CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 10 Straight Lines

Miscellaneous Exercise

1. Find the values of k for which the line (k-3)x -(4-k^2)y + k^2-7k+6 = 0 is:

(a) parallel to the x-axis

(b) parallel to the y-axis

(c) passing through the origin.

Ans. Given: Equation of line **(k-3)x - (4-k^2)y + k^2-7k+6 = 0 can be written as**

(4-k^2)y = (k-3)x + k^2-7k+6

$m = \frac{k-3}{4-k^2}$

(a) If the line parallel to x-axis, then $m = 0$

$=> \frac{k-3}{4-k^2} = 0$

$\Rightarrow$ k-3 = 0

$\Rightarrow$ k = 3

(b) If the line parallel to $y-$axis, then $\frac{1}{m} = 0$

$=> \frac{1}{\frac{k-3}{4-k^2}} = 0$

$=> \frac{4-k^2}{k-3} = 0$

$\Rightarrow$ 4- k^2 = 0

$\Rightarrow$ k^2 = 4

$\Rightarrow k = \pm 2$

(c) If the line passes through origin then

$=> (k-3) \times 0 - (4-k^2) \times 0 + k^2 - 7k + 6 = 0$

$\Rightarrow k^2 - 7k + 6 = 0$

$\Rightarrow (k-1)(k-6) = 0$

$\Rightarrow$ k = 1 or k = 6

2. Find the values of θ and p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Ans. Given: $\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow -\sqrt{3}x - y = 2$

Dividing both sides by $\sqrt{\left(-\sqrt{3}\right)^2 + \left(-1\right)^2} = 2$, we have

$\frac{-\sqrt{3}}{2}x - \frac{1}{2}y = 1$ ....(i)

comparing equation (i) with $x\cos\theta + y\sin\theta = p$, we get

$\cos\theta = \frac{-\sqrt{3}}{2}$, $\sin\theta = \frac{-1}{2}$ and $p = 1$

$\therefore \cos\theta = -\cos 30^\circ = \cos(180^\circ + 30^\circ) = \cos 210^\circ = \cos\frac{7\pi}{6}$

$\Rightarrow \theta = \frac{7\pi}{6}$

$\therefore \theta = \frac{7\pi}{6}$ and $p = 1$

3. Find the equations of the lines which cut-off intercepts on the axes whose sum and

product are 1 and -6 respectively.

Ans. Let the intercepts be a and b.

Then we have equation of line be $\frac{x}{a}+\frac{y}{b}=1$ and

it is given that $a+b=1$ and $ab=-6$

Now $(a-b)^2=(a+b)^2-4ab$

$\Rightarrow (a-b)^2=(1)^2-4(-6)$

$\Rightarrow (a-b)^2=1+24=25$

$\Rightarrow a-b=\pm 5$

Solving $a+b=1$ and $a-b=5$,

we get $a=3, b=-2$

$\therefore$ Equation of the line is $\frac{x}{3}+\frac{y}{-2}=1$

$\Rightarrow -2x+3y=-6$

$\Rightarrow 2x-3y=6$

Solving $a+b=1$ and $a-b=-5$, we get $a=-2, b=3$

$\therefore$ Equation of the line is $\frac{x}{-2}+\frac{y}{3}=1$

$\Rightarrow 3x-2y=-6$

$\Rightarrow -3x+2y=6$

Hence required equations of the lines are $2x-3y=6$ and $-3x+2y=6$

4. What are the points on the $y-$axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units?

Ans. Let point on y- axis be (0,y)

Given equation of line is $\frac{x}{3}+\frac{y}{4}=1$

$\Rightarrow 4x+3y=12$

$\Rightarrow 4x+3y-12=0$

We have perpendicular distance from (x_1, y_1) to the line $ax+by+c=0$ is

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ Perpendicular distance from point (0,y) to $4x+3y-12=0$ =

$$\left|\frac{4\times 0+3y-12}{\sqrt{(4)^2+(3)^2}}\right| = \left|\frac{3y-12}{5}\right|$$

According to question $\left|\frac{3y-12}{5}\right|=4$

$\Rightarrow \frac{3y-12}{5}=\pm 4$

Taking $\frac{3y-12}{5}=4$

$\Rightarrow 3y-12=20$

$\Rightarrow y=\frac{32}{3}$

Taking $\frac{3y-12}{5}=-4$

$\Rightarrow 3y-12=-20$

$\Rightarrow y=\frac{-8}{3}$

Therefore, required points are $\left(0,\frac{32}{3}\right)$ and $\left(0,\frac{-8}{3}\right)$.

