CBSE Class–11 Mathematics
NCERT Solutions
Chapter - 4 Principle of Mathematical Induction
Exercise 4.1

Prove the following by using the principle of mathematical induction for all $n \in$ N:

1. $1+3+3^2+..........+3^{n-1}=\frac{(3^n-1)}{2}$

Ans. Let $P(n)=1+3+3^2+..........+3^{n-1}=\frac{(3^n-1)}{2}$

For $n=1$

$\Rightarrow$ $P(1)=1=\frac{(3^1-1)}{2}$

$\Rightarrow$1 = 1

$\therefore P(1)$ is true.

Now, let $P(n)$ be true for $n=k$

$\Rightarrow P(k)=1+3+3^2+.......+3^{k-1}=\frac{(3^k-1)}{2}$ ....(i)

For $n=k+1$

$\Rightarrow$ $P(k+1)=1+3+3^2+..........+3^{k-1}+3^k=\frac{(3^k-1)}{2}+3^k$ [Using eq. (i)]

$\Rightarrow P(k+1)=\frac{3^k-1+2.3^k}{2}=\frac{3.3^k-1}{2}=\frac{3^{k+1}-1}{2}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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2. $1^3+2^3+3^3+\ldots\ldots\ldots.+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$

Ans. Let $\mathrm{P}(n)=1^3+2^3+3^3+\ldots\ldots\ldots.+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$

For $n=1$

$$\Rightarrow \mathrm{P}(1)=1=\left[\frac{1(1+1)}{2}\right]^2$$

$\Rightarrow 1 = 1$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$$\Rightarrow \mathrm{P}(k)=1^3+2^3+3^3+\ldots\ldots\ldots.+k^3=\left[\frac{k(k+1)}{2}\right]^2 \quad \ldots\ldots\ldots.(i)$$

For $n=k+1$

$$\mathrm{P}(k+1)=1^3+2^3+3^3+\ldots\ldots\ldots.+k^3+(k+1)^3$$
$$=\left[\frac{k(k+1)}{2}\right]^2+(k+1)^3$$

$\Rightarrow \mathrm{P}(k+1)=(k+1)^2\left[\frac{k^2}{4}+k+1\right]$

= $(k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

= $\frac{(k+1)^2(k+2)^2}{4}$

$\Rightarrow \mathrm{P}(k+1)=\left[\frac{(k+1)(k+2)}{2}\right]^2$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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3. $1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots\ldots\ldots+\frac{1}{(1+2+3+\ldots\ldots+n)}=\frac{2n}{(n+1)}$

Ans. Let $\mathrm{P}(n)=1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots\ldots\ldots+\frac{1}{(1+2+3+\ldots\ldots+n)}=\frac{2n}{(n+1)}$

For $n=1$

$\Rightarrow \mathrm{P}(1)=1=\frac{2\times 1}{1+1}$

$\Rightarrow 1 = 1$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\therefore\ \mathrm{P}(k)=1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+..........+\frac{1}{(1+2+3+......+k)}=\frac{2k}{(k+1)}$ ....(i)

For $n=k+1$

$\Rightarrow \mathrm{P}(k+1)=1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+..........+\frac{1}{(1+2+3+......+k)}$

$+\frac{1}{(1+2+3+......+k+1)}=\frac{2k}{(k+1)}+\frac{1}{(1+2+3+......+k+1)}$ [Using (i)]

$\Rightarrow \mathrm{P}(k+1)=\frac{2k}{k+1}+\frac{1}{\frac{(k+1)(k+2)}{2}}$

$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$

$=\frac{2}{k+1}\left[\frac{(k+1)^2}{k+2}\right]=\frac{2(k+1)}{k+2}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true..

Hence by Principle of Mathematical Induction, P(n) is true for all $n\in$ N.

