$ and $c=\frac{5}{3}$

(iii) Given: $y=0$

$\Rightarrow y=0x+0$ ....(i)

Comparing with $y = mx + c$, we have $m = 0$ and $c = 0$

2. Reduce the following equations into intercept form and find their intercepts on the axes:

(i) $3x + 2y - 12 = 0$

(ii) $4x - 3y = 6$

(iii) $3y + 2 = 0$

Ans. (i) Given: $3x + 2y - 12 = 0$

$\Rightarrow 3x + 2y = 12$

$\Rightarrow \frac{3x}{12} + \frac{2y}{12} = 1$

$\Rightarrow \frac{x}{4} + \frac{y}{6} = 1$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$, we have $a = 4$ and $b = 6$

(ii) Given: $4x - 3y = 6$

$\Rightarrow \frac{4x}{6} - \frac{3y}{6} = 1$

$\Rightarrow \frac{x}{3/2} + \frac{y}{-2} = 1$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$, we have $a = \frac{3}{2}$ and $b = -2$

(iii) Given: $3y + 2 = 0$

$\Rightarrow 3y = -2$

$\Rightarrow \frac{3y}{-2}=1$

$\Rightarrow \frac{y}{\frac{-2}{3}}=1$

Comparing with $\frac{x}{a}+\frac{y}{b}=1$, we have $b=\frac{-2}{3}$ and no inercept with x-axis.

3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive $x-$ axis:

(i) $x-\sqrt{3}y+8=0$

(ii) $y-2=0$

(iii) $x-y=4$

Ans. (i) Given: $x-\sqrt{3}y+8=0$

$\Rightarrow -x+\sqrt{3}y=8$

Dividing both sides by $\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=2,$ we have

$\frac{-x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$

$\Rightarrow \frac{-1}{2}x+\frac{\sqrt{3}}{2}y=4$

Putting $\cos\alpha=\frac{-1}{2}$ and $\sin\alpha=\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\alpha=-\cos 60^\circ$

$\Rightarrow \cos\alpha=\cos\left(180^\circ-60^\circ\right)$

$\Rightarrow \cos\alpha = \cos 120^\circ$

$\Rightarrow \alpha = 120^\circ = \frac{2\pi}{3}$

$\therefore$ Equation of line in normal form is $x\cos\frac{2\pi}{3} + y\sin\frac{2\pi}{3} = 4$

Comparing with $x\cos\alpha + y\sin\alpha = p$, we have $\alpha = \frac{2\pi}{3}$ and $p = 4$

(ii) Given: $y - 2 = 0 \Rightarrow y = 2$

$\Rightarrow 0x + y = 2$

Dividing both sides by $\sqrt{(0)^2 + (1)^2} = 1$, we have $0x + y = 2$

Putting $\cos\alpha = 0$ and $\sin\alpha = 1$

$\Rightarrow \alpha = \frac{\pi}{2}$

$\therefore$ Equation of line in normal form is $x\cos\frac{\pi}{2} + y\sin\frac{\pi}{2} = 2$

Comparing with $x\cos\alpha + y\sin\alpha = p$, we have $\alpha = \frac{\pi}{2}$ and $p = 2$

(iii) Given: $x - y = 4$

Dividing both sides by $\sqrt{(1)^2 + (-1)^2} = \sqrt{2}$, we have

$\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y = 2\sqrt{2}$

Putting $\cos\alpha = \frac{1}{\sqrt{2}}$ and $\sin\alpha = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos\alpha = \cos\left(2\pi - \frac{\pi}{4}\right)$

$\Rightarrow \alpha = \frac{7\pi}{4}$

$\therefore$ Equation of line in normal form is $x\cos\frac{7\pi}{4} + y\sin\frac{7\pi}{4} = 2\sqrt{2}$

Comparing with $x\cos\alpha + y\sin\alpha = p$, we have $\alpha = \frac{7\pi}{4}$ and $p = 2\sqrt{2}$

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4. Find the distance of the point $(-1, 1)$ from the line $12(x+6) = 5(y-2)$.

