CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 13 Limits and Derivative

Miscellaneous Exercise

1. Find the derivative of following functions from first principle:

(i) $-x$

(ii) $(-x)^{-1}$

(iii) $\sin(x+1)$

(iv) $\cos\left(x-\frac{\pi}{8}\right)$

Ans. (i) Given: $f(x)=-x$ then $f(x+h)=-(x+h)$

$\because \ f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

we get $f'(x)=\lim\limits_{h\to 0}\frac{-(x+h)-(-x)}{h}$

$= \lim\limits_{h\to 0}\frac{-x-h+x}{h} = \lim\limits_{h\to 0}\frac{-h}{h} = -1$

(ii) Given: $f(x)=(-x)^{-1}=\frac{-1}{x}$

then $f(x+h)=\frac{-1}{x+h}$

We have $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore \quad f'(x)=\lim\limits_{h\to 0}\dfrac{\dfrac{-1}{x+h}-\dfrac{-1}{x}}{h}$

$= \lim\limits_{h\to 0}\dfrac{-x+x+h}{hx(x+h)} = \lim\limits_{h\to 0}\dfrac{h}{hx(x+h)} = \dfrac{1}{x^2}$

(iii) Given: $f(x)=\sin(x+1)$

then $f(x+h)=\sin(x+h+1)$

$\because \quad f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

we get $f'(x)=\lim\limits_{h\to 0}\dfrac{\sin(x+h+1)-\sin(x+1)}{h}$

$\left[\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$= \lim\limits_{h\to 0}\dfrac{2\cos\left(\dfrac{2x+h+2}{2}\right)\sin\left(\dfrac{h}{2}\right)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\cos\left(x+1+\dfrac{h}{2}\right)\sin\left(\dfrac{h}{2}\right)}{\dfrac{h}{2}}$

$= \lim\limits_{h\to 0}\cos\left(x+1+\frac{h}{2}\right)\lim\limits_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$ $\left[\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$ $\left[h\to 0\Rightarrow\frac{h}{2}\to 0\right]$

$= \cos(x+1)$

(iv) Given: $f(x)=\cos\left(x-\dfrac{\pi}{8}\right)$

then $f(x+h)=\cos\left(x+h-\frac{\pi}{8}\right)$

$\because\ f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore$ $f'(x)=\lim\limits_{h\to 0}\frac{\cos\left(x+h-\frac{\pi}{8}\right)-\cos\left(x-\frac{\pi}{8}\right)}{h}$

$\left[\cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$=\lim\limits_{h\to 0}\frac{-2\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$=\lim\limits_{h\to 0}\frac{-\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

$=\lim\limits_{h\to 0}-\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\lim\limits_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$ $\left[\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]\left[h\to 0\Rightarrow\frac{h}{2}\to 0\right]$

$=-\sin\left(x-\frac{\pi}{8}\right)$

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

2. $(x+a)$

Ans. Given: $f(x)=(x+a)$

$\therefore\ f'(x)=\frac{d}{dx}(x+a)=\frac{d}{dx}(x)+\frac{d}{dx}(a)=1+0=1$

3. $\left(px+q\right)\left(\frac{r}{x}+s\right)$

Ans. Given: $f\left(x\right)=\left(px+q\right)\left(\frac{r}{x}+s\right)$

$\therefore\ f'(x)=\frac{d}{dx}\left[\left(px+q\right)\left(\frac{r}{x}+s\right)\right]$ [product rule]

$\Rightarrow\ f'(x)=\left(px+q\right)\frac{d}{dx}\left(\frac{r}{x}+s\right)+\left(\frac{r}{x}+s\right)\frac{d}{dx}\left(px+q\right)$ $\frac{d}{dx}\left(x^{-1}\right)=-1x^{-2}$

= $\left(px+q\right)\left(\frac{-r}{x^2}\right)+\left(\frac{r}{x}+s\right)\left(p\right)$

