CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 11 Conic Sections

Exercise 11.3

In each of the Exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $\frac{x^2}{36}+\frac{y^2}{16}=1$

Ans. Given: Equation of ellipse: $\frac{x^2}{36}+\frac{y^2}{16}=1$

$\because$ 36 > 16 (it's a horizontal ellipse)

$\therefore a^2=36, b^2=16$

$\Rightarrow a=6, b=4$

Now $c=\sqrt{a^2-b^2}$ = $\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow (\pm 2\sqrt{5}, 0)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 6, 0)$

Length of major axis = $2a$ = $2\times 6=12$ units

Length of minor axis = $2b$ = $2\times 4=8$ units

Eccentricity $\left(e\right)=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 16}{6}=\frac{16}{3}$ units

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2. $\frac{x^2}{4}+\frac{y^2}{25}=1$

Ans. Given: Equation of ellipse: $\frac{x^2}{4}+\frac{y^2}{25}=1$

$\because$ 25 > 4 (it's a vertical ellipse)

$\therefore a^2=25, b^2=4$

$\Rightarrow a=5, b=2$

Now $c=\sqrt{a^2-b^2}$ = $\sqrt{25-4}=\sqrt{21}$

$\therefore$ Coordinates of foci are $\left(0,\pm c\right)$

$\Rightarrow \left(0,\pm\sqrt{21}\right)$

Coordinates of vertices are $\left(0,\pm a\right)$

$\Rightarrow \left(0,\pm 5\right)$

Length of major axis = $2a$ = $2\times 5=10$ units

Length of minor axis = $2b$ = $2\times 2=4$ units

Eccentricity $\left(e\right)=\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$ units

3. $\frac{x^2}{16}+\frac{y^2}{9}=1$

Ans. Given: Equation of ellipse: $\frac{x^2}{16}+\frac{y^2}{9}=1$

$\because$ 16 > 9 (it's a horizontal ellipse)

$\therefore a^2=16, b^2=9$

$\Rightarrow a=4, b=3$

Now $c=\sqrt{a^2-b^2}$ = $\sqrt{16-9}=\sqrt{7}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow (\pm\sqrt{7}, 0)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 4, 0)$

Length of major axis = $2a$ = 2×4 $=$ 8 units

Length of minor axis = $2b$ = $2\times 3=6$ units

Eccentricity $(e)=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$ units

4. $\frac{x^2}{25}+\frac{y^2}{100}=1$

Ans. Given: Equation of ellipse: $\frac{x^2}{25}+\frac{y^2}{100}=1$

$\because$ 100 > 25 (it's a vertical ellipse)

$\therefore a^2=100, b^2=25$

$\Rightarrow a=10, b=5$

Now $c=\sqrt{a^2-b^2}$ = $\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$

$\Rightarrow \left(0, \pm 5\sqrt{3}\right)$

Coordinates of vertices are $(0, \pm a)$

$\Rightarrow (0, \pm 10)$

Length of major axis = $2a$ = $2\times 10=20$ units

Length of minor axis = $2b$ = $2\times 5=10$ units

Eccentricity $(e)=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 25}{10}=5$ units

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5. $\frac{x^2}{49}+\frac{y^2}{36}=1$

Ans. Given: Equation of ellipse: $\frac{x^2}{49}+\frac{y^2}{36}=1$

$\because$ 49 > 36 (it's a horizontal ellipse)

$\therefore a^2 = 49, b^2 = 36$

$\Rightarrow a = 7, b = 6$

Now $c = \sqrt{a^2 - b^2}$ = $\sqrt{49-36} = \sqrt{13}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow (\pm\sqrt{13}, 0)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 7, 0)$

Length of major axis = $2a$ = $2 \times 7 = 14$ units

Length of minor axis = $2b$ = $2 \times 6 = 12$ units

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 36}{7} = \frac{72}{7}$ units

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6. $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Ans. Given: Equation of ellipse: $\frac{x^2}{100} + \frac{y^2}{400} = 1$

$\because$ 400 > 100 (it's a vertical ellipse)

$\therefore a^2 = 400, b^2 = 100$

$\Rightarrow a = 20, b = 10$

Now $c = \sqrt{a^2 - b^2}$ = $\sqrt{400-100} = \sqrt{300} = 10\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$