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4. $1.2.3+2.3.4+..........+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

Ans. Let $\mathrm{P}(n)=1.2.3+2.3.4+..........+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

For $n=1$

$\Rightarrow \mathrm{P}(1)=1\times2\times3=\frac{1\times2\times3\times4}{4}$

$\Rightarrow$ 6 = 6

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\therefore \ \mathrm{P}(k)=1.2.3+2.3.4+..........+k(k+1)(k+2)=\dfrac{k(k+1)(k+2)(k+3)}{4}$ ....(i)

For $n=k+1$

$\therefore \mathrm{P}(k+1)=1.2.3+2.3.4+..........+k(k+1)(k+2)+(k+1)(k+2)(k+3)$

$=\dfrac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$ [Using eq. (i)]

$\Rightarrow \mathrm{P}(k+1)=(k+1)(k+2)(k+3)\left(\dfrac{k}{4}+1\right)$

= $(k+1)(k+2)(k+3)\left(\dfrac{k+4}{4}\right)$

$\Rightarrow \mathrm{P}(k+1)=\dfrac{(k+1)(k+2)(k+3)(k+4)}{4}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n\in$ N.

5. $1.3+2.3^2+3.3^3+..........+n.3^n=\dfrac{(2n-1)3^{n+1}+3}{4}$

Ans. Let $\mathrm{P}(n)=1.3+2.3^2+3.3^3+..........+n.3^n=\dfrac{(2n-1)3^{n+1}+3}{4}$

For $n=1$

$$\Rightarrow \mathrm{P}(1)=1\times 3=\frac{(2\times 1-1)3^{1+1}+3}{4}$$

$\Rightarrow$ 3 = 3

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$$\therefore \mathrm{P}(k)=1.3+2.3^2+3.3^3+\dots\dots\dots+k.3^k=\frac{(2k-1)3^{k+1}+3}{4}$$

For $n=k+1$

$\Rightarrow$

$$\mathrm{P}(k+1)=1.3+2.3^2+3.3^3+\dots\dots\dots+k.3^k+(k+1)3^{k+1}=\frac{(2k-1)3^{k+1}+3}{4}+(k+1)3^{k+1}$$

$$\Rightarrow \mathrm{P}(k+1)=\frac{(2k-1)3^{k+1}}{4}+\frac{3}{4}+(k+1)3^{k+1}$$

$$=3^{k+1}\left[\frac{2k-1}{4}+k+1\right]+\frac{3}{4}$$

$$\Rightarrow \mathrm{P}(k+1)=3^{k+1}\left[\frac{2k-1+4k+4}{4}\right]+\frac{3}{4}$$

$$=3^{k+1}\left(\frac{6k+3}{4}\right)+\frac{3}{4}$$

$$\Rightarrow \mathrm{P}(k+1)=\frac{3^{k+1}.3(2k+1)}{4}+\frac{3}{4}$$

$$=\frac{(2k+1)3^{k+2}+3}{4}$$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

6. $1.2+2.3+3.4+\dots\dots\dots+n(n+1)=\left[\dfrac{n(n+1)(n+2)}{3}\right]$

Ans. Let $\mathrm{P}(n)=1.2+2.3+3.4+\dots\dots\dots+n(n+1)=\left[\dfrac{n(n+1)(n+2)}{3}\right]$

For $n=1$

$$\Rightarrow \mathrm{P}(1)=1(1+1)=\frac{1(1+1)(1+2)}{3}$$

$\Rightarrow 2 = 2$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$$\therefore \mathrm{P}(k)=1.2.+2.3.+\dots\dots\dots+k(k+1)=\frac{k(k+1)(k+2)}{3} \quad \dots\dots\dots\text{(i)}$$

For $n=k+1$

$$\therefore \mathrm{P}(k+1)=1.2+2.3+\dots\dots\dots+k(k+1)+(k+1)(k+2)$$

$$=\frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$$

$$\Rightarrow \mathrm{P}(k+1)=(k+1)(k+2)\left(\frac{k}{3}+1\right)$$

= $(k+1)(k+2)\left(\frac{k+3}{3}\right)$

= $\frac{(k+1)(k+2)(k+3)}{3}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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7. $1.3+3.5+5.7+\ldots\ldots\ldots.+(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$

Ans. Let $\mathrm{P}(n)=1.3+3.5+5.7+\ldots\ldots\ldots.+(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$

For $n=1$ $\mathrm{P}(1)=(2\times1-1)(2\times1+1)=\frac{1\left[4(1)^2+6\times1-1\right]}{3}$

$\Rightarrow$ 3 = 3

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\therefore \mathrm{P}(k)=1.3+3.5+5.7+\ldots\ldots\ldots.+(2k-1)(2k+1)=\frac{k\left(4k^2+6k-1\right)}{3}$