= $\frac{-pr}{x}+\frac{-qr}{x^2}+\frac{pr}{x}+ps$ = $\frac{-qr}{x^2}+ps$

4. $\left(ax+b\right)\left(cx+d\right)^2$

Ans. Given: $f\left(x\right)=\left(ax+b\right)\left(cx+d\right)^2$

$\therefore\ f'(x)=\frac{d}{dx}\left[\left(ax+b\right)\left(cx+d\right)^2\right]$ [product rule]

$\Rightarrow\ f'(x)=\left(ax+b\right)\frac{d}{dx}\left(cx+d\right)^2+\left(cx+d\right)^2\frac{d}{dx}\left(ax+b\right)$

= $(ax+b)\frac{d}{dx}\left(c^2x^2+2cdx+d^2\right)+(cx+d)^2\frac{d}{dx}(ax+b)$

= $(ax+b)\left[c^2\frac{d}{dx}(x^2)+2cd\frac{d}{dx}(x)+\frac{d}{dx}(d^2)\right]+(cx+d)^2\left[a\frac{d}{dx}(x)+\frac{d}{dx}(b)\right]$

= $(ax+b)\left(2c^2x+2cd+0\right)+(cx+d)^2(a\cdot 1+0)$ $\left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$

= $2c\left(ax+b\right)\left(cx+d\right)+a\left(cx+d\right)^2$

5. $\dfrac{ax+b}{cx+d}$

Ans. Given: $f(x)=\dfrac{ax+b}{cx+d}$

$$\therefore\ f'(x)=\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right)$$

$$\Rightarrow\ f'(x)=\frac{(cx+d)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$$ [Quotient rule]

$$\Rightarrow\ f'(x)=\frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}$$

$$=\frac{acx+ad-acx-bc}{(cx+d)^2}=\frac{ad-bc}{(cx+d)^2}$$

6. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Ans. Given: $f(x)=\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{x+1}{x-1}$

$$\therefore\ f'(x)=\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$$

$$\Rightarrow\ f'(x)=\frac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$ [Quotient rule]

= $\dfrac{(x-1)\times 1-(x+1)\times 1}{(x-1)^2}$

= $\dfrac{x-1-x-1}{(x-1)^2}$ = $\dfrac{-2}{(x-1)^2}, x\neq 0,1$

7. $\dfrac{1}{ax^2+bx+c}$

Ans. Given: $f(x)=\dfrac{1}{ax^2+bx+c}$

$\therefore\ f'(x)=\dfrac{d}{dx}\left(\dfrac{1}{ax^2+bx+c}\right)$

$\Rightarrow\ f'(x)=\dfrac{\left(ax^2+bx+c\right)\dfrac{d}{dx}(1)-1.\dfrac{d}{dx}\left(ax^2+bx+c\right)}{\left(ax^2+bx+c\right)^2}$ [Quotient rule]

$\Rightarrow\ f'(x)=\dfrac{\left(ax^2+bx+c\right)(0)-1(2ax+b)}{\left(ax^2+bx+c\right)^2}$

= $\dfrac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$

8. $\dfrac{ax+b}{px^2+qx+r}$

Ans. Given: $f(x)=\dfrac{ax+b}{px^2+qx+r}$

$\therefore\ f'(x)=\dfrac{d}{dx}\left(\dfrac{ax+b}{px^2+qx+r}\right)$

$$\Rightarrow f'(x)=\frac{\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$ [Quotient rule]

$$\Rightarrow f'(x)=\frac{\left(px^2+qx+r\right)(a)-(ax+b)(2px+q)}{\left(px^2+qx+r\right)^2}$$

$$\Rightarrow f'(x)=\frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{-apx^2-2bpx+ar-bq}{\left(px^2+qx+r\right)^2}$$

