CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 16 Probability

Exercise 16.3

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space S = $\{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$

Assignment	w_1	w_2	w_3	w_4	w_5	w_6	w_7
a	0.1	0.01	0.05	0.03	0.01	0.2	0.6
b	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
c	0.1	0.2	0.3	0.4	0.5	0.6	0.7
d	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
e	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Ans. (a) Here probability of each outcome is positive and less than 1 and sum of probabilities is = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1

$\therefore$ Both the conditions of axiomatic approach are satisfied. Therefore, the assignment is valid.

(b) Here probability of each outcome is positive and less than 1 and sum of probabilities is

$$= \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=1$$

$\therefore$ Both the conditions of axiomatic approach are satisfied. Therefore, the assignment is valid.

(c) Here probability of each outcome is positive and less than 1 and sum of probabilities is

= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 > 1

$\therefore$ One of the conditions of axiomatic approach is not satisfied. Therefore, the assignment is not valid.

(d) Here probabilities of two events w_1 and w_5 are negative. Therefore, the assignment is not valid.

(e) Here probability $\frac{15}{14}$ is more than 1

Therefore, the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Ans. Here S = {HH, HT, TH, TT}

$\therefore$ Number of possible outcomes n (S) = 4

Let E be the event of getting at least one tail, therefore, n (E) = 3

$\therefore$ Probability of getting at least one tail P(E) = $\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{3}{4}$

3. A die is thrown, find the probability of following events:

(i) A prime number will appear.

(ii) A number greater than or equal to 3 will appear.

(iii) A number less than or equal to one will appear.

(iv) A number more than 6 will appear.

(v) A number less than 6 will appear.

Ans. Here S = {1, 2, 3, 4, 5, 6} $\therefore$ n(S) = 6

(i) Let A be the event of getting a prime number,

then A = {2, 3, 5} and n(A) = 3

$\therefore$ P(A) = $\frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{3}{6} = \frac{1}{2}$

(ii) Let B be the event of getting a number greater than or equal to 3,

Then B = {3, 4, 5, 6}

$\Rightarrow n(\mathrm{B})$ = 4

$\therefore$ P(B) = $\frac{n(\mathrm{B})}{n(\mathrm{S})} = \frac{4}{6} = \frac{2}{3}$

(iii) Let C be the event of getting a number less than or equal to 1, then

C = {1} $\Rightarrow n(\mathrm{C})$ = 1

$\therefore$ P(C) = $\frac{n(\mathrm{C})}{n(\mathrm{S})} = \frac{1}{6}$

(iv) Let D be the event of getting a number more than 6, then D = ϕ and $n(\mathrm{D})$ = 0

$\therefore$ P(D) = $\frac{n(\mathrm{D})}{n(\mathrm{S})} = \frac{0}{6} = 0$

(v) Let E be the event of getting a number less than 6, then

E = {1, 2, 3, 4, 5}

$\Rightarrow n(\mathrm{E})$ = 5

$\therefore$ P(E) = $\frac{n(\mathrm{E})}{n(\mathrm{S})} = \frac{5}{6}$

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4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Ans. (a) Number of points in the sample space (S) = 52

(b) Let A be the event of drawing an ace of spades. There is only one ace of spade.

$\therefore$ P(A) = $\frac{1}{52}$

(c) (i) Let B be the event of drawing an ace. There are four aces.

$\therefore$ P(B) = $\frac{4}{52} = \frac{1}{13}$

(ii) Let C be the event of drawing a black card. There are 26 black cards.

$\therefore$ P(C) = $\frac{26}{52} = \frac{1}{2}$

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12.

Ans. The coin and die are tossed together.

$\therefore$ S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}

$\Rightarrow n(S)$ = 12

(i) Let A be the event having sum of numbers is 3.

$\therefore$ A = {(1, 2)}

$\Rightarrow n(A)$ = 1

$\therefore$ P(A) = $\frac{1}{12}$

(ii) Let B be the event having sum of number is 12.

$\therefore$ B = {(6, 6)}

$\Rightarrow n(\mathrm{B})$ = 1

$\therefore$ P(B) = $\frac{1}{12}$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Ans. Here total members in the council = 4 + 6 = 10

$\therefore n(\mathrm{S}) = {}^{10}\mathrm{C}_1$ = 10

Let A be the event that the member is a woman.

