CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 11 Conic Sections

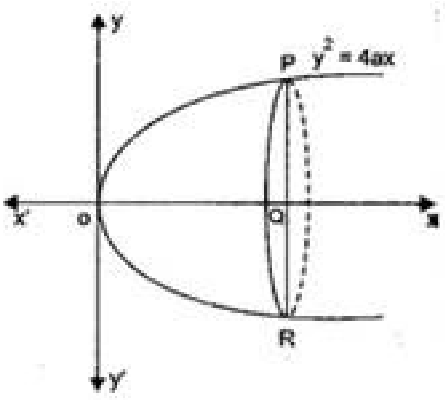
Miscellaneous Exercise

1. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

Ans. A parabolic reflector with diameter PR = 20 cm and OQ = 5 cm

Vertex of parabola is (0, 0)

Let focus of the parabola be $(a, 0)$.



Now, PR = 20 cm

$\Rightarrow$ PQ = 10 cm

$\therefore$ Coordinate of the point P are (5, 10)

Since the point P lies on the parabola $y^2 = 4ax$.

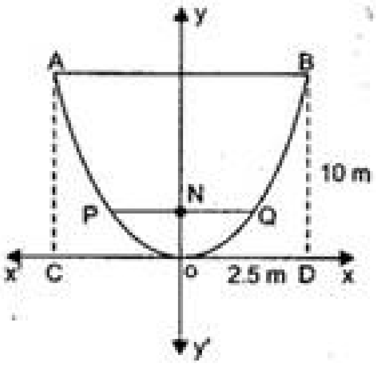
$\therefore (10)^2 = 4a \times 5$

$\Rightarrow a = \dfrac{100}{20} = 5$

Hence the focus i.e (5,0) is the mid point of the given diameter.

2. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

Ans. Let AB be the parabolic arch having O as the vertex and OY as the axis.



The parabola is of the form $x^2 = 4ay$.

Now, CD = 5 m

$\Rightarrow$ OD = 2.5 m

And BD = 10 m

Therefore, coordinates of point B are (2.5, 10).

Since the point B lies on the parabola $x^2 = 4ay$.

$\therefore (2.5)^2 = 4a \times 10$

$\Rightarrow a = \frac{6.25}{40}$

$\Rightarrow a = \frac{5}{32}$

$\therefore$ Equation of the parabola is $x^2 = 4 \times \frac{5}{32} y$

$\Rightarrow x^2 = \frac{5}{8} y$

Let PQ = d

$\Rightarrow$ NQ = $\frac{d}{2}$

$\therefore$ Coordinate of the point Q are $\left(\frac{d}{2}, 2\right)$

Since the point Q lies on the parabola $x^2 = \frac{5}{8}y$

$\therefore \left(\frac{d}{2}\right)^2 = \frac{5}{8} \times 2$

$\Rightarrow \frac{d^2}{4} = \frac{5}{4}$

$\Rightarrow d^2 = 5$

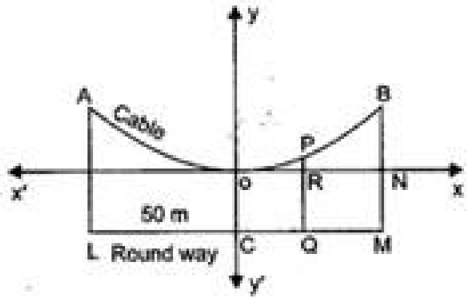
$\Rightarrow d = \sqrt{5}$

Therefore, width of arch is $\sqrt{5}$ m = 2.24 m approx.

3. he cable Tof a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Ans. Let AOB be the cable of uniformly loaded suspension bridge. Let AL and BM be the longest wires of length 30 m each. Let OC be the shortest wire of length 6 m and LM be the roadway.

Now AL = BM = 30 m, OC = 6 m and LM = 100 m



(in figure LM is roadway not roundway)

$\therefore$ LC = CM = $\frac{1}{2}$ LM = 50 m

Let O be the vertex and axis of the parabola be $y-$axis.

Therefore, the equation of the parabola in standard form is $x^2 = 4ay$.

$\therefore$ Coordinates of the point B are (50, 24)

Since point B lies on the parabola $x^2 = 4ay$.

$$\therefore (50)^2 = 4a \times 24$$

$$\Rightarrow a = \frac{2500}{4 \times 24} = \frac{625}{24}$$

Therefore, equation of parabola is $x^2 = \frac{4 \times 625}{24} y$

$$\Rightarrow x^2 = \frac{625}{6} y$$

Let length of the supporting wire PQ at a distance of 18 m be h.

