CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 13 Limits and Derivative

Exercise 13.2

1. Find the derivative of x^2-2 at $x=10$.

Ans. $\frac{d}{dx}\left(x^2-2\right)=\frac{d}{dx}\left(x^2\right)-\frac{d}{dx}(2)=2x-0=2x \qquad \left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$

Therefore, Derivative of x^2-2 at $x=10$ is $2\times 10 = 20$

2. Find the derivative of $99x$ at $x=100$.

Ans. $\frac{d}{dx}(99x)=99\frac{d}{dx}(x)=99\times 1=99$

Therefore, Derivative of $99x$ at $x=100$ is 99

3. Find the derivative of x at $x=1$.

Ans. Here $\frac{d}{dx}(x)=1$

Therefore, Derivative of x at $x=1$ is 1

4. Find the derivatives of the following functions from first principle:

(i) x^3-27

(ii) $(x-1)(x-2)$

(iii) $\frac{1}{x^2}$

(iv) $\frac{x+1}{x-1}$

Ans. (i) Given: $f(x) = x^3 - 27$

$\therefore f(x+h) = (x+h)^3 - 27$

Now, since $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 27 - x^3 + 27}{h}$

$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 27 - x^3 + 27}{h}$

$= \lim_{h \to 0} \frac{h(h^2 + 3x^2 + 3xh)}{h} = 3x^2$

(ii) Given: $f(x) = (x-1)(x-2) = x^2 - 3x + 2$

Now $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\left[(x+h)^2 - 3(x+h) + 2\right] - \left[x^2 - 3x + 2\right]}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$

$= \lim_{h \to 0} \frac{h(h + 2x - 3)}{h} = 2x - 3$

(iii) Given: $f(x) = \frac{1}{x^2}$

$\therefore f(x+h) = \frac{1}{(x+h)^2}$

Now, since $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow$ $f'(x)=\lim\limits_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

$=\lim\limits_{h\to 0}\frac{x^2-(x+h)^2}{hx^2(x+h)^2}=\lim\limits_{h\to 0}\frac{x^2-x^2-h^2-2xh}{hx^2(x+h)^2}$

$=\lim\limits_{h\to 0}\frac{h(-h-2x)}{hx^2(x+h)^2}=\frac{-2x}{x^2\times x^2}=\frac{-2}{x^3}$

(iv) $f(x)=\frac{x+1}{x-1}$

$\therefore f(x+h)=\frac{x+h+1}{x+h-1}$

Now, since $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow$ $f'(x)=\lim\limits_{h\to 0}\frac{\frac{x+h+1}{x+h-1}-\frac{x+1}{x-1}}{h}$

$=\lim\limits_{h\to 0}\frac{(x+h+1)(x-1)-(x+1)(x+h-1)}{h(x+h-1)(x-1)}$

$=\lim\limits_{h\to 0}\frac{(x^2-x+hx-h+x-1)+(x^2+hx-x+x+h-1)}{h}$

$=\lim\limits_{h\to 0}\frac{x^2-x+xh-h+x-1-x^2-xh+x-x-h+1}{h(x+h-1)(x-1)}$

= $\lim_{h\to 0} \frac{-2h}{h(x+h-1)(x-1)} = \frac{-2}{(x-1)^2}$

5. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$, **prove that** $f'(1) = 100 f'(0)$.

Ans. Given: $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$

$\therefore f'(x) = \frac{d}{dx}\left[\frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1\right]$

= $\frac{1}{100}\frac{d}{dx}(x^{100}) + \frac{1}{99}\frac{d}{dx}(x^{99}) + ... + \frac{1}{2}\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$
$\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$

= $\frac{1}{100} \times 100x^{99} + \frac{1}{99}99x^{98} + ... + \frac{1}{2}2x + 1 + 0$

= $x^{99} + x^{98} + ... + x + 1$

Now $f'(1) = (1)^{99} + (1)^{98} + ... + 1 + 1 = 100$

And $f'(0) = (0)^{99} + (0)^{98} + ... + 0 + 1 = 1$

Now $f'(1) = 100 f'(0)$

$\Rightarrow 100 = 1 \times 100$

$\Rightarrow$ 100 = 100 Proved.

6. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + ...a^{n-1}x + a^n$ **for some fixed real number** a.

Ans. Given: $f(x) = x^n + ax^{n-1} + a^2x^{n-2} + a^{n-1}x + a^n$

$\therefore f'(x) = \frac{d}{dx}\left[x^n + ax^{n-1} + a^2x^{n-2} + \ldots\ldots + a^{n-1}x + a^n\right]$

= $\frac{d}{dx}x^n + \frac{d}{dx}\left(ax^{n-1}\right) + \frac{d}{dx}\left(a^2x^{n-2}\right) + \ldots\ldots + \frac{d}{dx}\left(a^{n-1}x\right) + \frac{d}{dx}a^n$
$\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$

= $nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \ldots\ldots\ldots\ldots\ldots + a^{n-1}$ $\left[\frac{d}{dx}(K) = 0\right]$, K is a constant

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7. For some constants a and b, find the derivative of:

(i) $(x-a)(x-b)$

(ii) $\left(ax^2+b\right)^2$

(iii) $\frac{x-a}{x-b}$

Ans. (i) Given: $f(x) = (x-a)(x-b)$

$\therefore f'(x) = \frac{d}{dx}(x-a)(x-b)$

$\Rightarrow f'(x) = (x-a)\frac{d}{dx}(x-b) + (x-b)\frac{d}{dx}(x-a)$ $\left[\begin{matrix}\frac{d}{dx}(K) = 0 \\ \frac{d}{dx}(x) = 1\end{matrix}\right]$ K is a constant

= $(x-a)\times 1 + (x-b)\times 1$

= $x - a + x - b = 2x - a - b$

(ii) Given: $f(x) = \left(ax^2+b\right)^2$ = $a^2x^4 + b^2 + 2abx^2$

$\therefore f'(x) = \frac{d}{dx}\left[a^2x^4 + b^2 + 2abx^2\right]$

= $a^2 \frac{d}{dx}\left(x^4\right)+\frac{d}{dx}\left(b^2\right)+2ab\frac{d}{dx}\left(x^2\right)$ $\left[\frac{d}{dx}(K)=0\right]$, K is a constant

= $a^2 \times 4x^3+0+2ab\times 2x$ = $4a^2x^3+4abx$ $\left[\frac{d}{dx}(x^n)=nx^{n-1}\right]$

= $4ax\left(ax^2+b\right)$

(iii) Given: $f(x)=\frac{x-a}{x-b}$ $\therefore$ $f'(x)=\frac{d}{dx}\left(\frac{x-a}{x-b}\right)$

$\Rightarrow f'(x)=\frac{(x-b)\frac{d}{dx}(x-a)-(x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$ $\left[\begin{array}{l}\frac{d}{dx}(K)=0\\ \frac{d}{dx}(x)=1\end{array}\right]$ K is a constant

= $\frac{(x-b)\times 1-(x-a)\times 1}{(x-b)^2}$

= $\frac{x-b-x+a}{(x-b)^2}$ = $\frac{a-b}{(x-b)^2}$

8. Find the derivative of $\frac{x^n-a^n}{x-a}$ for some constant a.

Ans. Given: $\frac{x^n-a^n}{x-a}$

$\therefore f'(x)=\frac{d}{dx}\left(\frac{x^n-a^n}{x-a}\right)$

$\Rightarrow f'(x)=\frac{(x-a)\frac{d}{dx}\left(x^n-a^n\right)-\left(x^n-a^n\right)\frac{d}{dx}(x-a)}{(x-a)^2}$ $\left[\frac{d}{dx}(x^n)=nx^{n-1}\right]$

$= \frac{(x-a)nx^{n-1} - (x^n - a^n) \times 1}{(x-a)^2}$ $\left[\frac{d}{dx}(K) = 0\right]$,K is a constant

$= \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$

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9. Find the derivative of:

(i) $2x - \frac{3}{4}$

(ii) $(5x^3 + 3x - 1)(x-1)$

(iii) $x^3(5+3x)$

(iv) $x^5(3-6x^{-9})$

(v) $x^{-4}(3-4x^{-5})$

(vi) $\frac{2}{x+1} - \frac{x^2}{3x-1}$

Ans. (i) Given: $f(x) = 2x - \frac{3}{4}$

$\therefore f'(x) = \frac{d}{dx}\left(2x - \frac{3}{4}\right)$ $\left[\frac{d}{dx}(K) = 0\right]$,K is a constant

$\Rightarrow f'(x) = 2\frac{d}{dx}x - \frac{d}{dx}\left(\frac{3}{4}\right)$ = $2 \times 1 - 0 = 2$

(ii) Given: $f(x) = (5x^3 + 3x - 1)(x-1)$

$\therefore f'(x) = \frac{d}{dx}\left[(5x^3 + 3x - 1)(x-1)\right]$ [Product rule]

= $\left(5x^3+3x-1\right)\frac{d}{dx}(x-1)+(x-1)\frac{d}{dx}\left(5x^3+3x-1\right)$ $\left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$

= $\left(5x^3+3x-1\right)\times1+(x-1)\left(15x^2+3\right)$

= $5x^3+3x-1+15x^3+3x-15x^2-3$

= $20x^3-15x^2+6x-4$

(iii) Given: $f(x)=x^{-3}(5+3x)$

$\therefore\ f'(x)=\frac{d}{dx}\left[x^{-3}(5+3x)\right]$ [Product rule]

$\Rightarrow f'(x)=x^{-3}\frac{d}{dx}(5+3x)+(5+3x)\frac{d}{dx}\left(x^{-3}\right)$ $\left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$

= $x^{-3}\times3+(5+3x)\left(-3x^{-4}\right)$ = $\frac{3}{x^3}-\frac{3}{x^4}(5+3x)$

= $\frac{3}{x^3}\left(1-\frac{5+3x}{x}\right)=\frac{3}{x^3}\left(\frac{x-5-3x}{x}\right)=\frac{3}{x^3}\left(\frac{-5-2x}{x}\right)=\frac{-3}{x^4}(5+2x)$

(iv) Given: $f(x)=x^5\left(3-6x^{-9}\right)$

$\therefore\ f'(x)=\frac{d}{dx}\left[x^5\left(3-6x^{-9}\right)\right]$ [Product rule]

$\Rightarrow f'(x)=x^5\frac{d}{dx}\left(3-6x^{-9}\right)+\left(3-6x^{-9}\right)\frac{d}{dx}x^5$ $\left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$

$=x^5\left(54x^{-10}\right)+\left(3-6x^{-9}\right)\times5x^4$ = $54x^{-5}+15x^4-30x^{-5}$

= $24x^{-5}+15x^4$ = $\frac{24}{x^5}+15x^4$

(v) Given: $f(x)=x^{-4}\left(3-4x^{-5}\right)$

$\therefore f'(x)=\frac{d}{dx}\left[x^{-4}\left(3-4x^{-5}\right)\right]$ [Product rule]

$\Rightarrow f'(x)=x^{-4}\frac{d}{dx}\left(3-4x^{-5}\right)+\left(3-4x^{-5}\right)\frac{d}{dx}x^{-4}$ $\left[\frac{d}{dx}\left(x^{n}\right)=nx^{n-1}\right]$

$=x^{-4}\left(20x^{-6}\right)+\left(3-4x^{-5}\right)\left(-4x^{-5}\right)=20x^{-10}-12x^{-5}+16x^{-10}$

$=36x^{-10}-12x^{-5}=\frac{36}{x^{10}}-\frac{12}{x^{5}}$

(vi) Given: $f(x)=\frac{2}{x+1}-\frac{x^2}{3x-1}$

$\therefore f'(x)=\frac{d}{dx}\left[\frac{2}{x+1}-\frac{x^2}{3x-1}\right]$

$\Rightarrow f'(x)=\frac{d}{dx}\left(\frac{2}{x+1}\right)-\frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$