$\therefore n(\mathrm{A}) = {}^{6}\mathrm{C}_1$ = 6

$\therefore$ P(A) = $\frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{6}{10} = \frac{3}{5}$

7. A fair coin is tossed four times and a person with Re 1 each head and lose Rs. 1.50 for each tail that turns up. Form the sample space, calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Ans. Here the sample space is,

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, TTTH,

TTHT, THTT, HTTT, TTTT}

(i) For 4 heads = 1 + 1 + 1 + 1 = Rs. 4 = winning Rs. 4

(ii) For 3 heads 1 tail = 1 + 1 + 1 – 1.50 = Rs. 1.50 = winning Rs. 1.50

(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50 = Rs. (-1) = losing Re. 1

(iv) For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50 = Rs. (-3.50) = losing Rs. 3.50

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50 = Rs. (-6) = losing Rs. 6

Therefore, the sample space of amounts is

= {Winning Rs 4, Winning Rs 1.50, Losing Rs 1, Losing Rs 3.50, Losing Rs 6}

$\therefore$ P(winning Rs. 4) = $\frac{1}{16}$

P(winning Rs. 1.50) = $\frac{4}{16}=\frac{1}{4}$

P(losing Rs. 1) = $\frac{6}{16}=\frac{3}{8}$

P(losing Rs. 3.50) = $\frac{4}{16}=\frac{1}{4}$

P(losing Rs.6) = $\frac{1}{16}$

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8. Three coins are tossed once. Find the probability of getting:

(i) 3 heads **(ii) 2 heads**

(iii) at least 2 heads **(iv) at most 2 heads**

(v) no head **(vi) 3 tails**

(vii) exactly 2 tails **(viii) no tail**

(ix) at most two tails

Ans. When three coins are tossed then S = {HHH, HHT, HTH, THH, TTH, HTT,THT, TTT}

(i) P(getting 3 heads) = P(HHH) = $\frac{1}{8}$

(ii) P(getting 2 heads) = P(HHT, HTH, THH) = $\frac{3}{8}$

(iii) P(getting at least 2 heads) = P(HHH, HHT, HTH, THH) = $\frac{4}{8}=\frac{1}{2}$

(iv) P(getting at most 2 heads) = P(HHT, HTH, THH, TTH, THT, HTT, TTT) = $\frac{7}{8}$

(v) P(getting no heads) = P(TTT) = $\frac{1}{8}$

(vi) P(getting 3 tails) = P(TTT) = $\frac{1}{8}$

(vii) P(getting exactly 2 tails) = P(TTH, THT, HTT) = $\frac{3}{8}$

(viii) P(getting no tails) = P(HHH) = $\frac{1}{8}$

(ix) P(getting almost 2 tails) = P(HHH, TTH, THT, HTT,HHT, HTH, THH) = $\frac{7}{8}$

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9. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'?

Ans. Given: P(A) = $\frac{2}{11}$

$\therefore$ P(not A) = 1 – P(A) = $1-\frac{2}{11}=\frac{9}{11}$

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10. A letter is chosen at random from the word ASSASSINATION. Find the probability that letter is:

(i) a Vowel

(ii) a Consonant

Ans. There are 13 letters in the word ASSASSINATION which contains 6 vowels and 7 consonants.

$\therefore$ One letter is selected out of 13 letters in ${}^{13}C_1 = 13$ ways

(i) One vowel is selected out of 6 vowels in ${}^{6}C_1$ = 6 ways

$\therefore$ Probability of a vowel = $\frac{6}{13}$

(ii) One consonant is selected out of 7 consonants in ${}^{7}C_1$ = 7 ways

$\therefore$ Probability of a consonant = $\frac{7}{13}$

11. In a lottery, a person choses six different natural numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

Ans. Total numbers of numbers in the draw = 20 and numbers to be selected = 6

Let A be the event that six numbers match with the six numbers fixed by the lottery committee.

$\therefore n(A) = {}^{6}C_6$ =1

$\therefore$ Probability of winning the prize

P(A) = $\frac{n(A)}{n(S)} = \frac{{}^{6}C_6}{{}^{20}C_6}$ = $\frac{6! \times 14!}{20!}$ = $\frac{6\times5\times4\times3\times2\times1\times14!}{20\times19\times18\times17\times16\times15\times14!}$

= $\frac{1}{38760}$

12. Check whether the following probabilities P(A) and P(B) are consistently defined:

(i) P(A) = 0.5, P(B) = 0.7, P($A \cap B$) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P($A \cup B$) = 0.8

Ans. (i) Here P(A) = 0.5, P(B) = 0.7 and $P(A \cap B)$ = 0.6

Now $P(A \cap B) > P(A)$

Therefore, the given probabilities are not consistently defined.

(ii) Here P(A) = 0.5, P(B) = 0.4 and $P(A \cup B)$ = 0.8

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.8 = 0.5 + 0.4 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.9 - 0.8 = 0.1$

$\therefore P(A \cap B) < P(A)$ and $P(A \cap B) < P(B)$

Therefore, the given probabilities are consistently defined.

