CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 10 Straight Lines

Exercise 10.2

In Exercises 1 to 8, find the equations of the line which satisfy the given conditions:

1. Write the equations for the x-axis and y-axis.

Ans. Equation for x-axis is $y=0$ since the y-coordinate of every point on the x-axis is 0.

Equation for y-axis is $x=0$ since the x-coordinate of every point on the y-axis is 0.

2. Passing through the point $(-4, 3)$ with slope $\frac{1}{2}$.

Ans. We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m(x-x_0)$

Given: $x_0=-4$, $y_0=3$ and $m=\frac{1}{2}$

$\therefore$ Equation of required line is $y-3=\frac{1}{2}\{x-(-4)\}$

$\Rightarrow 2y-6=x+4$

$\Rightarrow x-2y+10=0$

3. Passing through (0, 0) with slope m.

Ans. We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m(x-x_0)$

Given: $x_0=0$, $y_0=0$ and slope = m

$\therefore$ Equation of the required line is $y-0=m(x-0) \Rightarrow y=mx$

4. Passing through $\left(2, 2\sqrt{3}\right)$ and inclined with the $x-$ axis at an angle of 75°.

Ans. We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y-y_0=m(x-x_0)$

Given: $x_\circ = 2, y_\circ = 2\sqrt{3}$ and $\theta = 75^\circ$

Now, $m=\tan 75^\circ$

$$= \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1+ \frac{1}{\sqrt{3}}}{1- \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$\therefore$ Equation of the required line is $y-2\sqrt{3} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$

$$\Rightarrow \left(\sqrt{3}-1\right)\left(y-2\sqrt{3}\right) = \left(\sqrt{3}+1\right)(x-2)$$

$$\Rightarrow (\sqrt{3}+1)x - (\sqrt{3}-1)y = 2\sqrt{3}+2-6+2\sqrt{3}$$

$$\Rightarrow (\sqrt{3}+1)x - (\sqrt{3}-1)y = 4\sqrt{3}-4$$

$$\Rightarrow (\sqrt{3}+1)x - (\sqrt{3}-1)y = 4(\sqrt{3}-1)$$

5. Intersecting the $x-$ axis at a distance of 3 units to the left of origin with slope -2.

Ans. We have if a line with slope m makes x-intercept d ,then equation of the line is $y=m(x-d)$

Given: $m=-2$ and $d=-3$

$\therefore$ Required line is $y = -2(x+3)$

$\Rightarrow 2x + y + 6 = 0$

6. Intersecting the $y-$axis at a distance of 2 units above the origin and making an angle of 30° with positive direction of the $x-$ axis.

Ans. We have if a line with slope m makes y-intercept c ,then equation of the line is $y = mx + c$

Given: $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$ and $c = 2$

$\therefore$ Equation of the required line is $y = \frac{1}{\sqrt{3}}x + 2$

$\Rightarrow y = \frac{x+2\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x - \sqrt{3}y + 2\sqrt{3} = 0$

7. **Passing through the points $(-1,1)$ and $(2,-4)$.**

Ans. We have the equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Given: $(x_1, y_1) = (-1,1)$ and $(x_2, y_2) = (2,-4)$

$\therefore$ Equation of the required line is $y - 1 = \left(\frac{-4-1}{2+1}\right)(x+1)$

$\Rightarrow y - 1 = \frac{-5}{3}(x+1)$

$\Rightarrow 3y - 3 = -5x - 5$

$\Rightarrow 5x+3y+2=0$

8. Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive $x-$ axis is 30°.