5. Find the perpendicular distance from the origin of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

Ans. Equation of the line joining points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is

$y-\sin\theta=\frac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta}(x-\cos\theta)$, using Two-point form

$\Rightarrow$

$(\cos\phi-\cos\theta)y-\sin\theta\cos\phi+\sin\theta\cos\theta=(\sin\phi-\sin\theta)x-\sin\phi\cos\theta+\sin\theta\cos\theta$

$\Rightarrow (\sin\phi-\sin\theta)x-(\cos\phi-\cos\theta)y+\sin\theta\cos\phi-\sin\phi\cos\theta=0$

$\Rightarrow (\sin\phi-\sin\theta)x-(\cos\phi-\cos\theta)y+\sin(\theta-\phi)=0$

Now, perpendicular distance from (0, 0) to this line,

$=\left|\frac{(\sin\phi-\sin\theta)\times 0-(\cos\phi-\cos\theta)\times 0+\sin(\theta-\phi)}{\sqrt{(\sin\phi-\sin\theta)^2+(\cos\phi-\cos\theta)^2}}\right|$ since $d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$=\left|\frac{\sin(\theta-\phi)}{\sqrt{\sin^2\phi+\sin^2\theta-2\sin\phi\sin\theta+\cos^2\phi+\cos^2\theta-2\cos\phi\cos\theta}}\right|$

$= \left|\frac{\sin(\theta-\phi)}{\sqrt{1+1-2(\cos\theta\cos\phi+\sin\theta\sin\phi)}}\right|$ since $\cos^2\theta+\sin^2\theta=1$

$= \left|\frac{\sin(\theta-\phi)}{\sqrt{2-2\cos(\theta-\phi)}}\right|$ using $\cos(\theta-\phi)=\cos\theta\cos\phi+\sin\theta\sin\phi$

$= \left|\frac{\sin(\theta-\phi)}{\sqrt{2[1-\cos(\theta-\phi)]}}\right|$

$= \left|\frac{\sin(\theta-\phi)}{\sqrt{2\left[2\sin^2\left(\frac{\theta-\phi}{2}\right)\right]}}\right|$ since $1-\cos\theta=2\sin^2\frac{\theta}{2}$

$= \left|\frac{\sin(\theta-\phi)}{2\sin\left(\frac{\theta-\phi}{2}\right)}\right|$

6. Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines $x-7y+5=0$ and $3x+y=0$.

Ans. The equation of any line parallel to y-axis is of the form x= k,a constant

GIven lines are $x-7y+5=0$ ....(i)

$3x+y=0$ ....(ii)

From (ii) we get $y=-3x$

substituting y in equation(i),$x-7(-3x)+5=0\Rightarrow 22x+5=0\Rightarrow x=\frac{-5}{22}$

and $y=-3x=-3\times\frac{-5}{22}=\frac{15}{22}$

$\therefore$ Point of intersection of lines (i) and (ii) is $\left(\frac{-5}{22},\frac{15}{22}\right)$

Since x= k ,passes through the point $\left(\frac{-5}{22}, \frac{15}{22}\right)$ we get $k = \frac{-5}{22}$

Therefore the equation of required line is $x = \frac{-5}{22}$.

7. Find the equation of a line drawn perpendicular to the line $\frac{x}{4}+\frac{y}{6}=1$ through the point, where it meets the $y-$axis.

Ans. Given: Equation of line $\frac{x}{4}+\frac{y}{6}=1$

$\Rightarrow 6x+4y-24=0$ ....(i)

Slope of given line $= m = \frac{-6}{4} = \frac{-3}{2}$

Therefore slope of line perpendicular to given line $= \frac{-1}{m} = \frac{-1}{\frac{-3}{2}} = \frac{2}{3}$

Let the given line meet the y-axis at (0, y)

Now by substituting x=0 in equation (i) ,$4y - 24 = 0 \Rightarrow y = \frac{24}{4} = 6$

$\therefore$ The given line meets the $y-$axis at (0, 6).