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9. $\frac{px^2+qx+r}{ax+b}$

Ans. Given: $f(x)=\frac{px^2+qx+r}{ax+b}$

$$\therefore f'(x)=\frac{d}{dx}\left(\frac{px^2+qx+r}{ax+b}\right)$$

$$\Rightarrow f'(x)=\frac{(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)-\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)}{(ax+b)^2}$$ [Quotient rule]

$$\Rightarrow f'(x)=\frac{(ax+b)(2px+q)-\left(px^2+qx+r\right)(a)}{(ax+b)^2}$$

$$\Rightarrow f'(x)=\frac{2apx^2+aqx+2bpx+bq-apx^2-aqx-ar}{(ax+b)^2}$$

$\Rightarrow f'(x) = \dfrac{apx^2 + 2bpx + bq - ar}{(ax+b)^2}$

10. $\dfrac{a}{x^4} - \dfrac{b}{x^2} + \cos x$

Ans. Given: $f(x) = \dfrac{a}{x^4} - \dfrac{b}{x^2} + \cos x$

$= ax^{-4} - bx^{-2} + \cos x$

$\therefore f'(x) = \dfrac{d}{dx}\left(ax^{-4} - bx^{-2} + \cos x\right)$ $\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$

$[\frac{d}{dx}(\cos x) = -\sin x]$

$\Rightarrow f'(x) = a\dfrac{d}{dx}\left(x^{-4}\right) - b\dfrac{d}{dx}\left(x^{-2}\right) + \dfrac{d}{dx}\cos x$

$\Rightarrow f'(x) = -4ax^{-5} + 2bx^{-3} - \sin x$

11. $4\sqrt{x} - 2$

Ans. Given: $f(x) = 4\sqrt{x} - 2$

$\therefore f'(x) = \dfrac{d}{dx}\left(4\sqrt{x} - 2\right)$

$\Rightarrow f'(x) = 4\dfrac{d}{dx}\sqrt{x} - \dfrac{d}{dx}(2)$ $[\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}]$

$= 4 \times \dfrac{1}{2\sqrt{x}} - 0 = \dfrac{2}{\sqrt{x}}$

12. $(ax+b)^n$

Ans. Given: $f(x)=(ax+b)^n$

$\therefore\ f'(x)=\frac{d}{dx}\left[(ax+b)^n\right]$

$\Rightarrow\ f'(x)=n(ax+b)^{n-1}\times\frac{d}{dx}(ax+b)$

$= n(ax+b)^{n-1}\times(a)$ = $na(ax+b)^{n-1}$

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13. $(ax+b)^n(cx+d)^m$

Ans. Given: $(ax+b)^n(cx+d)^m$

$\therefore\ f'(x)=\frac{d}{dx}\left[(ax+b)^n(cx+d)^m\right]$ [product rule]

$\Rightarrow\ f'(x)=(ax+b)^n\frac{d}{dx}(cx+d)^m+(cx+d)^m\frac{d}{dx}(ax+b)^n$

$\left[\frac{d}{dx}(x^n)=nx^{n-1}\right]$

$\Rightarrow\ f'(x)=(ax+b)^n.m(cx+d)^{m-1}\frac{d}{dx}(cx+d)+(cx+d)^m n(ax+b)^{n-1}\frac{d}{dx}(ax+b)$

$\Rightarrow\ f'(x)=(ax+b)^n.m(cx+d)^{m-1}(c)+(cx+d)^m n(ax+b)^{n-1}(a)$

$\Rightarrow\ f'(x)=cm(ax+b)^n(cx+d)^{m-1}+an(cx+d)^m(ax+b)^{n-1}$

$\Rightarrow\ f'(x)=(ax+b)^{n-1}(cx+d)^{m-1}\left[cm(ax+b)+an(cx+d)\right]$

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14. $\sin(x+a)$

Ans. Given: $f(x)=\sin(x+a)$

$\therefore\ f'(x)=\frac{d}{dx}\left[\sin(x+a)\right]$

$\Rightarrow\ f'(x)=\cos(x+a).\frac{d}{dx}(x+a) = \cos(x+a)$

15. $\cos ec\ x\cot x$

Ans. Given: $f(x)=\cos ec\ x\cot x$

$\therefore\ f'(x)=\frac{d}{dx}(\cos ec\ x\cot x)$

$\Rightarrow\ f'(x)=\cos ec\ x\frac{d}{dx}\cot x+\cot x\frac{d}{dx}\cos ec\ x$

$\Rightarrow\ f'(x)=\cos ec\ x\left(-\cos ec^2x\right)+\cot x\left(-\cos ec\ x\cot x\right)$