Ans. Given equation of the line is $12(x+6) = 5(y-2)$

$\Rightarrow 12x + 72 = 5y - 10$

$\Rightarrow 12x - 5y + 82 = 0$

We have the distance of a given point (x_1, y_1) from a given line Ax+BY+C=0 is $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$

$\therefore$ Perpendicular distance of the point $(-1, 1)$ from the line $12x - 5y + 82 = 0$ is

$\left|\frac{12(-1) - 5(1) + 82}{\sqrt{(12)^2 + (-5)^2}}\right|$ = $\left|\frac{-12 - 5 + 82}{\sqrt{144+25}}\right|$ = $\left|\frac{65}{13}\right|$ = 5 units

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5. Find the points on the $x-$ axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Ans. Let the coordinates of the point on $x-$ axis be $(\alpha, 0)$.

Given line is $\frac{x}{3}+\frac{y}{4}=1 \Rightarrow 4x+3y-12=0$

We have the distance of a given point (x_1, y_1) from a given line Ax+BY+C=0 is $d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

Therefore perpendicular distance of the point $(\alpha, 0)$ from the line $4x+3y-12=0$ is

$$\left|\frac{4(\alpha)+3(0)-12}{\sqrt{(4)^2+(3)^2}}\right| = \left|\frac{4\alpha-12}{\sqrt{16+9}}\right| = \left|\frac{4\alpha-12}{5}\right|$$

According to question, $\left|\frac{4\alpha-12}{5}\right|=4$

$$\Rightarrow \frac{4\alpha-12}{5}=\pm 4$$

$\Rightarrow \frac{4\alpha-12}{5}=4$ or $\frac{4\alpha-12}{5}=-4$

$\Rightarrow 4\alpha-12=20$ or $4\alpha-12=-20$

$\Rightarrow 4\alpha=32$ or $4\alpha=-8$

$\Rightarrow \alpha=8$ or $\alpha=-2$

Therefore, the points on $x-$ axis are (8, 0) and $(-2, 0)$.

6. Find the distance between parallel lines:

(i) $15x+8y-34=0$ **and** $15x+8y+31=0$

(ii) $l(x+y)+p=0$ **and** $l(x+y)-r=0$

Ans. (i) Given: Two equations $15x+8y-34=0$ and $15x+8y+31=0$

Here, $a=15, b=8, c_1=-34$ and $c_2=31$

Distance between two parallel lines $(d)=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$ = $\frac{|-34-31|}{\sqrt{(15)^2+(8)^2}}$

= $\frac{|-65|}{\sqrt{225+64}}$ = $\frac{65}{\sqrt{289}}$ = $\frac{65}{17}$ units

(ii) Given: Two equations $lx+ly+p=0$ and $lx+ly-r=0$

Here, $a=l, b=l, c_1=p$ and $c_2=-r$

Distance between two parallel lines $(d)=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$ = $\frac{|p+r|}{\sqrt{(l)^2+(l)^2}}$

= $\frac{|p+r|}{\sqrt{2l^2}}$ = $\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|$ units

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7. Find equation of the line parallel to the line $3x-4y+2=0$ and passing through the point $(-2,3)$.

Ans. We have equation of any line parallel to Ax+BY+C=0 is of the form Ax+By+K=0, K is a constant

$\therefore$ Equation of a line which is parallel to the line $3x-4y+2=0$ is $3x-4y+k=0$.

Since the line passes through the point $(-2,3)$.

we have $3\times(-2)-4\times3+k=0$

$\Rightarrow -6-12+k=0$

$\Rightarrow k=18$

Therefore, the equation of required line is $3x-4y+18=0$.

8. Find the equation of the line perpendicular to the line $x-7y+5=0$ and having x intercept 3.

Ans. We have equation of any line perpendicular to Ax+BY+C=0 is of the form Bx-Ay+K=0, K is a constant

$\therefore$ Equation of a line which is perpendicular to the line $x-7y+5=0$ is $7x+y+k=0$.