Now the equation of a line with slope $\frac{2}{3}$ and passes through (0,6) is

$y-6=\frac{2}{3}(x-0)$

$\Rightarrow 3y-18=2x$

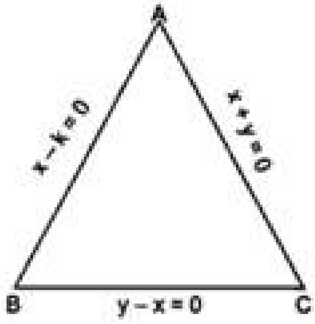
$\Rightarrow 2x-3y+18=0$

$\therefore$ Equation of required line is $2x-3y+18=0$

8. Find the area of the triangle formed by the lines $y-x=0$, $x+y=0$ and $x-k=0$.

Ans. Given: Equations of lines are $y-x=0$.....(i)

$x+y=0$.....(ii)



And $x-k=0$.....(iii)

On solving eq. (i) and (ii), we get the point of intersection C =(0, 0)

On solving eq. (ii) and (iii), we get the point of intersection A=$(k,-k)$

On solving eq. (i) and (iii), we get the point of intersection B=(k,k)

We have the area of the triangle whose vertices are $(x_1,y_1),(x_2,y_2) and (x_3,y_3)$ is

$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$\therefore$ Area of ΔABC = $=\frac{1}{2}\left|k(k-0)+k(0+k)+0(-k-k)\right|=\frac{1}{2}\left|k^2+k^2+0\right|=\frac{1}{2}\left|2k^2\right|=k^2$

Hence area of triangle formed by the three given lines is k^2 sq. units

9. Find the value of p so that three lines $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Ans. We know three lines $a_1x+b_1y+c_1z=0, a_2x+b_2y+c_2z=0 and a_3x+b_3y+c_3z=0$ intersect at one point if $a_3(b_1c_2-b_2c_1)+b_3(c_1a_2-c_2a_1)+c_3(a_1b_2-a_2b_1)=0$

Given lines are $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$

$\therefore 2\left[1\times(-3)-2\times(-2)\right]+(-1)\left[-2\times p-(-3)\times 3\right]+(-3)\left[3\times 2-p\times 1\right]=0$

$\Rightarrow 2\left[-3+4\right]-1\left[-2p+9\right]-3\left[6-p\right]=0$

$\Rightarrow 2+2p-9-18+3p=0$

$\Rightarrow 5p-25=0$

$\Rightarrow p=5$

Hence the required value of p is 5.

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10. If three lines whose equations are $y=m_1x+c_1$, $y=m_2x+c_2$ and $y=m_3x+c_3$ are concurrent., then show that $m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$.

Ans. We know three lines $a_1x+b_1y+c_1z=0, a_2x+b_2y+c_2z=0 and a_3x+b_3y+c_3z=0$ are concurrent if $a_3(b_1c_2-b_2c_1)+b_3(c_1a_2-c_2a_1)+c_3(a_1b_2-a_2b_1)=0$

Given lines are $y=m_1x+c_1$, $y=m_2x+c_2$ and $y=m_3x+c_3$

$\Rightarrow m_1x-y+c_1=0, m_2x-y+c_2=0$ and $m_3x-y+c_3=0$

Since the lines are concurrent ,we have

$m_3(-1.c_2+1.c_1)-1(c_1.m_2-c_2m_1)+c_3(m_1.-1+m_2.1)=0$

$\Rightarrow m_3(-c_2+c_1)-c_1.m_2+c_2m_1-c_3m_1+c_3m_2=0$

$\Rightarrow m_3(c_1-c_2)+m_2(c_3-c_1)+m_1(c_2-c_3)=0$

$\Rightarrow m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$

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11. Find the equations of the lines through the point (3, 2) which make an angle of 45° with the line $x-2y=3$.

Ans. Let m be the slope of required line which passes through point (3, 2), Then the equation of required line is $y-2=m(x-3)$ ....(i)

The equation of given line $x-2y=3$

$\Rightarrow y=\frac{x}{2}-\frac{3}{2}$ ....(ii)

$\therefore$ Slope of given line = $\frac{1}{2}$

We know that if θ is the acute angle between two lines with slopes m_1 and m_2 respectively

then $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

According to question, $\tan 45^\circ=\left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|$

$\Rightarrow 1=\left|\frac{2m-1}{2+m}\right|$

$\Rightarrow \frac{2m-1}{2+m}=\pm 1$

Taking $\frac{2m-1}{2+m}=1$

$\Rightarrow 2m-1=2+m$

$\Rightarrow m=3$

Then the equation of required line is $y-2=3(x-3)$

$\Rightarrow y-2=3x-9$

$\Rightarrow 3x-y-7=0$

Taking $\frac{2m-1}{2+m}=-1$

$\Rightarrow 2m-1=-2-m$

$\Rightarrow m=\frac{-1}{3}$

Then the equation of required line is $y-2=\frac{-1}{3}(x-3)$

$\Rightarrow 3y-6=-x+3$

$\Rightarrow x+3y-9=0$

Thus the equations of the lines are $3x-y-7=0$ and $x+3y-9=0$

12. Find the equation of the line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axis.

Ans. Let the equal intercepts be a. Equation of a line in intercept form is $\frac{x}{a}+\frac{y}{a}=1 \Rightarrow x+y=a$ ....(A)