For $n=k+1$

$\mathrm{P}(k+1)=1.3+3.5+5.7+\ldots\ldots\ldots.+(2k-1)(2k+1)+\left[2(k+1)-1\right]\left[2(k+1)+1\right]$

= $\dfrac{k\left(4k^2+6k-1\right)}{3}+\left[2(k+1)-1\right]\left[2(k+1)+1\right]$

$\Rightarrow \mathrm{P}(k+1)=\dfrac{4k^3+6k^2-k}{3}+(2k+1)(2k+3)$

= $\dfrac{4k^3+6k^2-k+3\left(4k^2+8k+3\right)}{3}$

$\Rightarrow \mathrm{P}(k+1)=\dfrac{4k^3+6k^2-k+12k^2+24k+9}{3}$

= $\dfrac{4k^3+18k^2+23k+9}{3}$

= $\dfrac{(k+1)\left(4k^2+14k+9\right)}{3}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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8. $1.2+2.2^2+3.2^3+\ldots\ldots\ldots.+n.2^n=(n-1)2^{n+1}+2$

Ans. Let $\mathrm{P}(n)=1.2+2.2^2+3.2^3+\ldots\ldots\ldots.+n.2^n=(n-1)2^{n+1}+2$

For $n=1$ $\mathrm{P}(1)=1\times 2^1=(1-1)2^{1+1}+2$

$\Rightarrow$ 2 = 2

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\therefore \mathrm{P}(k)=1.2+2.2^2+3.2^3+\ldots\ldots\ldots+k.2^k=(k-1)2^{k+1}+2$

For $n=k+1$

$\mathrm{P}(k+1)=1.2+2.2^2+3.2^3+\ldots\ldots\ldots+k.2^k+(k+1).2^{k+1}=(k-1)2^{k+1}+2+(k+1).2^{k+1}$

$\Rightarrow \mathrm{P}(k+1)=(k-1).2^{k+1}+2+(k+1).2^{k+1}$

= $2^{k+1}(k-1+k+1)+2$

= $2^{k+1}\times 2k+2$ = $k.2^{k+2}+2$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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9. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

Ans. Let $\mathrm{P}(n)=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

For $n=1$ $\mathrm{P}(1)=\frac{1}{2^1}=1-\frac{1}{2^1}$

$\Rightarrow \frac{1}{2}=\frac{1}{2}$

$\therefore$ P(1) is true.

Now, let P(n) be true for $n=k$

$\therefore \mathrm{P}(k)=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots+\frac{1}{2^k}=1-\frac{1}{2^k}$

For $n=k+1$

$$\Rightarrow P(k+1)=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots\dots\dots+\frac{1}{2^k}+\frac{1}{2^{k+1}}=1-\frac{1}{2^k}+\frac{1}{2^{k+1}}$$

$$\Rightarrow P(k+1)=1-\left(\frac{1}{2^k}-\frac{1}{2^{k+1}}\right)$$

$$= 1-\left(\frac{2-1}{2^{k+1}}\right) = 1-\frac{1}{2^{k+1}}$$

$\therefore P(k+1)$ is true.

Therefore, $P(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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10. $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots\dots\dots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$

Ans. Let $P(n)=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots\dots\dots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$

For $n=1$

$$\Rightarrow P(1)=\frac{1}{(3\times1-1)(3\times1+2)}=\frac{1}{(6\times1+4)}$$

$$\Rightarrow \frac{1}{10}=\frac{1}{10}$$

$\therefore P(1)$ is true.

Now, let $P(n)$ be true for $n=k$

$$\therefore P(k)=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots\dots\dots+\frac{1}{(3k-1)(3k+2)}=\frac{k}{(6k+4)}$$

For $n=k+1$

$\Rightarrow$

$$P(k+1)=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots\dots\dots+\frac{1}{(3k-1)(3k+2)}+\frac{1}{[3(k+1)-1][3(k+1)+2]}$$

$$\Rightarrow P(k+1)=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)}$$

$$\Rightarrow P(k+1)=\frac{1}{(3k+2)}\left[\frac{k}{2}+\frac{1}{3k+5}\right]$$

$$=\frac{1}{(3k+2)}\left[\frac{3k^2+5k+2}{2(3k+5)}\right]$$

$$=\frac{1}{(3k+2)}\left[\frac{(k+1)(3k+2)}{2(3k+5)}\right]$$

$$\Rightarrow P(k+1)=\frac{k+1}{6k+10}$$

$\therefore P(k+1)$ is true.