Ans. If the length of the perpendicular from origin to the line is p and the angle made by the perpendicular with the positive direction of x-axis is ω , then the equation of the line is $x\cos\omega + y\sin\omega = p$

Given: $p = 5$ and $\omega = 30^\circ$

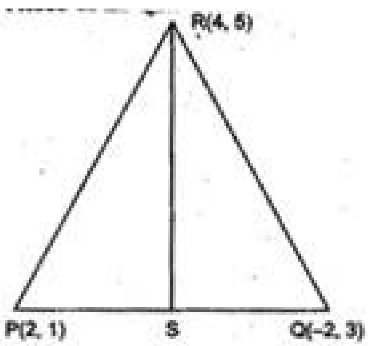
$\therefore$ Equation of the required line is $x\cos 30^\circ + y\sin 30^\circ = 5$

$\Rightarrow \frac{\sqrt{3}}{2}x+\frac{1}{2}y=5$

$\Rightarrow \sqrt{3}x+y=10$

9. The vertices of $\triangle$PQR are P (2 , 1), Q(-2 , 3) and R (4 , 5). Find equation of the median through the vertex R.

Ans. Given: P (2, 1), Q(-2, 3) and R (4, 5) are the vertices of ΔPQR . Let RS be the median through vertex R. Then S is the mid-point of PQ.



$\therefore$ Using midpoint formula coordinates of S are $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) \Rightarrow$ (0, 2)

We have the equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$$

$\therefore$ Equation of required median RS is

$$y-2=\left(\frac{5-2}{4-0}\right)(x-0)$$

$$\Rightarrow\ y-2=\frac{3}{4}x$$

$$\Rightarrow\ 4y-8=3x$$

$$\Rightarrow\ 3x-4y+8=0$$

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10. Find the equation of the line passing through $(-3,5)$ and perpendicular to the line through the points (2, 5) and $(-3,6)$.

Ans.

Slope of the line passing through te points A(2,5) and B $(-3,6)$ is $\frac{y_2-y_1}{x_2-x_1}=\frac{6-5}{-3-2}=\frac{1}{-5}=\frac{-1}{5}$

Since, the required line is perpendicular to AB, therefore slope of required line $m=5$

$\therefore$ Equation of the required line passing through point $(-3,5)$ having slope 5 is

$$y-y_0=m(x-x_0)$$

$$\Rightarrow y-5=5(x+3)$$

$$\Rightarrow y-5=5x+15$$

$$\Rightarrow\ 5x-y+20=0$$

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11. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it

in the ratio $1:n$. Find the equation of the line.

Ans. Let point C divides the join of A (1, 0) and B (2, 3) in the ratio $1:n$.

Now using section formula we have coordinates of C are $\left(\frac{n(1)+1(2)}{1+n}, \frac{n(0)+1(3)}{1+n}\right) = \left(\frac{n+2}{1+n}, \frac{3}{1+n}\right)$ And slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{3-0}{2-1} = 3$

Since, the required line is perpendicular to AB, slope of required line $m = \frac{-1}{3}$

We have the equation of a line passing through $(x_\circ, y_\circ)$ and slope m is $y - y_0 = m(x - x_0)$

$\therefore$ The equation of the line passing through point $\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$ having slope $m = \frac{-1}{3}$ is

$$y - \frac{3}{1+n} = \frac{-1}{3}\left(x - \frac{2+n}{1+n}\right)$$

$$\Rightarrow \frac{(1+n)y-3}{1+n} = \frac{-1}{3}\left\{\frac{(1+n)x-2-n}{1+n}\right\}$$

$$\Rightarrow 3(1+n)y - 9 = -(1+n)x + 2 + n$$

$$\Rightarrow (1+n)x + 3(1+n)y = n + 11$$

12. Find the equation of the line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Ans. Let equal intercepts on the coordinate axes be α

$\therefore$ Equation of the line in intercept form is $\frac{x}{\alpha} + \frac{y}{\alpha} = 1 \Rightarrow x + y = \alpha$

Since this line passes through point (2, 3), we get $\alpha = 2 + 3 = 5$

Therefore, the equation of required line is $\frac{x}{5} + \frac{y}{5} = 1$

$\Rightarrow x+y=5$

13. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Ans. Let the intercepts be a and b.