Given: The required equation passes through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$

Consider $4x+7y=3$.....................(i) and $2x-3y=-1$..........................(ii)

Multiplying equation(ii) by 2, we get $4x-6y=-2$.........................(iii)

subtracting equation (iii) from equation(i), $13y=5 \Rightarrow y=\frac{5}{13}$

substituting y in equation(ii) we get $2x=-1+3.\frac{5}{13}=\frac{-13+15}{13}=\frac{2}{13} \Rightarrow x=\frac{1}{13}$

Hence the point of intersection of (i) and (ii) is $\left(\frac{1}{13},\frac{5}{13}\right)$

Now since the required line $x+y=a$ passes through the point $\left(\frac{1}{13},\frac{5}{13}\right)$

we have, $a=\frac{1}{13}+\frac{5}{13}=\frac{6}{13}$

$\therefore$ From equation(A) we get the required equation of the line is $x+y=\frac{6}{13} \Rightarrow 13x+13y=6$

13. Show that the equation of the line passing through the origin and making an angle θ with the line $y=mx+c$ is $\frac{y}{x}=\frac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

Ans. Let m_1 be the slope of required line which passes through (0, 0).

$\therefore$ Equation of line is $y-0=m_1(x-0)$

$\Rightarrow y=m_1x$ ....(i)

Now, let θ be the angle between $y=mx+c$ and $y=m_1x$

We know that if θ is the acute angle between two lines with slopes m_1 and m_2 respectively then $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

$$\therefore \tan\theta=\left|\frac{m_1-m}{1+m_1m}\right|$$

$$\Rightarrow \tan\theta=\pm\frac{m_1-m}{1+m_1m}$$

$$\Rightarrow \tan\theta=\frac{m_1-m}{1+m_1m} \text{ or } \tan\theta=-\frac{m_1-m}{1+m_1m}$$

$$\Rightarrow \tan\theta+m_1m\tan\theta=m_1-m \text{ or } \tan\theta+m_1m\tan\theta=m-m_1$$

$$\Rightarrow m_1(1-m\tan\theta)=m+\tan\theta \text{ or } m_1(1+m\tan\theta)=m-\tan\theta$$

$$\Rightarrow m_1=\frac{m+\tan\theta}{1-m\tan\theta} \text{ or } m_1=\frac{m-\tan\theta}{1+m\tan\theta}$$

Putting $m_1=\frac{m+\tan\theta}{1-m\tan\theta}$ in eq. (i), we get $y=\frac{m+\tan\theta}{1-m\tan\theta}x$.................(ii)

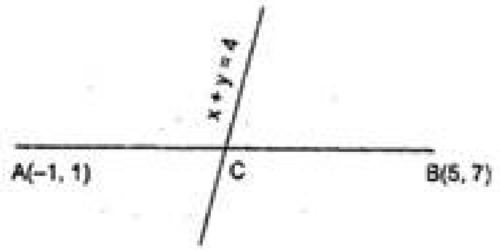
Putting $m_1 = \frac{m-\tan\theta}{1+m\tan\theta}$ in eq. (i), we get $y = \frac{m-\tan\theta}{1+m\tan\theta}x$................(iii)

Therefore from (ii) and (iii) we get $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

14. In what ratio, the line joining $(-1, 1)$ and (5, 7) is divided by the line $x+y=4$?

Ans. Given: Equation of line $x+y-4=0$

Let the given line divide the line joining A $(-1, 1)$ and B (5, 7) in the ratio $k:1$ at a point C.



$\therefore$ Using section formula we have coordinates of C are $\left(\frac{k(5)+1(-1)}{k+1}, \frac{k(7)+1(1)}{k+1}\right)$

$= \left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$

Since the point C lies on the given line.

$\therefore \frac{5k-1}{k+1} + \frac{7k+1}{k+1} - 4 = 0$

$\Rightarrow 5k-1+7k+1-4k-4=0$

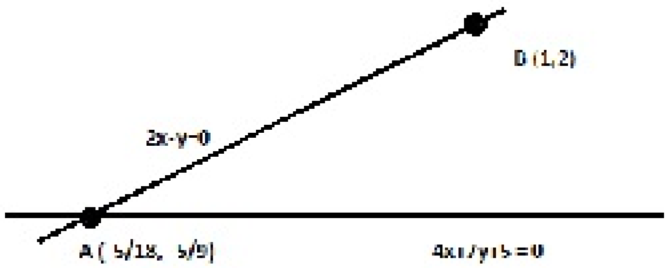
$\Rightarrow 8k=4$

$\Rightarrow k=\frac{1}{2}$

Therefore, the required ratio is 1: 2.