Since the line passes through the point $(3,0)$.

we have $7\times 3+0+k=0$

$\Rightarrow 21+k=0$

$\Rightarrow k=-21$

Therefore, the equation of required line is $7x+y-21=0$.

9. Find the angles between the lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$.

Ans. Given: $\sqrt{3}x+y=1$

$\Rightarrow y=-\sqrt{3}x+1$

$\therefore m_1=-\sqrt{3}$

Also $x+\sqrt{3}y=1$

$$\Rightarrow y=-\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}$$

$$\therefore m_2=-\frac{1}{\sqrt{3}}$$

An acute angle θ between the two lines is given by $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\therefore \tan\theta = \left|\frac{-\sqrt{3}+\frac{1}{\sqrt{3}}}{1+(-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)}\right|$

= $\left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right|$ = $\left|\frac{-2}{\sqrt{3}}\times\frac{1}{2}\right|$ = $\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^\circ$ and $\tan\left(180^\circ - 30^\circ\right) = \tan 150^\circ$

$\Rightarrow \theta = 30^\circ$ and 150°

10. The line through the points $(h, 3)$ and (4, 1) intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h.

Ans. Slope of the line passing through the points $(h, 3)$ and (4, 1) = $\frac{1-3}{4-h} = \frac{-2}{4-h}$

Also slope of the line $7x - 9y - 19 = 0$ is $\frac{7}{9}$

We have if two lines are perpendicular to each other then products of their slopes = -1

$\therefore \frac{-2}{4-h}\times\frac{7}{9} = -1$

$\Rightarrow \frac{-14}{36-9h} = -1$

$\Rightarrow -14 = -36 + 9h$

$\Rightarrow 9h = 36 - 14$

$\Rightarrow h = \frac{22}{9}$

11. Prove that the line through the point (x_1, y_1) and parallel to the line $Ax+By+C=0$ is $A(x-x_1)+B(y-y_1)=0$.

Ans. Equation of any line parallel to the line $Ax+By+C=0$ is $Ax+By+K=0$ .....(i)

Since line (i) passes through (x_1, y_1), we get $Ax_1+By_1+K=0$ .....(ii)

Subtracting eq. (ii) from eq. (i), we have $A(x-x_1)+B(y-y_1)=0$

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12. Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Ans. Given: $m_1=2$ and $\theta=60^\circ$

An acute angle θ between the two lines is given by $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$$\Rightarrow \tan 60^\circ=\left|\frac{2-m_2}{1+2m_2}\right|$$

$$\Rightarrow \sqrt{3}=\left|\frac{2-m_2}{1+2m_2}\right|$$

$$\Rightarrow \frac{2-m_2}{1+2m_2}=\pm\sqrt{3}$$

Taking $\frac{2-m_2}{1+2m_2}=\sqrt{3}$

$$\Rightarrow 2-m_2=\sqrt{3}+2\sqrt{3}m_2$$

$$\Rightarrow \left(2\sqrt{3}+1\right)m_2=2-\sqrt{3}$$

$$\Rightarrow m_2=\frac{2-\sqrt{3}}{2\sqrt{3}+1}$$

$\therefore$ Equation of required line is $y-3=\dfrac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$

$\Rightarrow \left(2\sqrt{3}+1\right)y-6\sqrt{3}-3=\left(2-\sqrt{3}\right)x-4+2\sqrt{3}$

$\Rightarrow \left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-4+2\sqrt{3}+6\sqrt{3}+3$

$\Rightarrow \left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=8\sqrt{3}-1$

Taking $\dfrac{2-m_2}{1+2m_2}=-\sqrt{3}$

$\Rightarrow 2-m_2=-\sqrt{3}-2\sqrt{3}m_2$

$\Rightarrow \left(2\sqrt{3}-1\right)m_2=-\left(2+\sqrt{3}\right)$

$\Rightarrow m_2=\dfrac{-\left(2+\sqrt{3}\right)}{2\sqrt{3}-1}$

$\therefore$ Equation of required line is $y-3=\dfrac{-\left(2+\sqrt{3}\right)}{2\sqrt{3}-1}(x-2)$