Given $a+b=9 \Rightarrow a=9-b$

Equation of the line in intercept form is $\frac{x}{9-b}+\frac{y}{b}=1$

Since this line passes through (2,2),we get $\frac{2}{9-b}+\frac{2}{b}=1$

$\Rightarrow \frac{2b+2(9-b)}{b(9-b)}=1$

$\Rightarrow 2b+18-2b=9b-b^2$

$\Rightarrow b^2-9b+18=0$

$\Rightarrow (b-3)(b-6)=0$

$\Rightarrow b=3$ and $b=6$

$\therefore a=9-3=6$ and $a=9-6=3$

Therefore, required equations of the lines are

$\frac{x}{6}+\frac{y}{3}=1$ and $\frac{x}{3}+\frac{y}{6}=1$

$\Rightarrow x+2y=6$ and $2x+y=6$

14. Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive $x-$ axis. Also find the equation of the line parallel to it and crossing the $y-$ axis at a distance of 2 units below the origin.

Ans. Given: $m=\tan\frac{2\pi}{3}=\tan 120^{\circ}=\tan\left(90^{\circ}+30^{\circ}\right)=-\cot 30^{\circ}=-\sqrt{3}$

We have the equation of a line passing through $(x_{\circ},y_{\circ})$ and slope m is $y-y_0=m\left(x-x_0\right)$

$\therefore$ Equation of the line passing through point (0, 2) having slope $-\sqrt{3}$ is

$y-2=-\sqrt{3}\left(x-0\right)$

$\Rightarrow \sqrt{3}x+y-2=0$

Now we have to find the equation of the line parallel to $\sqrt{3}x+y-2=0$ and crossing the y-axis at(0,-2)

Since parallel lines have same slope, slope of required line= $-\sqrt{3}$

Therefore, the equation of required line is $y-y_0=m\left(x-x_0\right)$

$\Rightarrow y-(-2)=-\sqrt{3}(x-0)$

$\Rightarrow y+2=-\sqrt{3}x$

$\Rightarrow \sqrt{3}x+y+2=0$

15. The perpendicular from the origin to a line meets it at the point $\left(-2,9\right)$, find the equation of the line.

Ans. Let O(0,0) be the origin and P be the given point (-2,9)

Now slope of the line OP = $\frac{9-0}{-2-0}=\frac{-9}{2}$

Since the required line is perpendicular to OP, it's Slope = $\frac{2}{9}$

We have the equation of a line passing through $(x_{\circ},y_{\circ})$ and slope m is $y-y_0=m\left(x-x_0\right)$

$\therefore$ Equation of the required line is $y-9=\frac{2}{9}(x+2)$

$\Rightarrow 9y-81=2x+4$

$\Rightarrow 2x-9y+85=0$

16. The length L (in centimeter) of a copper rod is a linear function of its Celsius temperature C. In an experiment if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

Ans. According to Question, when C=20, the value of L is 124.942 and when C=110, the value of L is 125.134

So the point (20, 124.942) and (110, 125.134) ll satisfy the linear function of L and C.

Considering C along x-axis and L along y axis, we have two points stated above in XY-plane. .i.e. (20, 124.942) and (110, 125.134)

Linear equation of L and C ll be the line passing through these two points.

$=> (L-124.942)=\frac{125.134-124.942}{110-20}(C-20)$

$=> (L-124.942)=\frac{0.192}{90}(C-20)$

$=> L=\frac{0.192}{90}(C-20)+124.942$

17. The owner of a milk store finds that he can sell 980 litres of milk each week at Rs. 14 per litre and 1220 litres of milk each week at Rs. 16 per litre. Assuming a linear relationship between selling price and demand, how many liters could he sell weekly at Rs. 17 per litre?

Ans. Let x be the demand in litres and y be the selling price per litre

Now the linear relationship between demand and selling price is the equation of the line passing through the points (980, 14) and (1220, 16)

We have the equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Here, (x_1, y_1) = (980, 14) and (x_2, y_2) = (1220, 16)