Therefore, $P(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n\in$ N.

11. $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\dots\dots\dots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

Ans. Let $P(n)=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\dots\dots\dots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

For $n=1$

$$\Rightarrow \mathrm{P}(1)=\frac{1}{1(1+1)(1+2)}=\frac{1(1+3)}{4(1+1)(1+2)}$$

$$\Rightarrow \frac{1}{6}=\frac{1}{6}$$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$$\Rightarrow \mathrm{P}(k)=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\text{..........}+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)} \text{ ....(i)}$$

For $n=k+1$

$$\Rightarrow \text{R.H.S.} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

And L.H.S. = $\frac{k(k+3)}{4(k+2)(k+3)}+\frac{1}{(k+1)(k+2)(k+3)}$ [Using eq. (i)]

$$= \frac{1}{(k+1)(k+2)}\left[\frac{k^2+3k}{4}+\frac{1}{k+3}\right]$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{k^3+6k^2+9k+4}{4(k+3)}\right]$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{(k+1)^2(k+4)}{4(k+3)}\right]$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

12. $a+ar+ar^2+\ldots\ldots\ldots.+ar^{n-1}=\dfrac{a\left(r^n-1\right)}{r-1}$

Ans. Let $\mathrm{P}(n)=a+ar+ar^2+\ldots\ldots\ldots.+ar^{n-1}=\dfrac{a\left(r^n-1\right)}{r-1}$

For $n=1$

$\Rightarrow \mathrm{P}(1)=ar^{1-1}=\dfrac{a\left(r^1-1\right)}{r-1}$

$\Rightarrow a=a$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\Rightarrow \mathrm{P}(k)=a+ar+ar^2+\ldots\ldots\ldots.+ar^{k-1}=\dfrac{a\left(r^k-1\right)}{r-1}$ ....(i)

For $n=k+1$

R.H.S. = $\dfrac{a\left(r^{k+1}-1\right)}{r-1}$

$\therefore$ L.H.S. = $\frac{a(r^k-1)}{r-1}+ar^k$ [Using eq. (i)]

L.H.S. = $\dfrac{ar^k}{r-1}-\dfrac{a}{r-1}+ar^k$

= $ar^k\left(\dfrac{1}{r-1}+1\right)-\dfrac{a}{r-1}$

= $ar^k\left(\frac{r}{r-1}\right)-\frac{a}{r-1}$

= $\frac{ar^{k+1}}{r-1}-\frac{a}{r-1}$

= $\frac{ar^{k+1}-a}{r-1}$

= $\frac{a\left(r^{k+1}-1\right)}{r-1}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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13. $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)$..........$\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$

Ans. Let $\mathrm{P}(n)=\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)$..........$\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$

For $n=1$ $\mathrm{P}(1)=1+\frac{(2\times1+1)}{1^2}=(1+1)^2$

$\Rightarrow 4=4$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\Rightarrow \mathrm{P}(k)=\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)$..........$\left(1+\frac{2k+1}{k^2}\right)=(k+1)^2$ ....(i)

For $n = k+1$

R.H.S. = $(k+2)^2$

$\therefore$ L.H.S. = $(k+1)^2\left(1+\dfrac{2k+3}{(k+1)^2}\right)$ [Using eq. (i)]

L.H.S. = $(k+1)^2\left(\dfrac{(k+1)^2+2k+3}{(k+1)^2}\right)$

= $k^2+4k+4=(k+2)^2$

$\therefore$ $\mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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14. $\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)..........\left(1+\dfrac{1}{n}\right)=(n+1)$

Ans. Let $\mathrm{P}(n)=\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)..........\left(1+\dfrac{1}{n}\right)=(n+1)$

For $n=1$ $\mathrm{P}(1)=\left(1+\dfrac{1}{1}\right)=1+1$

$\Rightarrow 2=2$

$\therefore$ $\mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\Rightarrow$ $\mathrm{P}(k)=\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)..........\left(1+\dfrac{1}{k}\right)=(k+1)$

For $n = k+1$

R.H.S. = $k+2$

$\therefore$ L.H.S. = $(k+1)\left(1+\frac{1}{k+1}\right)$ [Using eq. (i)]

