CBSE Class–11 Mathematics

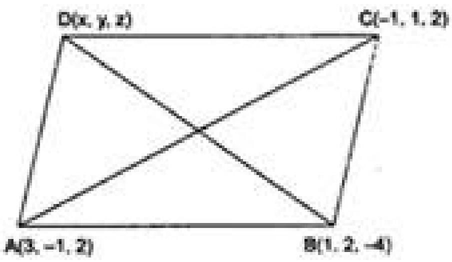
NCERT Solutions

Chapter - 12 Introduction to Three Dimensional Geometry

Miscellaneous Exercise

1. Three vertices of a parallelogram ABCD are A$(3,-1,2)$, B$(1,2,-4)$ and C$(-1,1,2)$. Find the coordinates of the fourth vertex.

Ans. Let D(x,y,z) be the fourth vertex of parallelogram ABCD.



Since, the diagonals of a parallelogram bisect each other. Therefore, the mid-point of AC and BD coincide.

$\therefore$ Coordinates of mid-point of AC = $\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$ = $(1,0,2)$

Also Coordinates of mid-point of BD = $\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$

$\therefore \frac{x+1}{2}=1 \Rightarrow x+1=2 \Rightarrow x=1$

$\frac{y+2}{2}=0 \Rightarrow y+2=0 \Rightarrow y=-2$

$\frac{z-4}{2}=2 \Rightarrow z-4=4 \Rightarrow z=8$

Therefore, the coordinates of point D are $(1,-2,8)$.

2. Find the length of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

Ans. Given: A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0) are vertices of ΔABC.

Let D, E and F be the mid-points of BC, AC and AB respectively. Then

Coordinates of D = $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ = $(3, 2, 0)$

And AD = $\sqrt{(0-3)^2+(0-2)^2+(6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$ units

Again, Coordinates of E = $\left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right)$ = $(3, 0, 3)$

And BE = $\sqrt{(0-3)^2+(4-0)^2+(0-3)^2} = \sqrt{9+16+9} = \sqrt{34}$ units

Also Coordinates of F = $\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right)$ = $(0, 2, 3)$

And CF = $\sqrt{(6-0)^2+(0-2)^2+(0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$ units

3. If the origin is the centroid of the triangle PQR with vertices P $(2a, 2, 6)$, Q $(-4, 3b, -10)$ and R $(8, 14, 2c)$, then find the values of a, b and c.

Ans. Given: P $(2a, 2, 6)$, Q $(-4, 3b, -10)$ and R $(8, 14, 2c)$ are the vertices of triangle PQR.

$\therefore$ Coordinates of centroid of ΔPQR = $\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$

= $\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

According to question,

$\frac{2a+4}{3}=0 \Rightarrow 2a+4=0 \Rightarrow a=-2$

$\frac{3b+16}{3}=0 \Rightarrow 3b+16=0 \Rightarrow b=\frac{-16}{3}$

$\frac{2c-4}{3}=0 \Rightarrow 2c-4=0 \Rightarrow c=2$

Therefore, $a=-2$, $b=\frac{-16}{3}$, $c=2$

4. Find the coordinates of a point on $y-$ axis which are at a distance of $5\sqrt{2}$ from the point P$\left(3,-2,5\right)$.

Ans. Let Q$\left(0,y,0\right)$ be any point on $y-$ axis. Then according to question,

PQ = $\sqrt{\left(0-3\right)^2+\left(y+2\right)^2+\left(0-5\right)^2}$ = $5\sqrt{2}$

$\Rightarrow \sqrt{9+y^2+4+4y+25}=5\sqrt{2}$

$\Rightarrow \sqrt{y^2+4y+38}=5\sqrt{2}$

Squaring both sides,

$\Rightarrow y^2+4y+38=50$

$\Rightarrow y^2+4y-12=0$

$\Rightarrow \left(y-2\right)\left(y+6\right)=0$

$\Rightarrow y=2, y=-6$

Therefore, the coordinates of point Q are (0, 2, 0) and $\left(0,-6,0\right)$.

5. A point R with $x-$ coordinate 4 lies on the line segment joining the points P$\left(2,-3,4\right)$ and Q (8, 0, 10). Find the coordinates of the point R.

Ans. Let R$\left(4,y,z\right)$ be any point which divides the line segment joining P$\left(2,-3,4\right)$ and Q

$(8,0,10)$ in the ratio $k:1$ internally.

$\therefore$ Coordinates of R = $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

But according to question, $\frac{8k+2}{k+1} = 4$

$\Rightarrow 8k + 2 = 4k + 4 \Rightarrow k = \frac{1}{2}$

$\therefore y = \frac{-3}{\frac{1}{2}+1} = \frac{-3}{\frac{3}{2}} = -2$ and $z = \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1} = \frac{9}{\frac{3}{2}} = 6$

Therefore, coordinates of R is $(4,-2,6)$.

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6. If A and B be the points (3, 4, 5) and $(-1,3,-7)$ respectively. Find the equation of the set of points P such that $\mathrm{PA}^2 + \mathrm{PB}^2 = k^2$ where k is a constant.

Ans. Let P(x, y, z) be any point.

$\therefore \mathrm{PA}^2 + \mathrm{PB}^2 = k^2 \Rightarrow$

$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = k^2$

$\Rightarrow$

$x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x + y^2 - 6y + 9 + z^2 + 49 +$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$

$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$