∴ Linear relation relationship between selling price and demand is given by

$$y - 14 = \left(\frac{16 - 14}{1220 - 980}\right)(x - 980)$$

$$\Rightarrow y - 14 = \frac{2}{240}(x - 980)$$

$$\Rightarrow y - 14 = \frac{1}{120}(x - 980)$$

$$\Rightarrow 120(y - 14) = x - 980$$

Putting $y = 17$, we have

$$\Rightarrow 120(17 - 14) = x - 980$$

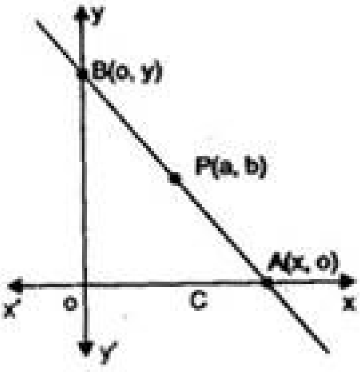
$$\Rightarrow 120 \times 3 = x - 980$$

$\Rightarrow x = 1340$ liters

Hence the owner could sell 1340 litres of milk weekly at rupees 17 per litre.

18. P (a, b) is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

Ans. Let A$(x, 0)$ and B$(0, y)$ be two points where the line intersect x and $y-$ axis respectively and P$\left(a, b\right)$ be the mid-point of AB.



Then $\frac{0+x}{2}=a$

$\Rightarrow x=2a$

And $\frac{0+y}{2}=b$

$\Rightarrow \ y=2b$

Hence the coordinates of A and B are (2a,0) and (0,2b) respectively

We have the equation of a line passing through the points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)\left(x-x_1\right)$$

$\therefore$ Equation of the required line is $y-0=\frac{(2b-0)}{(0-2a)}(x-2a)$

$\Rightarrow y=\frac{-b}{a}(x-2a)$

$\Rightarrow ay=-bx+2ab$

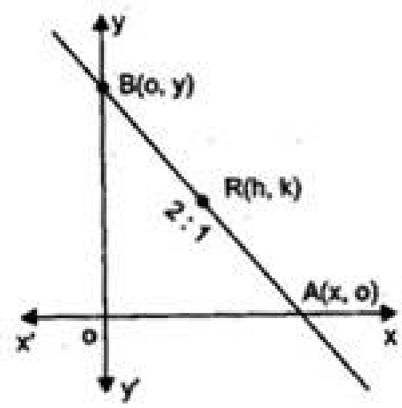
$\Rightarrow bx+ay=2ab$

On dividing both sides by ab,we get

$\frac{x}{a}+\frac{y}{b}=2$ Hence Proved

19. Point R(h, k) divides a line segment between the axes in the ratio 1: 2. Find equation of the line.

Ans. Let A$(x, 0)$ and B$(0, y)$ be two points where the line intersect x and $y-$ axes respectively and let R(h, k) be the point that divides AB in the ratio 1: 2.



$\therefore$ using section formula $\frac{2x+0}{2+1}=h$ and $\frac{0+y}{2+1}=k$

$\Rightarrow x=\frac{3}{2}h$ and $y=3k$

Therefore coordinates of A and B are $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$

Hence we have the intercepts on the coordinate axes are $\frac{3h}{2}$ and $3k$ respectively

$\therefore$ Using intercept form equation of the required line is

$$\frac{x}{3/2\,h}+\frac{y}{3k}=1$$

$$\Rightarrow \frac{2x}{3h}+\frac{y}{3k}=1$$

$$\Rightarrow 2kx+hy=3kh$$

20. By using concept of equation of a line, prove that the three points (3, 0), $(-2, -2)$ and (8, 2) are collinear.

Ans.

We have the equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Here, (x_1, y_1) = (3, 0) and (x_2, y_2) = $(-2, -2)$

Therefore equation of a line joining the points (3,0) and $(-2, -2)$ is

$$y - 0 = \left(\frac{-2 - 0}{-2 - 3}\right)(x - 3)$$

$$\Rightarrow\ y = \frac{2}{5}(x - 3)$$

$$\Rightarrow\ 5y = 2x - 6$$

$$\Rightarrow\ 2x - 5y = 6$$

Putting the coordinates of third point in in the above equation , we have $2 \times 8 - 5 \times 2 = 6$

$\Rightarrow$ 16 – 10 = 6

$\Rightarrow$ 6 = 6

Therefore,line joining the points (3,0) and $(-2, -2)$ also passes through the point (8,2)

Hence we have the given points are collinear.