L.H.S. = $(k+1)\left(\frac{k+1+1}{k+1}\right)$ = $(k+2)$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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15. $1^2+3^2+5^2+\ldots\ldots\ldots\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

Ans. Let $\mathrm{P}(n)=1^2+3^2+5^2+\ldots\ldots\ldots\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

For $n=1$

$\Rightarrow \mathrm{P}(1)=(2\times1-1)^2=\frac{1(2\times1-1)(2\times1+1)}{3}$

$\Rightarrow 1=1$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$\Rightarrow \mathrm{P}(k)=1^2+3^2+5^2+\ldots\ldots\ldots\ldots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$ ....(i)

For $n=k+1$

R.H.S. = $\frac{(k+1)(2k+1)(2k+3)}{3}$

$\therefore$ L.H.S. = $\frac{k(2k-1)(2k+1)}{3}+(2k+1)^2$ [Using eq. (i)]

$=(2k+1)\left(\frac{k(2k-1)}{3}+(2k+1)\right)$

$= (2k+1)\left(\frac{2k^2-k+6k+3)}{3}\right)$

$= \frac{(2k+1)(2k^2+5k+3)}{3}$

$= \frac{(2k+1)(k+1)(2k+3)}{3}$

$= \frac{(k+1)(2k+1)(2k+3)}{3}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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16. $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\text{..........}+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

Ans. Let $\mathrm{P}(n)=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\text{..........}+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

For $n=1$

$\Rightarrow \mathrm{P}(1)=\frac{1}{(3\times1-2)(3\times1+1)}=\frac{1}{(3\times1+1)}$

$$\Rightarrow \frac{1}{4}=\frac{1}{4}$$

$\therefore \mathrm{P}(1)$ is true.

Now, let $\mathrm{P}(n)$ be true for $n=k$

$$\Rightarrow \mathrm{P}(k)=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots\ldots\ldots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{(3k+1)} \quad \ldots\ldots\ldots\text{(i)}$$

For $n=k+1$

R.H.S. = $\frac{k+1}{3k+1}$

$\therefore$ L.H.S. = $\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$

L.H.S. = $\frac{1}{3k+1}\left[k+\frac{1}{3k+4}\right]$

= $\frac{1}{3k+1}\left[\frac{3k^2+4k+1}{3k+4}\right]$

= $\frac{1}{3k+1}\left[\frac{(3k+1)(k+1)}{3k+4}\right]=\frac{k+1}{3k+4}$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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17. $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots\ldots\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$

Ans. Let $P(n) = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

For $n = 1$

$\Rightarrow P(1) = \frac{1}{(2\times1+1)(2\times1+3)} = \frac{1}{3(2\times1+3)}$

$\Rightarrow \frac{1}{15} = \frac{1}{15}$

$\therefore P(1)$ is true.

Now, let $P(n)$ be true for $n = k$

$\Rightarrow P(k) = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$ ....(i)

For $n = k+1$

$\Rightarrow$ R.H.S. = $\frac{k+1}{3(2k+5)}$

$\Rightarrow$ L.H.S. = $\frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)}$

$\Rightarrow$ L.H.S. = $\frac{1}{(2k+3)}\left[\frac{k}{3} + \frac{1}{2k+5}\right]$

= $\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$

= $\frac{1}{(2k+3)}\left[\frac{(k+1)(2k+3)}{3(2k+5)}\right]$

= $\frac{k+1}{3(2k+5)}$

$\therefore P(k+1)$ is true.

Therefore, $P(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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18. $1+2+3+..........+n<\frac{1}{8}(2n+1)^2$

Ans. Let $P(n)=1+2+3+..........+n<\frac{1}{8}(2n+1)^2$

For $n=1$

$\Rightarrow P(1)=1<\frac{1}{8}(2\times 1+1)^2$

$\Rightarrow 1<\frac{9}{8}$

$\therefore P(1)$ is true.

