CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 10 Straight Lines

Exercise 10.1

1. Draw a quadrilateral in the Cartesian plane, whose vertices are $(-4,5),(0,7),(5,-5)$ and $(-4,-2)$. Also, find its area.

Ans. Let $A(-4,5), B(0,7), C(5,-5)$ and $D(-4,-2)$ be the vertices of the quadrilateral

Join the vertices A and C to obtain the diagonal AC

Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD....................(i)

But we have area of a triangle with vertices $(x_1, y_1), (x_2, y_2)$and (x_3, y_3) is

$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

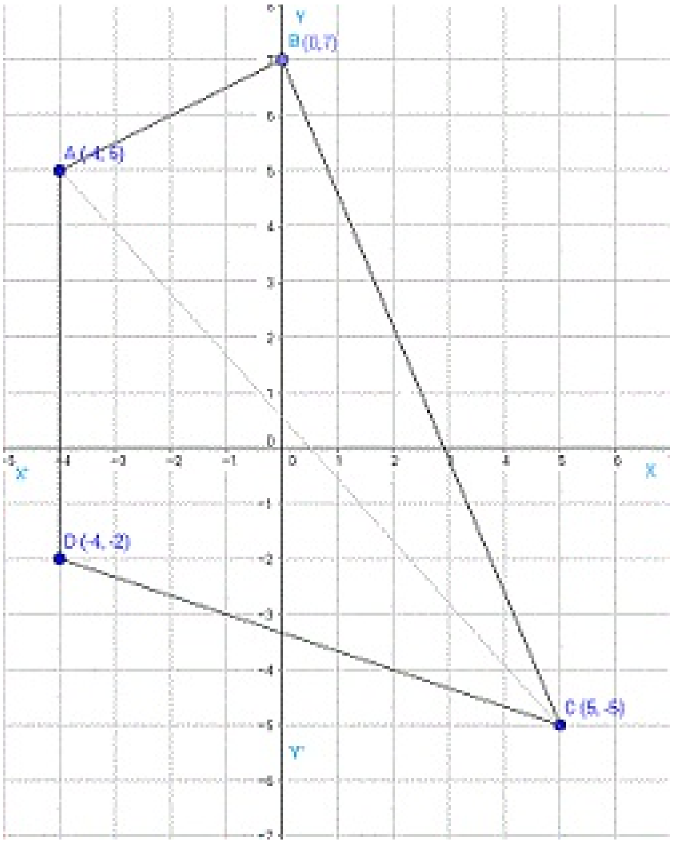
$\therefore$ Area of triangle ABC = $\frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\right|$

= $\frac{1}{2}\left|-4(12)+0+5(-2)\right| = \frac{1}{2}\left|-48+0-10\right| = \frac{58}{2}$ square units

Also Area of triangle ACD = $\frac{1}{2}\left|-4(-5+2)+5(-2-5)+(-4)(5+5)\right|$

= $\frac{1}{2}\left|-4(-3)+5(-7)+(-4)(10)\right| = \frac{1}{2}\left|12-35-40\right| = \frac{63}{2}$ square units

$\therefore$ From equation (i) we haveArea of quadrilateral ABCD = $\frac{58}{2}+\frac{63}{2}=\frac{121}{2}$ square units



2. The base of an equilateral triangle with side $2a$ lies along the $y-$axis such that the mid-point of the base is at origin. Find the vertices of the triangle.

Ans. Given: Length of side of equilateral triangle = $2a$. The base of triangle lies along $y-$axis and the mid-point of base is at origin so that the coordinates of vertices are $(0, a)$ and $(0, -a)$.

We have for an equilateral triangle line from the vertex to the mid point of the base will be perpendicular to the base.So we can say the third vertex lies on X axis (positive or negative)

Now let the third vertex be $(\pm x, 0)$.

It is known that area of an equilaterl triangle = $\frac{\sqrt{3}}{4}a^2$,where a is the common length of the sides

$\therefore$ Area of theequilaterl triangle with side 2a= $\frac{\sqrt{3}}{4}(2a)^2$

$\Rightarrow \frac{1}{2} \times 2a \times (\pm x) = \frac{\sqrt{3}}{4} \times (2a)^2$

$\Rightarrow x = \pm\sqrt{3}a$

Therefore the third vertex can be $(\sqrt{3}a \quad ,0)$ or $(-\sqrt{3}a, 0)$

$\therefore$ The vertices of triangle are $(0, a)$, $(0, -a)$ and $(\sqrt{3}a \quad ,0)$ or $(0, a), (0, -a)$ and $(-\sqrt{3}a, 0)$

3. Find the distance between P (x_1, y_1) and Q (x_2, y_2) when (i) PQ is parallel to the $y-$ axis (ii) PQ is parallel to the $x-$ axis.