$\Rightarrow \left(0, \pm 10\sqrt{3}\right)$

Coordinates of vertices are $(0, \pm a)$

$\Rightarrow (0, \pm 20)$

Length of major axis = $2a$ = $2 \times 20 = 40$ units

Length of minor axis = $2b$ = $2 \times 10 = 20$ units

Eccentricity $(e) = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$ units

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7. $36x^2 + 4y^2 = 144$

Ans. Given: Equation of ellipse: $36x^2 + 4y^2 = 144$

$\Rightarrow \frac{36x^2}{144} + \frac{4y^2}{144} = 1$ (divide both sides of equation by 144)

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$

$\because$ 36 > 4 (it's a vertical ellipse)

$\therefore a^2 = 36, b^2 = 4$

$\Rightarrow a = 6, b = 2$

Now $c = \sqrt{a^2 - b^2}$ = $\sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$

$\therefore$ Coordinates of foci are $(0, \pm c)$

$\Rightarrow \left(0, \pm 4\sqrt{2}\right)$

Coordinates of vertices are $\left(0, \pm a\right)$

$\Rightarrow \left(0, \pm 6\right)$

Length of major axis = $2a$ = 2×6 $=$ 12 units

Length of minor axis = $2b$ = 2×2 $= 4$ units

Eccentricity $\left(e\right) = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$ units

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8. $16x^2 + y^2 = 16$

Ans. Given: Equation of ellipse: $16x^2 + y^2 = 16$

$\Rightarrow \frac{16x^2}{16} + \frac{y^2}{16} = 1$ (divide both sides of equation by 16)

$\Rightarrow \frac{x^2}{1} + \frac{y^2}{16} = 1$

$\because$ 16 > 1 (it's a vertical ellipse)

$\therefore$ $a^2 = 16, b^2 = 1$

$\Rightarrow a = 4, b = 1$

Now $c = \sqrt{a^2 - b^2}$ = $\sqrt{16 - 1} = \sqrt{15}$

$\therefore$ Coordinates of foci are $\left(0, \pm c\right)$

$\Rightarrow \left(0, \pm\sqrt{15}\right)$

Coordinates of vertices are $(0, \pm a)$

$\Rightarrow (0, \pm 4)$

Length of major axis = $2a$ = $2 \times 4 = 8$ units

Length of minor axis = $2b$ = $2 \times 1 = 2$ units

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{15}}{4}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$ units

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9. $4x^2 + 9y^2 = 36$

Ans. Given: Equation of ellipse: $4x^2 + 9y^2 = 36$

$\Rightarrow \frac{4x^2}{36} + \frac{9y^2}{36} = 1$ (divide both sides of equation by 36)

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

$\because$ 9 > 4 (it's a horizontal ellipse)

$\therefore a^2 = 9, b^2 = 4$

$\Rightarrow a = 3, b = 2$

Now $c = \sqrt{a^2 - b^2}$ = $\sqrt{9-4} = \sqrt{5}$

$\therefore$ Coordinates of foci are $(\pm c, 0)$

$\Rightarrow \left(\pm\sqrt{5}, 0\right)$

Coordinates of vertices are $(\pm a, 0)$

$\Rightarrow (\pm 3, 0)$

Length of major axis = $2a$ = $2\times 3 = 6$ units

Length of minor axis = $2b$ = $2\times 2 = 4$ units

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 4}{3} = \frac{8}{3}$ units

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In each of the Exercises 10 to 20, find the equation of the ellipse that satisfies the given conditions:

10. Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Ans. Since foci $(\pm 4, 0)$ lie on $x-$ axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now Vertices $(\pm a, 0) = (\pm 5, 0)$

$\Rightarrow a = 5$

And Foci $(\pm c, 0) = (\pm 4, 0)$

$\Rightarrow c = 4$

$\because c^2 = a^2 - b^2 \therefore (4)^2 = (5)^2 - b^2$

$\Rightarrow b^2 = 25 - 16 = 9$

Therefore, the required equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

11. Vertices $(0,\pm 13)$, **foci** $(0,\pm 5)$

Ans. Since foci $(0,\pm 5)$ lie on $y-$axis, therefore equation of ellipse is $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$

Now Vertices $(0,\pm a)=(0,\pm 13)$

$\Rightarrow a=13$

And Foci $(0,\pm c)=(0,\pm 5)$

$\Rightarrow c=5$

$\because c^2=a^2-b^2$

$\therefore (5)^2=(13)^2-b^2$

$\Rightarrow b^2=169-25=144$

Therefore, the required equation of ellipse is $\frac{x^2}{144}+\frac{y^2}{169}=1$.