Now, let $P(n)$ be true for $n=k$

$P(k)=1+2+3+..........+k<\frac{1}{8}(2k+1)^2$ ....(i)

For $n=k+1$,

$\Rightarrow P(k+1)=1+2+3+..........+k+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$

Now, adding $(k+1)$ on both sides of eq. (i), we have

$1+2+3+..........+k+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$

$\Rightarrow 1+2+3+..........+k+(k+1)<\frac{1}{8}(4k^2+4k+1+8k+8)$

$\Rightarrow 1+2+3+..........+k+(k+1)<\frac{1}{8}(4k^2+12k+9)$

$\Rightarrow 1+2+3+..........+k+(k+1)<\frac{1}{8}(2k+3)^2$

$\Rightarrow \frac{(k+1)(k+2)}{2}<\frac{1}{8}(2k+3)^2$

$\Rightarrow 4(k+1)(k+2)<4k^2+9+6k$

$\Rightarrow 4(k^2+3k+2)<4k^2+9+6k$

$\Rightarrow 4k^2+12k+8<4k^2+9+6k$

$\Rightarrow$ 8 < 9

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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19. $n(n+1)(n+5)$ **is a multiple of 3.**

Ans. Let $\mathrm{P}(n)=n(n+1)(n+5)$ is a multiple of 3.

For $n=1$,

$\Rightarrow$ $\mathrm{P}(1)=$ 1 (1 + 1) (1 + 5) is a multiple of 3 = 12 is a multiple of 3

$\therefore$ P (1) is true.

Let $\mathrm{P}(n)$ be true for $n=k$,

$\Rightarrow \mathrm{P}(k)=k(k+1)(k+5)$ is a multiple of 3.

$\Rightarrow k(k+1)(k+5)=3\lambda$

$\Rightarrow k^3+6k^2+5k=3\lambda$

$\Rightarrow k^3=3\lambda-6k^2-5k$ ....(i)

For $n=k+1$,

$\Rightarrow \mathrm{P}(k+1)=(k+1)(k+2)(k+6)$ is a multiple of 3

Now, $(k+1)(k+2)(k+6)$

= $k^3+9k^2+20k+12$

= $3\lambda-6k^2-5k+9k^2+20k+12$ [Using (i)]

= $3\lambda+3k^2+15k+12$

= $3(\lambda+k^2+5k+4)$

= $(k+1)(k+2)(k+6)$ is a multiple of 3

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n\in$ N.

20. $10^{2n-1}+1$ **is divisible by 11.**

Ans. Let $\mathrm{P}(n)=10^{2n-1}+1$ is divisible by 11.

For $n=1$, $P(1)=10^{2\times1-1}+1$ is divisible by 11

= 11 is divisible by 11

$\therefore$ P (1) is true.

Let $P(n)$ be true for $n=k$,

$\Rightarrow$ $P(k)=10^{2k-1}+1$ is divisible by 11

$\Rightarrow$ $10^{2k-1}+1=11\lambda$

$\Rightarrow 10^{2k-1}=11\lambda-1$ ....(i)

For $n=k+1$,

$\Rightarrow$ $P(k+1)=10^{2(k+1)-1}+1$ is divisible by 11

$\Rightarrow$ $P(k+1)=10^{2k+1}+1$ is divisible by 11

Now, $10^{2k+1}+1=10^{2k-1}.10^2+1$

= $(11\lambda-1).10^2+1=1100\lambda-100+1$

= $11(100\lambda-9)$

$\therefore$ $10^{2(k+1)-1}+1$ is divisible by 11

$\therefore$ $P(k+1)$ is true.

Therefore, $P(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n\in$ N.

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21. $x^{2n}-y^{2n}$ **is divisible by** $(x+y)$.

Ans. Let $P(n)=x^{2n}-y^{2n}$ is divisible by $(x+y)$

For $n=1$,

$\Rightarrow P(1)=x^{2\times1}-y^{2\times1}$ is divisible by $(x+y)$

$\Rightarrow (x+y)(x-y)$ is divisible by $(x+y)$

$\therefore$ P (1) is true.

Let $P(n)$ be true for $n=k$,

$\Rightarrow P(k)=x^{2k}-y^{2k}$ is divisible by $(x+y)$

$\Rightarrow x^{2k}-y^{2k}=\lambda(x+y)$

$\Rightarrow x^{2k}-y^{2k}=\lambda(x+y)$ ....(i)

For $n=k+1$,

$\Rightarrow P(k+1)=x^{2(k+1)}-y^{2(k+1)}$ is divisible by $(x+y)$

Now,

$\Rightarrow x^{2k+2}-y^{2k+2}=x^{2k+2}-x^{2k}y^2+x^{2k}y^2-y^{2k+2}$

$= x^{2k}.x^2-x^{2k}y^2+x^{2k}y^2-y^{2k}.y^2$

$= x^{2k}(x^2-y^2)+y^2(x^{2k}-y^{2k})$

$= x^{2k}(x^2-y^2)+y^2\lambda(x+y)$ [From eq. (i)]

$= (x+y)\left[x^{2k}(x-y)+\lambda y^2\right]$

$\therefore x^{2(k+1)}-y^{2(k+1)}$ is divisible by $(x+y)$

$\therefore P(k+1)$ is true.