15. Find the distance of the line $4x+7y+5=0$ from the point (1, 2) along the line $2x-y$

Ans. First we have to find the point of intersection of given lines $4x+7y+5=0$ ....(i) and $2x-y=0$ ....(ii)



From(ii) we get $y=2x$

Substituting $y=2x$ in equation(i),we get $18x+5=0 \Rightarrow x=\frac{-5}{18}$

$\therefore y=2x=2\left(\frac{-5}{18}\right)=\frac{-5}{9}$

Hence point of intersection of (i) and (ii) is A$\left(\frac{-5}{18},\frac{-5}{9}\right)$

$\therefore$ Distance between the points A $\left(\frac{-5}{18},\frac{-5}{9}\right)$ and B(1, 2)

$$=\sqrt{\left(1+\frac{5}{18}\right)^2+\left(2+\frac{5}{9}\right)^2}$$

$$=\sqrt{\left(\frac{23}{18}\right)^2+\left(\frac{23}{9}\right)^2}$$

$$=\frac{23}{18}\sqrt{(1)^2+(2)^2}=\frac{23}{18}\sqrt{5}\text{ units}$$

16. Find the direction in which a straight line must be drawn through the point $(-1,2)$ so that its point of intersection with the line $x+y=4$ may be at a distance of 3 units from this point.

Ans. Let the required equation of the line be $y-y_0=m(x-x_0)$ ....(i)

Since equation (i) passes through the point $(-1,2)$,we get $y-2=m(x+1)$

$\Rightarrow mx-y+(m+2)=0$.......................(ii)

Equation of the given line is $x+y=4$ ....(iii)

From(iii) we get $y=4-x$,substituting this in(ii),we get

$mx-(4-x)+m+2=0 \Rightarrow x(m+1)+(m-2)=0 \Rightarrow x=\frac{2-m}{m+1}$

$\therefore y=4-\left(\frac{2-m}{m+1}\right)=\frac{4m+4-2+m}{m+1}=\frac{5m+2}{m+1}$

Hence we get point of intersection of (ii) and (iii) is $\left(\frac{2-m}{m+1},\frac{5m+2}{m+1}\right)$

Given distance between $(-1,2)$ and $\left(\frac{2-m}{m+1},\frac{5m+2}{m+1}\right)$ is 3

Now using distance formula we get $\sqrt{\left(\frac{2-m}{m+1}+1\right)^2+\left(\frac{5m+2}{m+1}-2\right)^2}=3$

squaring on both sides $\left(\frac{2-m+m+1}{m+1}\right)^2+\left(\frac{5m+2-2m-2}{m+1}\right)^2=9$

$\Rightarrow\left(\frac{3}{m+1}\right)^2+\left(\frac{3m}{m+1}\right)^2=9 \Rightarrow 9\left[\frac{1+m^2}{(m+1)^2}\right]=9$

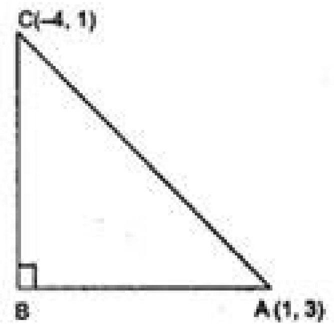
$\Rightarrow 1+m^2=(m+1)^2 \Rightarrow 1+m^2=m^2+1+2m$

$\Rightarrow 2m=0 \Rightarrow m=0$

Since the slope is zero,we have the required line is parallel to $x-$ axis .

17. The hypotenuse of a right angled triangle has its ends at the points (1, 3) and $(-4,1)$. Find the equation of the legs (perpendicular sides) of the triangle.

Ans. Let ABC be a right angled triangle with diagonal AC.