Ans. Given: P (x_1, y_1) and Q (x_2, y_2) are two points.

$\therefore$ PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) We have equation of a line parallel to $y-$ axis is X = K, a constant

PQ is parallel to $y-$ axis, then $x_2 = x_1 \Rightarrow x_2 - x_1 = 0$

$\therefore$ PQ = $\sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

(ii) We have equation of a line parallel to $x-$ axis is Y= K, a constant

PQ is parallel to $x-$ axis, then $y_2 - y_1 = 0$.

$\therefore$ PQ = $\sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$

4. Find the point on the $x-$ axis, which is equidistant from the points (7, 6) and (3, 4).

Ans. Let P $(x, 0)$ be any point on the $x-$ axis which is equidistant from Q (7, 6) and R (3, 4).

We have distnce between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ PQ = $\sqrt{(x-7)^2+(0-6)^2}$

= $\sqrt{x^2-14x+49+36}$

= $\sqrt{x^2-14x+85}$

And PR = $\sqrt{(x-3)^2+(0-4)^2}$

= $\sqrt{x^2-6x+9+16}$

= $\sqrt{x^2-6x+25}$

According to question, PQ = PR

$\Rightarrow \sqrt{x^2-14x+85}$ = $\sqrt{x^2-6x+25}$

Squaring both sides, $x^2-14x+85$ = $x^2-6x+25$

$\Rightarrow -14x+6x=25-85$

$\Rightarrow -8x=-60 \Rightarrow x=\frac{15}{2}$

Therefore, the required point is $\left(\frac{15}{2},0\right)$.

5. Find the slope of a line, which passes through the origin and the mid-point of the line segment of joining the points P $(0,-4)$ and B (8, 0).

Ans. Here, mid-point of the line segment joining P $(0,-4)$ and Q (8, 0) is

$\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$ = $(4,-2)$ using mid point formula

We have slope of a line passing through two points (x_1,y_1) and (x_2,y_2) is given by

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Therefore, Slope of the line passing through the points (0, 0) and $(4, -2)$ = $\frac{-2-0}{4-0} = \frac{-2}{4} = \frac{-1}{2}$

Hence slope of the required line is $\frac{-1}{2}$

6. Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and $(-1, -1)$ are the vertices of a right angled triangle.

Ans. Let A (4, 4), B (3, 5) and C$(-1, -1)$ be the three vertices of ΔABC.

We have slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of AB = $\frac{5-4}{3-4} = \frac{1}{-1} = -1$

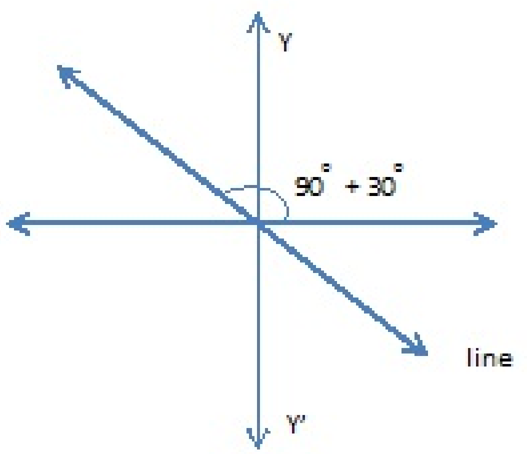
Slope of BC = $\frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of AC = $\frac{-1-4}{-1-4} = \frac{-5}{-5} = 1$

Now, Slope of AB x Slope of AC = $-1 \times 1 = -1$

This shows that AB $\perp$ AC. Thus ΔABC is right angled at point A.

7. Find the slope of the line, which makes an angle of 30° with the positive direction of $y-$ axis measured anticlockwise.

Ans. 

If the line makes an angle of 30° with the positive direction of $y-$axis then the line will make an angle of $\left(90^\circ + 30^\circ\right) = 120^\circ$ with the positive direction of $x-$axis.