= $-\cos ec^3x-\cos ecx.\cot^2x$

= $-\cos ecx(\cos ec^2x+\cot^2x)$

16. $\frac{\cos x}{1+\sin x}$

Ans. Given: $f(x)=\frac{\cos x}{1+\sin x}$

$\therefore\ f'(x)=\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right)$

$\Rightarrow\ f'(x)=\frac{(1+\sin x)\frac{d}{dx}\cos x-\cos x\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$ $\quad [\frac{d}{dx}(\cos x)=-\sin x]$

$$= \frac{(1+\sin x)(-\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$$ $[\frac{d}{dx}(\sin x) = \cos x]$

$$\Rightarrow f'(x) = \frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x-(\sin^2 x+\cos^2 x)}{(1+\sin x)^2} = \frac{-\sin x-1}{(1+\sin x)^2}$$ $[\sin^2 x + \cos^2 x = 1]$

$$\Rightarrow f'(x) = \frac{-(1+\sin x)}{(1+\sin x)^2} = \frac{-1}{1+\sin x}$$

17. $\frac{\sin x+\cos x}{\sin x-\cos x}$

Ans. Given: $f(x) = \frac{\sin x+\cos x}{\sin x-\cos x}$

$$\therefore f'(x) = \frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$$

$$\Rightarrow f'(x) = \frac{(\sin x-\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$\Rightarrow f'(x) = \frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$$

$[\frac{d}{dx}(\cos x) = -\sin x]$

$$\Rightarrow f'(x) = \frac{-\sin^2 x-\cos^2 x+2\sin x\cos x-\sin^2 x-\cos^2 x-2\sin x\cos x}{(\sin x-\cos x)^2}$$

$[\frac{d}{dx}(\sin x) = \cos x]$

$\Rightarrow f'(x) = \dfrac{-2(\sin^2 x + \cos^2 x)}{(\sin x - \cos x)^2}$ $[\sin^2 x + \cos^2 x = 1]$

$= \dfrac{-2}{(\sin x - \cos x)^2}$

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18. $\dfrac{\sec x - 1}{\sec x + 1}$

Ans. Given: $f(x) = \dfrac{\sec x - 1}{\sec x + 1}$

$\therefore f'(x) = \dfrac{d}{dx}\left(\dfrac{\sec x - 1}{\sec x + 1}\right)$

$\Rightarrow f'(x) = \dfrac{(\sec x + 1)\frac{d}{dx}(\sec x - 1) - (\sec x - 1)\frac{d}{dx}(\sec x + 1)}{(\sec x + 1)^2}$

$\Rightarrow f'(x) = \dfrac{(\sec x + 1)(\sec x \tan x) - (\sec x - 1)(\sec x \tan x)}{(\sec x + 1)^2}$

$[\frac{d}{dx}(\sec x) = \sec x \tan x]$

$\Rightarrow f'(x) = \dfrac{\sec^2 x \tan x + \sec x \tan x - \sec^2 x \tan x + \sec x \tan x}{(\sec x + 1)^2}$

$\Rightarrow f'(x) = \dfrac{2\sec x \tan x}{(\sec x + 1)^2}$

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19. $\sin^n x$

Ans. Given: $f(x) = \sin^n x$

$\therefore\ f'(x) = \frac{d}{dx}(\sin^n x)$

$\Rightarrow\ f'(x) = n\sin^{n-1} x.\frac{d}{dx}(\sin x)$ $[\frac{d}{dx}(\sin x) = \cos x]$

$= n\sin^{n-1} x\cos x$

20. $\frac{a+b\sin x}{c+d\cos x}$

Ans. Given: $f(x) = \frac{a+b\sin x}{c+d\cos x}$

$\therefore\ f'(x) = \frac{d}{dx}\left(\frac{a+b\sin x}{c+d\cos x}\right)$

$\Rightarrow\ f'(x) = \frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x)-(a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$