Then we have the legs of the triangle (perpendicular sides) are BA and BC

Let the slope of AB is m ,then since AB and BC are perpendicular to each other

we have, Slope of BC = $\frac{-1}{m}$

We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m(x-x_0)$

Now since AB passes through A**(1, 3)** and have slope m equation of AB is

$y-3=m(x-1)$..............................(i)

Also BC is a line through C$(-4,1)$. with slope $\frac{-1}{m}$ hence it'sequation is

$y-1=\frac{-1}{m}(x+4)$...........................(ii)

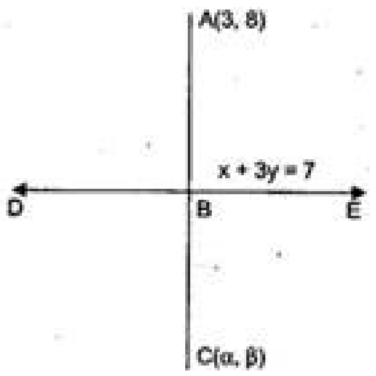
By putting different values for m in equations (i) and (ii) we get different equations for BA and BC

In particular for m=0,equation of AB is y=3 [which is a line parallel to X-axis] and

equation of BC is x= -4 [which is a line parallel to Y-axis]

18. Find the image of the point (3, 8) with respect to the line $x+3y=7$ assuming the line to be a plane mirror.

Ans. Let the image of the point A (3, 8) in the line mirror DE be C(α,β). Then AC is perpendicular bisector of DE.



$\therefore$ The coordinates of point B are $\left(\frac{\alpha+3}{2}, \frac{\beta+8}{2}\right)$

Since point B lies on the line $x+3y=7$.

$\therefore \frac{\alpha+3}{2}+\frac{3(\beta+8)}{2}=7$

$\Rightarrow \alpha+3+3\beta+24=14$

$\Rightarrow \alpha+3\beta+13=0$ .....(i)

Since AC is perpendicular on DE.

$\therefore$ Slope of AC x Slope of DE = -1

$\Rightarrow \frac{\beta-8}{\alpha-3}\times\frac{-1}{3}=-1$

$\Rightarrow \beta-8=3\alpha-9$

$\Rightarrow 3\alpha-\beta-1=0$ .....(ii)

Solving eq. (i) and (ii), we get $\alpha=-1$ and $\beta=-4$

Therefore, the image of point (3, 8) is $(-1, -4)$.

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19. If the lines $y=3x+1$ and $2y=x+3$ are equally inclined to the line $y=mx+4$, find the value of m.

Ans. Given lines are $y=3x+1$ ....(i)

$2y=x+3$ ....(ii) and $y=mx+4$ ....(iii)

Now we have slopes of lines (i) ,(ii) and (iii) are 3,1/2,m respectively

We know that if θ is the acute angle between two lines with slopes m_1 and m_2 respectively then $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

Let θ be the angle which the line $y=mx+4$ makes with the line $y=3x+1$ and $2y=x+3$.

$$\therefore \tan\theta=\left|\frac{m-3}{1+3m}\right| \text{ and } \tan\theta=\left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|=\left|\frac{2m-1}{2+m}\right|$$

Given that lines $y=3x+1$ and $2y=x+3$ are equally inclined to the line $y=mx+4$

$$\Rightarrow \left|\frac{m-3}{1+3m}\right|=\left|\frac{2m-1}{2+m}\right|$$

$$\Rightarrow \frac{m-3}{1+3m}=\pm\frac{2m-1}{2+m}$$

$$\Rightarrow m^2-m-6=\pm\left(6m^2-m-1\right)$$

$$\Rightarrow 5m^2+5=0 \text{ or } 7m^2-2m-7=0$$

$$\Rightarrow m=\frac{1\pm5\sqrt{2}}{7}$$

20. If sum of the perpendicular distances of a variable point P(x,y) from the lines $x+y-5=0$ and $3x-2y+7=0$ is always 10. Show that P must move on a line.

Ans. Given: Equations of lines are $x+y-5=0$ .....(i) and $3x-2y+7=0$ .....(ii)

Perpendicular distance of point P(x,y) from line (i)

= $\left|\frac{x+y-5}{\sqrt{(1)^2+(1)^2}}\right|=\left|\frac{x+y-5}{\sqrt{2}}\right|$

Perpendicular distance of point P(x,y) from line (ii)

$\left|\frac{3x-2y+7}{\sqrt{(3)^2+(-2)^2}}\right|=\left|\frac{3x-2y+7}{\sqrt{13}}\right|$

According to question, $\left|\frac{x+y-5}{\sqrt{2}}\right|+\left|\frac{3x-2y+7}{\sqrt{13}}\right|=10$

$\Rightarrow \left|\sqrt{13}(x+y-5)\right|+\left|\sqrt{2}(3x-2y+7)\right|=10\sqrt{26}$

When $x+y-5\geq 0$ and $3x-2y+7\geq 0$

Then $\left|\sqrt{13}(x+y-5)\right|+\left|\sqrt{2}(3x-2y+7)\right|=10\sqrt{26}$

$\Rightarrow \sqrt{13}x+\sqrt{13}y-5\sqrt{13}+3\sqrt{2}x-2\sqrt{2}y+7\sqrt{2}-10\sqrt{26}=0$

$\Rightarrow \left(\sqrt{13}+3\sqrt{2}\right)x+\left(\sqrt{13}-2\sqrt{2}\right)y-5\sqrt{13}+7\sqrt{2}-10\sqrt{26}=0$, which represent a line.