$\therefore$ Slope of the line = $\tan 120^\circ = \tan\left(90^\circ + 30^\circ\right) = -\cot 30^\circ = -\sqrt{3}$

8. Find the value of x for which the points $(x, -1), (2, 1)$ and $(4, 5)$ are collinear.

Ans. Let A$(x, -1)$, B (2, 1) and C (4, 5) be three collinear points.

$\therefore$ Slope of AB = $\dfrac{1-(-1)}{2-x} = \dfrac{1+1}{2-x} = \dfrac{2}{2-x}$

Slope of BC = $\dfrac{5-1}{4-2} = \dfrac{4}{2} = 2$

According to question, Slope of AB = Slope of BC

$\Rightarrow \dfrac{2}{2-x} = 2$

$\Rightarrow 2 = 4 - 2x$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

9. Without using distance formula, show that the points $(-2,-1),(4,0),(3,3)$ and $(-3,2)$ are the vertices of a parallelogram.

Ans. Let A$(-2,-1)$, B (4, 0), C (3, 3) and D$(-3,2)$ be vertices of a quadrilateral ABCD.

To prove a quadrilateral is a parallelogram it is enough to show that both pairs of opposite sides are parallel

We have slope of a line passing through two points (x_1,y_1) and (x_2,y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1}$

$\therefore$ Slope of AB = $\frac{0-(-1)}{4-(-2)}=\frac{1}{6}$ Slope of BC = $\frac{3-0}{3-4}=\frac{3}{-1}=-3$

Slope of DC = $\frac{3-2}{3-(-3)}=\frac{1}{6}$ Slope of AD = $\frac{2-(-1)}{-3-(-2)}=\frac{3}{-1}=-3$

Here Slope of AB = Slope of DC

$\Rightarrow$ AB $\|$ DC

And Slope of BC = Slope of AD

$\Rightarrow$ BC $\|$ AD

Therefore, ABCD is a parallelogram.

10. Find the angle between the $x-$ axis and the line joining the points $(3,-1)$ and $(4,-2)$

Ans. Let A$(3,-1)$ and B$(4,-2)$ be two points. Let θ be the angle which AB makes with positive direction of $x-$ axis.

$\therefore$ Slope of AB = $\tan\theta$ ....(i)

We have slope of a line passing through two points (x_1,y_1) and (x_2,y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1}$

$\therefore$ Slope of AB = $\dfrac{-2-(-1)}{4-3}=\dfrac{-1}{1}=-1$ ....(ii)

From (i) and (ii) we get $\tan\theta=-1$

$\Rightarrow \tan\theta=-\tan 45^\circ$

$\Rightarrow \tan\theta=\tan\left(180^\circ-45^\circ\right)$

$\Rightarrow \tan\theta=\tan 135^\circ$

$\Rightarrow \theta=135^\circ$

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11. The slope of a line is double of the slope of the another line. If tangent of the angle between them is $1/3$ find the slopes of the lines.

Ans. Given: $\tan\theta=\dfrac{1}{3}$. Let the slopes of two lines be m and $2m$.

We know that if θ is the acute angle between two lines with slopes m_1 and m_2 respectively

then $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

$\therefore \left|\dfrac{m-2m}{1+m\times 2m}\right|=\dfrac{1}{3}$

$\Rightarrow \left|\dfrac{-m}{1+2m^2}\right|=\dfrac{1}{3}$

$\Rightarrow \dfrac{-m}{1+2m^2}=\pm\dfrac{1}{3}$

Taking $\dfrac{-m}{1+2m^2}=\dfrac{1}{3}$

$\Rightarrow -3m=1+2m^2$

$\Rightarrow 2m^2+3m+1=0$

$\Rightarrow (m+1)(2m+1)=0$

$\Rightarrow m=-1$ and $m=\frac{-1}{2}$

When $m=-1$ the slopes of the lines are -1 and -2

when $m=\frac{-1}{2}$ the slopes of the lines are $\frac{-1}{2}$ and -1

Taking $\frac{-m}{1+2m^2}=-\frac{1}{3}$

$\Rightarrow -3m=-1-2m^2$

$\Rightarrow 2m^2-3m+1=0$

$\Rightarrow (m-1)(2m-1)=0$

$\Rightarrow m=1$ and $m=\frac{1}{2}$

When m=1 the slopes of the lines are 1 and 2

when $m=\frac{1}{2}$ the slopes of the lines are $\frac{1}{2}$ and 1

Therefore, the slopes of lines are -1 and -2 or 1 and 2 or -1 and $\frac{-1}{2}$ or 1 and $\frac{1}{2}$.

12. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k-y_1=$

Ans. Let A(x_1, y_1) and B(h, k) be two points. It is given that Slope of AB = m

We have slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1}$

$\therefore$ Slope of AB = $\frac{k-y_1}{h-x_1}$ = m (given)

$\Rightarrow k-y_1=m(h-x_1)$

Hence proved

13. If three points $(h,0),(a,b)$ and $(0,k)$ lies on a line, show that $\frac{a}{h}+\frac{b}{k}=1$.

Ans. Let A$(h,0)$, B(a,b) and C$(0,k)$ be three points lie on the line.

We have slope of a line passing through two points (x_1,y_1) and (x_2,y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1}$

$\therefore$ Slope of AB = $\frac{b-0}{a-h}=\frac{b}{a-h}$ Slope of BC = $\frac{k-b}{0-a}=\frac{b-k}{a}$

Slope of AB = Slope of BC (given)

$\Rightarrow \frac{b}{a-h}=\frac{b-k}{a}$

$\Rightarrow ab=(a-h)(b-k)$

$\Rightarrow ab=ab-ak-bh+hk$

$\Rightarrow ak+bh=hk$

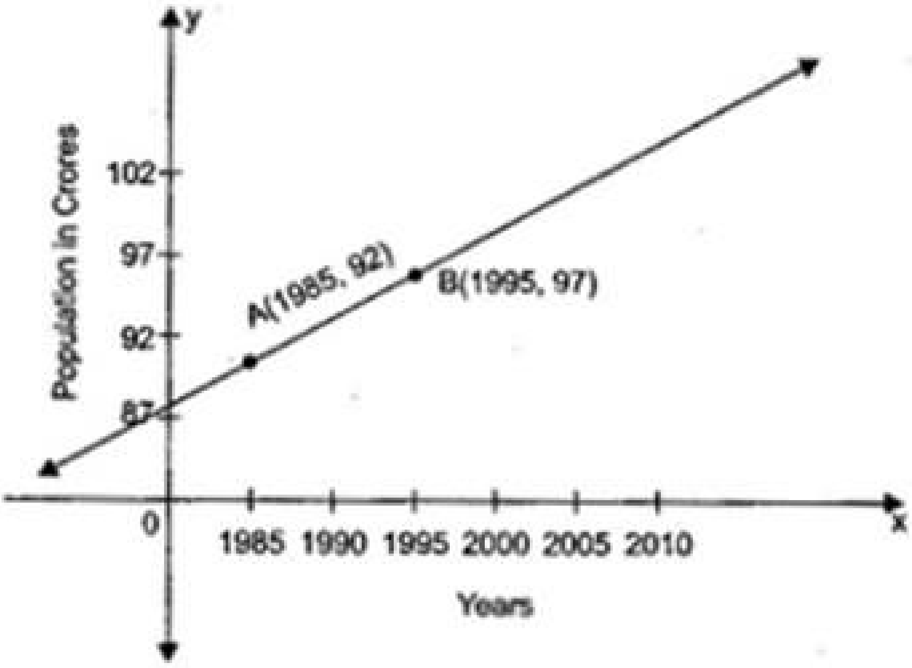
Dividing through out by hk

$\Rightarrow \frac{ak}{hk}+\frac{bh}{hk}=1$

$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$

14. Consider the following population and year graph, find the slope of the line AB and

using it, find what will be the population in the year 2010?



Ans. Given: The points on the line are A (1985, 92) and B (1995, 97).

We have slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of AB = $\dfrac{97-92}{1995-1985} = \dfrac{5}{10} = \dfrac{1}{2}$

Let the population in year 2010 be y crores. Then C$(2010, y)$ lies on the line AB.

Slope of BC = $\dfrac{y-97}{2010-1995} = \dfrac{y-97}{15}$

Since points A, B and C lie on the line(collinear points),we have

Slope of AB = Slope of BC

$\Rightarrow \dfrac{y-97}{15} = \dfrac{1}{2}$

$\Rightarrow 2y - 194 = 15$

$\Rightarrow 2y = 209$

$\Rightarrow y = \dfrac{209}{2} = 104.5$

Therefore, population in 2010 will be 104.5 crores.