

CBSE Class-11 Mathematics

NCERT Solutions

Chapter - 12 Introduction to Three Dimensional Geometry

Exercise 12.3

1. Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio:

(i) 2: 3 internally,

(ii) 2: 3 externally.

Ans. (i) Let $P(x, y, z)$ be any point which divides the line segment joining points A $(-2, 3, 5)$

and B $(1, -4, 6)$ in the ratio 2: 3 internally. Then

$$x = \frac{2 \times 1 + 3 \times (-2)}{2 + 3} = \frac{2 - 6}{5} = \frac{-4}{5}$$

$$y = \frac{2 \times (-4) + 3 \times 3}{2 + 3} = \frac{-8 + 9}{5} = \frac{1}{5}$$

$$z = \frac{2 \times 6 + 3 \times 5}{2 + 3} = \frac{12 + 15}{5} = \frac{27}{5}$$

Therefore, Coordinates of P are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

(ii) Let $P(x, y, z)$ be any point which divides the line segment joining points A $(-2, 3, 5)$ and B $(1, -4, 6)$ in the ratio 2: 3 externally. Then

$$x = \frac{2 \times 1 - 3 \times (-2)}{2 - 3} = \frac{2 + 6}{-1} = -8$$

$$y = \frac{2 \times (-4) - 3 \times 3}{2 - 3} = \frac{-8 - 9}{-1} = 17$$

$$z = \frac{2 \times 6 - 3 \times 5}{2 - 3} = \frac{12 - 15}{-1} = 3$$

Therefore, Coordinates of P are $(-8, 17, 3)$.

2. Given that P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Ans. Let Q(5, 4, -6) divides the line segment joining points P(3, 2, -4) and R(9, 8, -10) in the ratio $k:1$ internally. Then

$$\text{Coordinates of Q are } \left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1} \right)$$

$$\text{According to question, } \frac{9k+3}{k+1} = 5 \text{ (Since; collinear)}$$

$$\Rightarrow 9k+3 = 5k+5$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{1}{2}$$

Therefore, Q divides the line segment joining the points P and R in the ratio 1: 2.

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.

Ans. Let the line segment joining points A $(-2, 4, 7)$ and B $(3, -5, 8)$ be divided by YZ-plane at a point C in the ratio $k:1$ internally. Then

$$\text{Coordinates of C are } \left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1} \right)$$

According to question, C lies on YZ-plane, i.e., $x = 0$

$$\therefore \frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k - 2 = 0$$

$$\Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$$

Therefore, the required ratio is 2: 3.

4. Using section formula, show that the points A(2, -3, 4), B(-1, 2, 1) and C(0, $\frac{1}{3}$, 2) are collinear.

Ans. Let the point P (x,y,z) divides the line segment joining A(2,-3,4) and B(-1,2,1) in the ratio $k:1$ internally.

$$\text{Then coordinates of P} = \left(\frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1} \right)$$

Now, we ll find values of k at which point P coincides with point C,

$$\text{By taking } \frac{-k+2}{k+1} = 0,$$

$$\Rightarrow -k + 2 = 0,$$

$$\Rightarrow k = 2$$

$$\text{For } k=2, \text{ the coordinates of P are } \left(0, \frac{1}{3}, 2 \right)$$

i.e the point C $\left(0, \frac{1}{3}, 2 \right)$ is the point which divides AB in 2:1 internally and is same as of point P.

Hence A,B,C are collinear.

5. Find the coordinates of the points which trisect the line segment joining the points P

$(4, 2, -6)$ and $Q(10, -16, 6)$.

Ans. Let R and S be two points which trisect the line segment PQ.

Then $PR = RS = SQ$

\therefore Point R divides the line segment PQ in the ratio 1: 2.

Then Coordinates of R = $\left(\frac{1 \times 10 + 2 \times 4}{1+2}, \frac{1 \times (-16) + 2 \times 2}{1+2}, \frac{1 \times 6 + 2 \times (-6)}{1+2} \right) = (6, -4, -2)$

Again, Point S divides the line segment PQ in the ratio 2: 1.

Then Coordinates of S = $\left(\frac{2 \times 10 + 1 \times 4}{1+2}, \frac{2 \times (-16) + 1 \times 2}{1+2}, \frac{2 \times 6 + 1 \times (-6)}{1+2} \right) = (8, -10, 2)$