Similarly,we can get the equation of a line for any signs (positive or negative) of $x+y-5$ and $3x-2y+7$

Therefore, we can say that P(x,y) must move on a line.

21. Find equation of the line which is equidistant from parallel lines $9x+6y-7=0$ and $3x+2y+6=0$.

Ans. The equations of parallel lines are $9x+6-7=0$ and $3x+2y+6=0$

Let A (x_1, y_1) be any point which is equidistant from the parallel lines.

$$\therefore \left|\frac{9x_1+6y_1-7}{\sqrt{(9)^2+(6)^2}}\right| = \left|\frac{3x_1+2y_1+6}{\sqrt{(3)^2+(2)^2}}\right|$$

$$\Rightarrow \frac{9x_1+6y_1-7}{3\sqrt{13}} = \pm\frac{3x_1+2y_1+6}{\sqrt{13}}$$

Consider the case $\frac{9x_1+6y_1-7}{3\sqrt{13}} = +\frac{3x_1+2y_1+6}{\sqrt{13}}$

$\Rightarrow 9x_1+6y_1-7 = 3(3x_1+2y_1+6) \Rightarrow -7 = 18$ which leads to some wrong conclusion

so this case is not possible

Now consider $\frac{9x_1+6y_1-7}{3\sqrt{13}} = -\frac{3x_1+2y_1+6}{\sqrt{13}}$

$\Rightarrow 9x_1+6y_1-7 = -9x_1-6y_1-18$

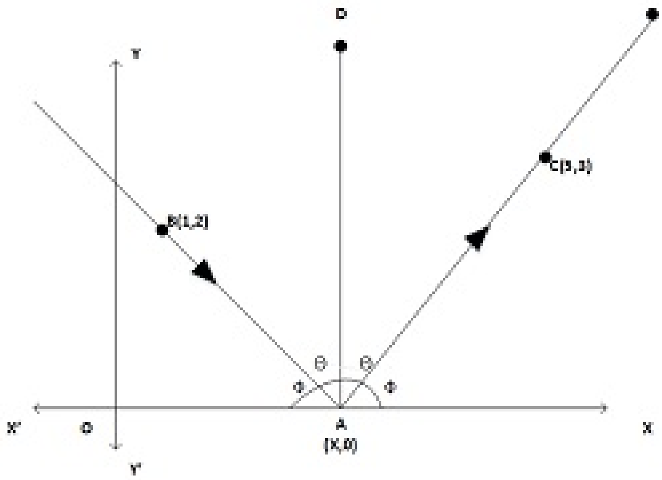
$\Rightarrow 18x_1+12y_1+11=0$

Therefore, the required line is $18x+12y+11=0$.

22. A ray of light passing through the point A and the reflected ray passes through the point (1, 2) reflects on the $x-$ axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

Ans. Let the coordinates of the point A be (x,0)

Also let BA be the incident ray and AC be the reflected ray and AD be the normal



We have angle of incidence = angle of reflection $\Rightarrow \angle BAD = \angle CAD$

Let $\angle BAD = \angle CAD = \theta$

Also let $\angle CAX = \phi$

Then we have $\phi + \theta = 90^\circ \Rightarrow \theta = 90^\circ - \phi$

Also $\angle OAB = 180^\circ - (\phi + 2\theta) = 180^\circ - [\phi + 2(90^\circ - \phi)] = 180^\circ - (180^\circ - \phi) = \phi$

Now, for line AC $\tan\phi = \dfrac{3-0}{5-x}$

$\Rightarrow \tan\phi = \dfrac{3}{5-x}$ .....(i)

For line BA $\tan\left(180^\circ - \phi\right) = \dfrac{2-0}{1-x}$

$\Rightarrow -\tan\phi = \dfrac{2}{1-x}$

$\Rightarrow \tan\phi = \frac{-2}{1-x}$ .....(ii)

From eq. (i) and (ii), $\frac{3}{5-x} = \frac{-2}{1-x}$

$\Rightarrow 3 - 3x = -10 + 2x$

$\Rightarrow -5x = -13$

$\Rightarrow\ x = \dfrac{13}{5}$

Therefore, coordinates of point A are $\left(\dfrac{13}{5}, 0\right)$.