$\therefore$ OR = 18 m and PR = PQ – QR = PQ – OC = $h-6$

$\therefore$ Coordinates of point P are $(18, h-6)$

Now, since the point P lies on parabola $x^2 = \frac{625}{6} y$

$\therefore (18)^2 = \frac{625}{6}(h-6)$

$\Rightarrow 324 \times 6 = 625h - 3750$

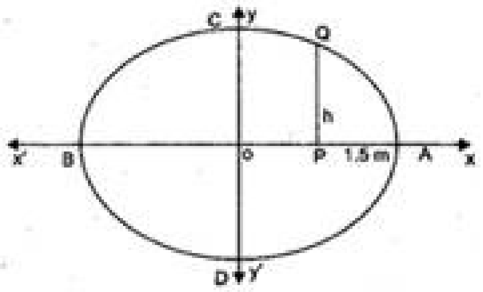
$\Rightarrow 625h = 1944 + 3750$

$\Rightarrow h = \frac{5694}{625}$ =9.11 m approx.

4. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Ans. Given: Width of elliptical arch (AB)

= $2a$ = 8 $\Rightarrow$ a = 4 m



Height of the centre (OB) = b = 2 m

The axis of ellipse is $x-$ axis.

Therefore, the equation of ellipse in standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$\therefore \frac{x^2}{(4)^2} + \frac{y^2}{(2)^2} = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

Now, AP = 1.5 m

$\therefore$ OP = OA – AP = 4 – 1.5 = 2.5 m

Let PQ = h, then coordinates of Q are $(2.5, h)$

Since the point Q lies on the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$

$\therefore \frac{(2.5)^2}{16} + \frac{h^2}{4} = 1$

$=> \frac{6.25}{16} + \frac{h^2}{4} = 1$

$=> \frac{h^2}{4} = 1 - \frac{6.25}{16}$

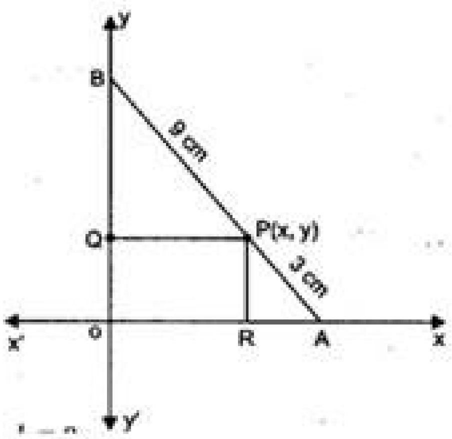
$\Rightarrow h^2 = \frac{9.75 \times 4}{16} = \frac{9.75}{4}$

$\Rightarrow h^2 = 2.44$

$\Rightarrow h$ = 1.56 m approx.

5. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the $x-$axis.

Ans. Let AB be a rod of length 12 cm and P(x, y) be any point on the rod such that PA = 3 cm and PB = 9 cm.



Let AR = a and BQ = b

Then $\Delta ARP \sim \Delta PQB$

$$\therefore \frac{AR}{PQ} = \frac{AP}{PB}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{9}$$

$$\Rightarrow 9a = 3x$$

$$\Rightarrow a = \frac{x}{3}$$

And $\frac{BQ}{BP} = \frac{PR}{PA}$

$$\Rightarrow \frac{b}{9} = \frac{y}{3}$$

$$\Rightarrow 3b = 9y$$

$$\Rightarrow b = 3y$$

Now, OA = OR + AR = $x + a = x + \frac{x}{3} = \frac{4x}{3}$

And OB = OQ + BQ = $y + b = y + 3y = 4y$

In right angled triangle AOB, $AB^2 = OA^2 + OB^2$

$$\Rightarrow (12)^2 = \left(\frac{4x}{3}\right)^2 + (4y)^2$$

$$\Rightarrow 144 = \frac{16x^2}{9} + 16y^2$$

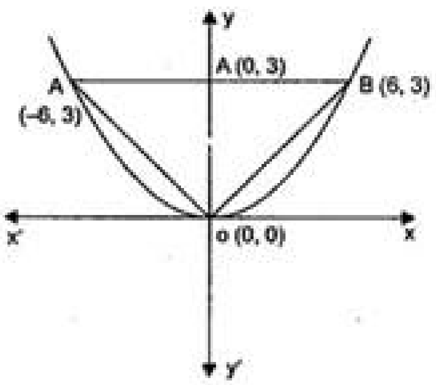
$$\Rightarrow \frac{16x^2}{9\times144}+\frac{16y^2}{144}=1$$

$$\Rightarrow \frac{x^2}{81}+\frac{y^2}{9}=1$$

which is required locus of point P and which represents an ellipse.

6. Find the area of the triangle formed by the lines the vertex of the parabola $x^2=12y$ to the ends of its latus rectum.

Ans. Given: Equation of parabola $x^2=12y$ which is in the form of $x^2=4ay$



$\therefore 4a=12$

$\Rightarrow a=3$

Focus of the parabola is (0, 3).

Let AB be the latus rectum of the parabola, then $y=3$

$\therefore x^2=4\times3\times3=36$

$\Rightarrow x=\pm6$

The coordinates of A are $(-6,3)$

and coordinates of B are (6, 3).