13. Fill in the blanks in following table:

	P(A)	**P(B)**	**P($A \cap B$)**	**P($A \cup B$)**
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	**.....**
(ii)	**0.35**	**....**	**0.25**	**0.6**
(iii)	**0.5**	**0.35**	**....**	**0.7**

Ans. (i) Here $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cap B) = \frac{1}{15}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{7}{15}$

(ii) Here $P(A) = 0.35, P(A \cap B) = 0.25$ and $P(A \cup B) = 0.6$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = 0.35 + P(B) - 0.25$

$\Rightarrow P(B) = 0.6 - 0.1 = 0.5$

(iii) Here P(A) = 0.5, P(B) = 0.35 and $P(A \cup B)$ = 0.7

$\Rightarrow 0.7 = 0.5 + 0.35 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.85 - 0.7 = 0.15$

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14. Given P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Ans. Given: $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$

Since A and B are mutually exclusive events.

$\therefore P(A \cup B) = P(A) + P(B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

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15. If E and F are events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$. Find:

(i) P (E or F)

(ii) P (not E and not F)

Ans. Given: P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and $P(E \cap F) = \frac{1}{8}$

(i) Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ = $\frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$

(ii) P(not E and not F) = $P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$ (by De' Morgan's law)

= $1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

16. Events E and F are such that P (not E or not F) = 0.25. State whether E and F are mutually exclusive.

Ans. Given: P(not E and not F) = 0.25

$\Rightarrow P(\overline{E} \cup \overline{F}) = 0.25$

$\Rightarrow P(\overline{E \cap F}) = 0.25$ (by De' Morgan's law)

$\Rightarrow = 1 - P(E \cap F) = 0.25$

$\Rightarrow P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

Therefore, E and F are not mutually exclusive events.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine:

(i) P (not A)

(ii) P (not B)

(iii) P (A or B)

Ans. Given: P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16

(i) P (not A) = $P\left(\overline{A}\right)=1-P\left(A\right)$ = 1 – 0.42 = 0.58

(ii) P (not B) = $P\left(\overline{B}\right)=1-P\left(B\right)$ = 1 – 0.48 = 0.52

(iii) $P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$ = 0.42 + 0.48 – 0.16 = 0.74

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18. In class XI of a school 40% of the students study Mathematics and 30% study Biology, 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Ans. Let A be the event that the student is studying mathematics and B be the event that the student is studying biology.

Then $P\left(A\right)=\frac{40}{100}=\frac{2}{5}$, $P\left(B\right)=\frac{30}{100}=\frac{3}{10}$ and $P\left(A\cap B\right)=\frac{10}{100}=\frac{1}{10}$

$\therefore P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$

= $\frac{2}{5}+\frac{3}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$ = 0.6

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19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Ans. Let A be the event that the student passes the first examination and B be the event that the students passes the second examination.

Then P(A) = 0.8, P(B) = 0.7 and $P\left(A\cup B\right)$ = 0.95

$\therefore P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$

$\Rightarrow 0.95=0.8+0.7-P\left(A\cap B\right)$

$\Rightarrow P(A \cap B)$ = 1.5 – 0.95 = 0.55

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing the Hindi examination?

Ans. Let A be the event that the student passes English examination and B be the event that the students passes Hindi examination.

Then $P(A \cap B) = 0.5, P(\overline{A} \cap \overline{B}) = 0.1$ and $P(A)$ = 0.75

Now $P(\overline{A} \cap \overline{B}) = 0.1$

$1 - P(A \cup B) = 0.1$

$\Rightarrow P(A \cup B) = 0.9$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow$ P(B) = 0.9 – 0.25 = 0.65

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that:

(i) The student opted for NCC or NSS

(ii) The student has opted neither NCC nor NSS

(iii) The student has opted NSS but not NCC

Ans. Given: Total number of students $n(S)$ = 60

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Then $n(\mathrm{A}) = 30$, $n(\mathrm{B}) = 32$ and $n(\mathrm{A} \cap \mathrm{B})$ = 24

$\therefore \mathrm{P}(\mathrm{A}) = \dfrac{n(\mathrm{A})}{n(\mathrm{S})} = \dfrac{30}{60} = \dfrac{1}{2}$ and $\mathrm{P}(\mathrm{B}) = \dfrac{n(\mathrm{B})}{n(\mathrm{S})} = \dfrac{32}{60} = \dfrac{8}{15}$

And $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \dfrac{n(\mathrm{A} \cap \mathrm{B})}{n(\mathrm{S})} = \dfrac{24}{60} = \dfrac{2}{5}$

$\because \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$

(i) P(Student opted for NCC or NSS) = $\dfrac{1}{2} + \dfrac{8}{15} - \dfrac{2}{5} = \dfrac{19}{30}$

(ii) P(Student has opted neither NCC nor NSS) = $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

= $1 - \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 1 - \dfrac{19}{30} = \dfrac{11}{30}$

(iii) P(Student has opted NSS but not NCC) = $\mathrm{P}(\mathrm{A}' \cap \mathrm{B}) = \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \dfrac{8}{15} - \dfrac{2}{5} = \dfrac{2}{15}$