23. Prove that the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ is b^2.

Ans. Let p_1 and p_2 be the length of perpendiculars from $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$.

$$\therefore\ p_1 = \left| \frac{\dfrac{\sqrt{a^2 - b^2}\cos\theta}{a} + \dfrac{0 \times \sin\theta}{b} - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\dfrac{\sqrt{a^2 - b^2}\cos\theta}{a} - 1}{\sqrt{\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2}}} \right|$$

And $$p_2 = \left| \frac{\dfrac{-\sqrt{a^2 - b^2}\cos\theta}{a} + \dfrac{0 \times \sin\theta}{b} - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\frac{-\sqrt{a^2-b^2}\cos\theta}{a} - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \right|$$

Now $p_1 p_2 = \left| \frac{\frac{\sqrt{a^2-b^2}\cos\theta}{a} - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \right| \left| \frac{\frac{-\sqrt{a^2-b^2}\cos\theta}{a} - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \right|$

$$= \frac{\left| \left[\frac{\sqrt{a^2-b^2}\cos\theta}{a} - 1 \right] \left[\frac{\sqrt{a^2-b^2}\cos\theta}{a} + 1 \right] \right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left| \left[\frac{(a^2-b^2)\cos^2\theta}{a^2} - 1 \right] \right|}{\frac{\cos^2\theta}{a^2} + \frac{1-\cos^2\theta}{b^2}}$$

$$= \frac{\left| \left[\frac{(a^2-b^2)\cos^2\theta}{a^2} - 1 \right] \right|}{\frac{b^2\cos^2\theta + a^2 - a^2\cos^2\theta}{a^2 b^2}}$$

$$= \frac{\left| a^2 - (a^2-b^2)\cos^2\theta \right|}{\frac{a^2 - (a^2-b^2)\cos^2\theta}{b^2}}$$

$$= a^2 - (a^2-b^2)\cos^2\theta \times \frac{b^2}{a^2 - (a^2-b^2)\cos^2\theta} = b^2$$

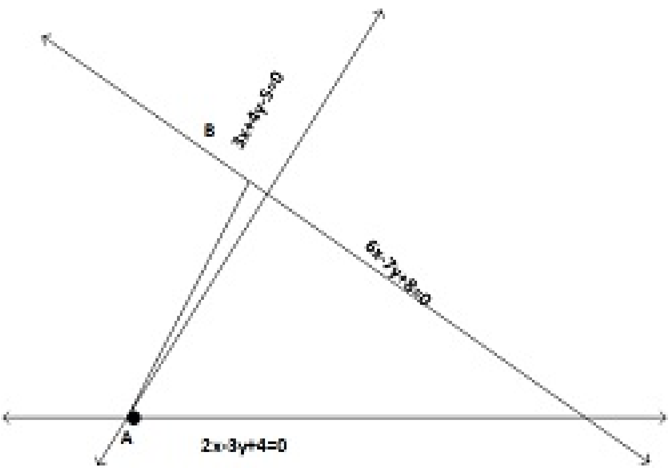
24. A person standing at the junction (crossing) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$ wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Find equation of the path that he should follow.

Ans. Consider $2x-3y+4=0$ ....(i) and $3x+4y-5=0$ ....(ii)

The point of intersection of lines (i) and (ii) is given by $A\left(\frac{22}{17},\frac{-2}{17}\right)$

It is given that the person is standing at A and wants to reach the path whose equation is $6x-7y+8=0$ ....(iii) in the least time

We have the shortest path from point A to line(iii) is the line AB which is the perpendicular distance from $A\left(\frac{-1}{17},\frac{22}{17}\right)$ to equation (iii)



$\therefore$ Slope of line $6x-7y+8=0$ is $\frac{6}{7}$

$\therefore$ Slope of required line AB is $\frac{-7}{6}$

Therefore, the equation of AB is $y-y_0=m(x-x_0)$

$\Rightarrow y-\frac{22}{17}=\frac{-7}{6}\left(x+\frac{1}{17}\right)$

$\Rightarrow \frac{17y-22}{17} = \frac{-7}{6}\left(\frac{17x+1}{17}\right)$

$\Rightarrow 6(17y-22) = -7(17x+1)$

$\Rightarrow 102y - 132 = -119x - 7$

$\Rightarrow 119x + 102y = 125$

Hence the equation of the path that the person should follow is $119x + 102y = 125$