$\Rightarrow \left(2\sqrt{3}-1\right)y-6\sqrt{3}+3=-\left(2+\sqrt{3}\right)x+4+2\sqrt{3}$

$\Rightarrow \left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=4+2\sqrt{3}+6\sqrt{3}-3$

$\Rightarrow \left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=8\sqrt{3}+1$

13. Find the equation of the right bisector of the line segment joining the points (3, 4) and $(-1, 2)$.

Ans.

Let A(3,4) and B $\left(-1,2\right)$ be the given points

Midpoint of the line segment joining the points A and B = $\left(\frac{3-1}{2},\frac{4+2}{2}\right)$ = (1, 3)

Slope of the line joining points A(3, 4) and B $\left(-1,2\right)$ = $\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

Since the required line is perpendiculat to the line AB,slope of the required line is -2

We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m\left(x-x_0\right)$

Since the required line passes through point (1, 3) and having slope -2.

We have ,equation of the required line is $y-3=-2\left(x-1\right)$

$\Rightarrow y-3=-2x+2$

$\Rightarrow 2x+y-5=0$

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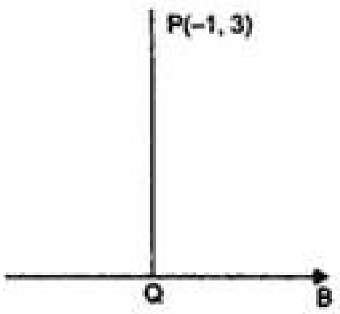
14. Find the coordinates of the foot of perpendicular from the point $\left(-1,3\right)$ to the line $3x$

Ans. Let Q be the foot of perpendicular drawn from P $\left(-1,3\right)$ on the line $3x-4y-16=0$



$\therefore$ Equation of a line perpendicular to $3x-4y-16=0$ is $4x+3y+k=0$

Since the line passes through $\left(-1,3\right)$

$\therefore 4\times\left(-1\right)+3\times 3+k=0$

$\Rightarrow -4+9+k=0$

$\Rightarrow k=-5$

Therefore, Q is a point of intersection of the lines $3x-4y-16=0$ and $4x+3y-5=0$

Solving both the equations, we have $x=\frac{68}{25}$ and $y=\frac{-49}{25}$

Therefore, coordinates of foot of perpendicular are $\left(\frac{68}{25},\frac{-49}{25}\right)$.

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15. The perpendicular from the origin to the line $y=mx+c$ meets it at the point $(-1,2)$. Find the value of m and c.

Ans. Given equation of line is $y=mx+c$.........................(i)

Since the perpendicular from the origin meets the above line (i) at $(-1,2)$,we have the line joining $(0,0)$ and $(-1,2)$ is perpendicular to the line (i)

Now slope of line joining $(0,0)$ and $(-1,2)$= $\frac{2-0}{-1-0}=-2$

and slope of given line =m

But we have if two lines are perpendicular the product of their slopes is equal to -1.

$\therefore (-2)m=-1 \Rightarrow m=\frac{-1}{-2}=\frac{1}{2}$

Since the point $(-1,2)$ lies on the line(i) we get $2=m(-1)+c \Rightarrow 2=\frac{1}{2}(-1)+c$

$\Rightarrow c=2+\frac{1}{2}=\frac{5}{2}$

Hence we have $m=\frac{1}{2}$ and $c=\frac{5}{2}$

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16. If p and q are the lengths of perpendiculars from the origin to the line $x\cos\theta-y\sin\theta=k\cos 2\theta$ and $x\sec\theta+y\operatorname{cosec}\theta=k$ respectively, prove that p^2+4

Ans. We have the length of the perpendicular from of a given point (x_1, y_1) to a given line Ax+BY+C=0 is $d = \left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