12. Vertices $(\pm 6,0)$, **foci** $(\pm 4,0)$

Ans. Since foci $(\pm 4,0)$ lie on $x-$axis, therefore equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Now Vertices $(\pm a,0)=(\pm 6,0)$

$\Rightarrow a=6$

And Foci $(\pm c,0)=(\pm 4,0)$

$\Rightarrow c = 4$

$\because c^2 = a^2 - b^2$

$\therefore (4)^2 = (6)^2 - b^2$

$\Rightarrow b^2 = 36 - 16 = 20$

Therefore, the required equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{20} = 1$.

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13. Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Ans. Ends of major axis $(\pm 3, 0)$ lie on $x-$axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now Ends of major axis $(\pm a, 0)$ =$(\pm 3, 0)$

$\Rightarrow a = 3$

And Ends of minor of axis $(0, \pm b)$ = $(0, \pm 2)$

$\Rightarrow b = 2$

Therefore, the required equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

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14. Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Ans. Ends of major axis $(0, \pm\sqrt{5})$ lie on $y-$axis, therefore equation of ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Now Ends of major axis $(0, \pm a)$ = $(0, \pm\sqrt{5})$

$\Rightarrow \ a = \sqrt{5}$

And Ends of minor of axis $(\pm b, 0) = (\pm 1, 0)$

$\Rightarrow \ b = 1$

Therefore, the required equation of ellipse is $\frac{x^2}{1} + \frac{y^2}{5} = 1$.

15. Length of major axis 26, foci $(\pm 5, 0)$

Ans. Since foci $(\pm 5, 0)$ lie on $x-$ axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore$ Length of major axis = $2a = 26$

$\Rightarrow a = 13$

Foci $(\pm c, 0)$ = $(\pm 5, 0)$

$\Rightarrow c = 5$

$c^2 = a^2 - b^2$

$\Rightarrow \ (5)^2 = (13)^2 - b^2$

$\Rightarrow \ b^2 = 169 - 25 = 144$

Therefore, the required equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

16. Length of minor axis 16, foci $(0, \pm 6)$

Ans. Since foci $(0, \pm 6)$ lie on $y-$ axis, therefore equation of ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\therefore$ Length of major axis = $2b = 16$

$\Rightarrow b = 8$

Foci $(0, \pm c)$ = $(0, \pm 6)$

$\Rightarrow c = 6$

$c^2 = a^2 - b^2$

$\Rightarrow (6)^2 = a^2 - (8)^2$

$\Rightarrow a^2 = 36 + 64 = 100$

Therefore, the required equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{100} = 1$.

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17. Foci $(\pm 3, 0)$, $a = 4$

Ans. Since foci $(\pm 3, 0)$ lie on $x-$ axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore$ Foci $(\pm c, 0)$ = $(\pm 3, 0)$

$\Rightarrow c = 3$
$c^2 = a^2 - b^2$
$\Rightarrow (3)^2 = (4)^2 - b^2$
$\Rightarrow b^2 = 16 - 9 = 7$
Therefore, the required equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1$.

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18. $b = 3, c = 4,$ centre at origin; foci on $x-$ axis

Ans. Since foci lie on $x-$ axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore c^2 = a^2 - b^2$

$\Rightarrow (4)^2 = a^2 - (3)^2$

$\Rightarrow a^2 = 16 + 9 = 25$

Therefore, the required equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

19. Centre at (0, 0), major axis on the $y-$axis and passes through the points (3, 2) and (1, 6).

Ans. Since the major axis is along $y-$axis, therefore equation of ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

And the ellipse passes through the point (3, 2) therefore $\frac{9}{b^2} + \frac{4}{a^2} = 1$ .....(i)

And the ellipse passes through the point (1, 6) therefore $\frac{1}{b^2} + \frac{36}{a^2} = 1$ .....(ii)

Solving eq. (i) and (ii), we have $a^2 = 40, b^2 = 10$

Therefore, the required equation of ellipse is $\frac{x^2}{10} + \frac{y^2}{40} = 1$.

20. Major axis on the $x-$ axis and passes through the points (4, 3) and (6, 2).

Ans. Since the major axis is along $x-$ axis, therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

And the ellipse passes through the point (4, 3) therefore $\frac{16}{a^2} + \frac{9}{b^2} = 1$ .....(i)

And the ellipse passes through the point (6, 2) therefore $\frac{36}{a^2} + \frac{4}{b^2} = 1$ .....(ii)

Solving eq. (i) and (ii), we have $a^2 = 52, b^2 = 13$

Therefore, the required equation of ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$.