$\therefore$ Area of ΔOAB

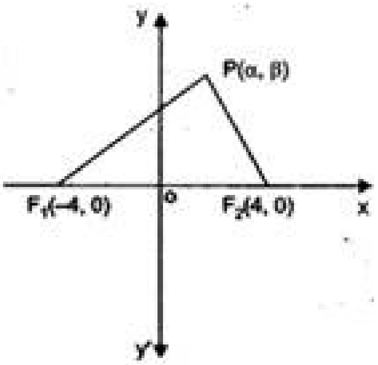
= $\frac{1}{2}|0(3-3)+6(3-0)+(-6)(0-3)|$

= $\frac{1}{2}|0+18+18|$

= $\frac{1}{2}\times|36|$ = 18 sq. units

7. A man running a race course notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

Ans. Let $F_1 and\ F_2$ be two points where the flag parts are fixed on the ground. The origin O is the mid-point of F_1F_2 .



$\therefore OF_1 = OF_2 = \frac{1}{2}F_1F_2$ = $\frac{1}{2}\times 8$ = 4 m

$\therefore$ Coordinates of F_1 are $(-4, 0)$ and F_2 are (4, 0).

Let P(α, β) be any point on the track.

$\therefore PF_1 + PF_2 = 10$

$\Rightarrow \sqrt{(\alpha+4)^2+(\beta-0)^2}+\sqrt{(\alpha-4)^2+(\beta-0)^2}=10$

$\Rightarrow \sqrt{\alpha^2+16+8\alpha+\beta^2}=10-\sqrt{\alpha^2+16-8\alpha+\beta^2}$

Squaring both sides, we have,

$\Rightarrow\ \alpha^2+\beta^2+8\alpha+16=100+\alpha^2+\beta^2-8\alpha+16-20\sqrt{\alpha^2+16-8\alpha+\beta^2}$

$\Rightarrow\ 16\alpha-100=-20\sqrt{\alpha^2+16-8\alpha+\beta^2}$

Squaring both sides again, we have,

$\Rightarrow\ 256\alpha^2+10000-3200\alpha=400\left(\alpha^2+\beta^2-8\alpha+16\right)$

$\Rightarrow\ 256\alpha^2+10000-3200\alpha=400\alpha^2+400\beta^2-3200\alpha+6400$

$\Rightarrow\ 144\alpha^2+400\beta^2=3600$

Divide both sides of equation by 3600

$\Rightarrow\ \dfrac{144\alpha^2}{3600}+\dfrac{400\beta^2}{3600}=1$

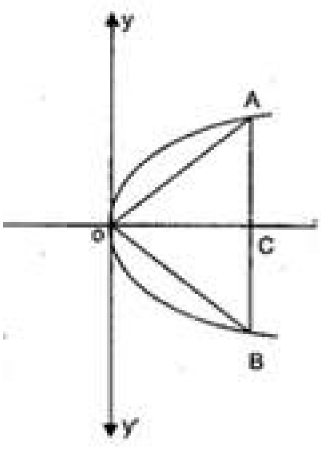
$\Rightarrow \dfrac{\alpha^2}{25}+\dfrac{\beta^2}{9}=1$

=> $\frac{x^2}{25}+\frac{y^2}{9}=1$

which is the required equation of locus of point P.

8. An equilateral triangle is inscribed in the parabola $y^2=4ax$ where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Ans. Given: Equation of the parabola $y^2=4ax$.



Let b be the side of an equilateral ΔOAB whose one vertex

is the vertex of parabola and let OC = x

Now, AB = b

$\therefore$ AC = BC = $\frac{1}{2} \times AB$ = $\frac{b}{2}$

Coordinates of point A are $\left(x, \frac{b}{2}\right)$.

Since, point A lies on the parabola $y^2 = 4ax$

$$\Rightarrow \left(\frac{b}{2}\right)^2 = 4ax$$

$$\Rightarrow x = \frac{b^2}{4 \times 4a}$$

$$\Rightarrow x = \frac{b^2}{16a}$$

In right angled triangle ΔOAC

$OA^2 = OC^2 + AC^2$

$\Rightarrow b^2 = x^2 + \left(\frac{b}{2}\right)^2$

$\Rightarrow b^2 = \left(\frac{b^2}{16a}\right)^2 + \frac{b^2}{4}$

$\Rightarrow b^2 = \frac{b^4}{256a^2} + \frac{b^2}{4}$

$\Rightarrow 1 = \frac{b^2}{256a^2} + \frac{1}{4}$

$\Rightarrow \frac{b^2}{256a^2} = 1 - \frac{1}{4}$

$\Rightarrow b^2 = \frac{3}{4} \times 256a^2$

$\Rightarrow b^2 = 192a^2$

$\Rightarrow b = 8\sqrt{3}a$

Therefore, the side of triangle is $8\sqrt{3}a$.