Length of perpendicular from origin to line $x\cos\theta - y\sin\theta - k\cos 2\theta = 0$ is

$$p = \left|\frac{0\times\cos\theta - 0\times\sin\theta - k\cos 2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}\right|$$

= $\left|\frac{-k\cos 2\theta}{1}\right|$ = $k\cos 2\theta$

And Length of perpendicular from origin to line $x\sec\theta + y\cos ec\theta - k = 0$ is

$$q = \left|\frac{0\times\sec\theta - 0\times\cos ec\theta - k}{\sqrt{\sec^2\theta+\cos ec^2\theta}}\right| = \left|\frac{-k}{\sqrt{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}}\right|$$

= $\left|\frac{-k}{\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}}}\right|$ = $\left|\frac{-k}{\sqrt{\left(\frac{1}{\cos^2\theta\sin^2\theta}\right)}}\right|$ since $\cos^2\theta+\sin^2\theta = 1$

= $\left|-k\sin\theta\cos\theta\right|$ = $\frac{k}{2}\sin 2\theta$ $[\sin 2\theta = 2\sin\theta\cos\theta]$

Now, $p^2+4q^2 = (k\cos 2\theta)^2 + 4\left(\frac{k}{2}\sin 2\theta\right)^2$

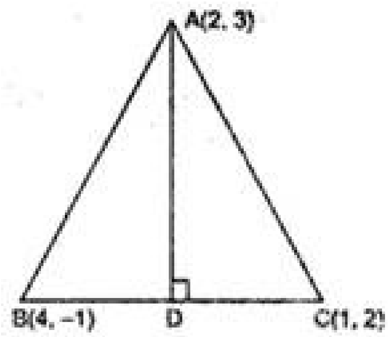
= $k^2\cos^2 2\theta + \frac{4}{4}k^2\sin^2 2\theta$ since $\cos^2\theta+\sin^2\theta = 1$

= $k^2\left(\cos^2 2\theta+\sin^2 2\theta\right) = k^2$

Therefore $p^2+4q^2 = k^2$.

17. In the triangle ABC with vertices A (2, 3), B $(4, -1)$ and C (1, 2), find the equation and length of altitude from the vertex A.

Ans. Slope of BC= $\dfrac{2-(-1)}{1-4}=\dfrac{2+1}{-3}=\dfrac{3}{-3}=-1$



Since AD $\perp$ BC, therefore slope of AD = 1

We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m(x-x_0)$

$\therefore$ Equation of altitude AD is

$$y-3=1(x-2)$$

$$\Rightarrow x-y+1=0$$

And Equation of BC is

$$y+1=-1(x-4)$$

$$\Rightarrow x+y-3=0$$

We have the length of the perpendicular from of a given point (x_1, y_1) to a given line Ax+BY+C=0 is $d=\left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$\therefore$ Length of AD = Perpendicular distance from **(2, 3)** to the line BC= $\left|\dfrac{2+3-3}{\sqrt{(1)^2+(1)^2}}\right|$

= $\left|\frac{2}{\sqrt{2}}\right| = \sqrt{2}$ units

$\therefore$ Equation of the altitude from A is $x - y + 1 = 0$ and it's length = $\sqrt{2}$ units

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans. Given: Line $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow bx + ay = ab$

$\Rightarrow bx + ay - ab = 0$

Now, p is the length of perpendicular from origin to $bx + ay - ab = 0$.

We have the length of the perpendicular from of a given point (x_1, y_1) to a given line Ax+BY+C=0 is $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$

$\therefore p = \left|\frac{b \times 0 + a \times 0 - ab}{\sqrt{b^2 + a^2}}\right| = \frac{ab}{\sqrt{b^2 + a^2}}$

$\Rightarrow p^2 = \frac{a^2 b^2}{b^2 + a^2}$

$\Rightarrow \frac{1}{p^2} = \frac{b^2 + a^2}{a^2 b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{b^2}{a^2 b^2} + \frac{a^2}{a^2 b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$