Therefore, $P(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

22. $3^{2n+2}-8n-9$ **is divisible by 8.**

Ans. Let $\mathrm{P}(n)=3^{2n+2}-8n-9$ is divisible by 8.

For $n=1$,

$\Rightarrow \mathrm{P}(1)=3^{2\times1+2}-8\times1-9$ is divisible by 8

$\Rightarrow$ 64 is divisible by 8

$\therefore$ P (1) is true.

Let $\mathrm{P}(n)$ be true for $n=k$,

$\Rightarrow \mathrm{P}(k)=3^{2k+2}-8k-9$ is divisible by 8

$\Rightarrow 3^{2k+2}-8k-9=8\lambda$

$\Rightarrow 3^{2k+2}=8\lambda+8k+9$ ....(i)

For $n=k+1$,

$\Rightarrow \mathrm{P}(k+1)=3^{2(k+1)+2}-8(k+1)-9$ is divisible by 8

$\Rightarrow \mathrm{P}(k+1)=3^{2k+2}.3^2-8k-8-9$ is divisible by 8

Now, $3^{2k+2}.9-8k-17$

= $(8\lambda+8k+9).9-8k-17$ [From eq. (i)]

= $72\lambda+72k+81-8k-17$

= $72\lambda+64k+64$

$\Rightarrow 8(9\lambda+8k+8)$

$\therefore 3^{2(k+1)+2}-8(k+1)-9$ is divisible by 8

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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23. 41^n-14^n **is a multiple of 27.**

Ans. Let $\mathrm{P}(n)=41^n-14^n$ is a multiple of 27.

For $n=1$, $\mathrm{P}(1)=41^1-14^1$ is a multiple of 27

$\Rightarrow$ 27 is a multiple of 27

$\therefore$ P (1) is true.

Let $\mathrm{P}(n)$ be true for $n=k$,

$\Rightarrow \mathrm{P}(k)=41^k-14^k$ is a multiple of 27

$\Rightarrow 41^k-14^k=27\lambda$ .....(i)

For $n=k+1$,

$\Rightarrow \mathrm{P}(k+1)=41^{k+1}-14^{k+1}$ is a multiple of 27

Now, $41^{k+1}-14^{k+1}$

= $41^{k+1}-41^k.14+41^{k}.14-14^{k+1}$

= $41^k(41-14)+14(41^k-14^k)$

= $41^k\times 27+14\times 27\lambda$ [From eq. (i)]

= $27(41^k+14\lambda)$

$\therefore 41^{k+1}-14^{k+1}$ is a multiple of 27

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.

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24. $(2n+7)<(n+3)^2$

Ans. Let $\mathrm{P}(n)=(2n+7)<(n+3)^2$

For $n=1$,

$\mathrm{P}(1)=(2\times1+7)<(1+3)^2$

$\Rightarrow$ 9 < 16

$\therefore$ P (1) is true.

Let $\mathrm{P}(n)$ be true for $n=k$

$\mathrm{P}(k)=(2k+7)<(k+3)^2$ ....(i)

For $n=k+1$

$\mathrm{P}(k+1)=2(k+1)+7<(k+1+3)^2$

$= 2(k+1)+7<(k+4)^2$

Now, adding 2 on both sides in eq. (i),

$(2k+7)+2<(k+3)^2+2$

$\Rightarrow 2(k+1)+7<k^2+9+6k+2$

$\Rightarrow 2(k+1)+7<k^2+6k+11$

$\Rightarrow 2(k+1)+7<k^2+6k+11$

Also $2(k+1)+7<k^2+6k+11<(k+4)^2$

$\therefore 2(k+1)+7<(k+4)^2$

$\therefore \mathrm{P}(k+1)$ is true.

Therefore, $\mathrm{P}(k)$ is true.

Hence by Principle of Mathematical Induction, P(n) is true for all $n \in$ N.