$=\frac{(x+1)\frac{d}{dx}(2)-2\frac{d}{dx}(x+1)}{(x+1)^2}-\frac{(3x-1)\frac{d}{dx}x^2-x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}$ (Quotient rule)

$=\frac{(x+1)\times 0-2\times 1}{(x+1)^2}-\frac{(3x-1)(2x)-x^2(3)}{(3x-1)^2}$

$=\frac{-2}{(x+1)^2}-\frac{6x^2-2x-3x^2}{(3x-1)^2}$

$=\frac{-2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$

$=\frac{-2}{(x+1)^2}-\frac{x(3x-2)}{(3x-1)^2}$

10. Find the derivative of $\cos x$ from first principle.

Ans. Given: $f(x) = \cos x$,

then $f(x+h) = \cos(x+h)$

Since, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$

$\left[\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

= $\lim_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

= $\lim_{h\to 0} \left[-\sin\left(\frac{2x+h}{2}\right) \cdot \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right]$

= $\lim_{h\to 0} -\sin\left(x + \frac{h}{2}\right) \lim_{\frac{h}{2}\to 0} \left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$ $\left[\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$

$\left[h \to 0 \Rightarrow \frac{h}{2} \to 0\right]$

= $-\sin x \times 1$= $-\sin x$

11. Find the derivative of the following functions:

(i) $\sin x \cos x$

(ii) $\sec x$

(iii) $5\sec x+4\cos x$

(iv) $\cos ec\ x$

(v) $3\cot x+5\cos ec\ x$

(vi) $5\sin x-6\cos x+7$

(vii) $2\tan x-7\sec x$

Ans. (i) Given: $f(x)=\sin x\cos x$

$\therefore f'(x)=\frac{d}{dx}(\sin x\cos x)$ [Product rule]

$\Rightarrow f'(x)=\sin x\frac{d}{dx}\cos x+\cos x\frac{d}{dx}\sin x$

= $\sin x(-\sin x)+\cos x\cos x$

= $\cos^2 x-\sin^2 x$ = $\cos 2x$

(ii) Given: $f(x)=\sec x$

$\therefore f'(x)=\frac{d}{dx}\sec x$

$\Rightarrow f'(x)=\sec x\tan x$

(iii) Given: $f(x)=5\sec x+4\cos x$

$\therefore f'(x)=\frac{d}{dx}(5\sec x+4\cos x)$

$\Rightarrow f'(x)=5\frac{d}{dx}\sec x+4\frac{d}{dx}\cos x$

= $5\sec x\tan x-4\sin x$

(iv) Given: $f(x) = \operatorname{cosec} x$

$$\therefore f'(x) = \frac{d}{dx}\operatorname{cosec} x$$

$$\Rightarrow f'(x) = -\operatorname{cosec} x \cot x$$

(v) Given: $f(x) = 3\cot x + 5\operatorname{cosec} x$

$$\therefore f'(x) = \frac{d}{dx}(3\cot x + 5\operatorname{cosec} x)$$

$$\Rightarrow f'(x) = 3\frac{d}{dx}\cot x + 5\frac{d}{dx}\operatorname{cosec} x$$

= $-3\operatorname{cosec}^2 x - 5\operatorname{cosec} x \cot x$

(vi) Given: $f(x) = 5\sin x - 6\cos x + 7$

$$\therefore f'(x) = \frac{d}{dx}(5\sin x - 6\cos x + 7)$$

$$\Rightarrow f'(x) = 5\frac{d}{dx}\sin x - 6\frac{d}{dx}\cos x + \frac{d}{dx}7$$

= $5\cos x - 6(-\sin x) + 0$

= $5\cos x + 6\sin x + 0$ = $5\cos x + 6\sin x$

(vii) Given: $f(x) = 2\tan x - 7\sec x$

$$\therefore f'(x) = \frac{d}{dx}(2\tan x - 7\sec x)$$

$$\Rightarrow f'(x) = 2\frac{d}{dx}\tan x - 7\frac{d}{dx}\sec x$$

= $2\sec^2 x - 7\sec x \tan x$