$[\frac{d}{dx}(\sin x) = \cos x]$

$= \frac{(c+d\cos x)(b\cos x)-(a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$ $[\frac{d}{dx}(\cos x) = -\sin x]$

$\Rightarrow\ f'(x) = \frac{bc\cos x+bd\cos^2 x+ad\sin x+bd\sin^2 x}{(c+d\cos x)^2}$

$= \frac{bc\cos x+ad\sin x+bd(\cos^2 x+\sin^2 x)}{(c+d\cos x)^2}$ $[\sin^2 x+\cos^2 x = 1]$

$= \frac{bc\cos x+ad\sin x+bd}{(c+d\cos x)^2}$

21. $\frac{\sin(x+a)}{\cos x}$

Ans. Given: $f(x) = \frac{\sin(x+a)}{\cos x}$

$$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{\sin(x+a)}{\cos x}\right]$$

$$\Rightarrow\ f'(x) = \frac{\cos x\frac{d}{dx}[\sin(x+a)] - \sin(x+a)\frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos x.\cos(x+a).\frac{d}{dx}(x+a) - \sin(x+a).\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$\Rightarrow\ f'(x) = \frac{\cos x\cos(x+a) - \sin(x+a)(-\sin x)}{\cos^2 x}$$

$$\Rightarrow\ f'(x) = \frac{\cos x\cos(x+a) + \sin x\sin(x+a)}{\cos^2 x}$$

$[\cos(A-B) = \cos A.\cos B + \sin A.\sin B]$

$$= \frac{\cos(x+a-x)}{\cos^2 x} = \frac{\cos a}{\cos^2 x}$$

22. $x^4(5\sin x - 3\cos x)$

Ans. Given: $f(x) = x^4(5\sin x - 3\cos x)$

$$\therefore\ f'(x) = \frac{d}{dx}\left[x^4(5\sin x - 3\cos x)\right]$$ [product rule]

$$\Rightarrow\ f'(x) = x^4\frac{d}{dx}(5\sin x - 3\cos x) + (5\sin x - 3\cos x)\frac{d}{dx}(x^4)$$ $[\frac{d}{dx}(\sin x) = \cos x]$

$$\Rightarrow\ f'(x) = x^4(5\cos x + 3\sin x) + (5\sin x - 3\cos x)(4x^3)$$ $[\frac{d}{dx}(\cos x) = -\sin x]$

$\Rightarrow f'(x) = x^3(5x\cos x + 3x\sin x + 20\sin x - 12\cos x)$ $\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$

23. $(x^2+1)\cos x$

Ans. Given: $f(x) = (x^2+1)\cos x$

$\therefore f'(x) = \frac{d}{dx}\left[(x^2+1)\cos x\right]$ [product rule]

$\Rightarrow f'(x) = (x^2+1)\frac{d}{dx}\cos x + \cos x\frac{d}{dx}(x^2+1)$ $[\frac{d}{dx}(\cos x) = -\sin x]$

$= (x^2+1)(-\sin x) + \cos x(2x)$ $\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$

$\Rightarrow f'(x) = -x^2\sin x - \sin x + 2x\cos x$

24. $(ax^2+\sin x)(p+q\cos x)$

Ans. Given: $f(x) = (ax^2+\sin x)(p+q\cos x)$

$\therefore f'(x) = \frac{d}{dx}\left[(ax^2+\sin x)(p+q\cos x)\right]$ [product rule]

$\Rightarrow f'(x) = (ax^2+\sin x)\frac{d}{dx}(p+q\cos x) + (p+q\cos x)\frac{d}{dx}(ax^2+\sin x)$
$[\frac{d}{dx}(\sin x) = \cos x]$

$\Rightarrow f'(x) = (ax^2+\sin x)(-q\sin x) + (p+q\cos x)(2ax+\cos x)$
$[\frac{d}{dx}(\cos x) = -\sin x]$

$\Rightarrow f'(x) = -q\sin x(ax^2+\sin x) + (p+q\cos x)(2ax+\cos x)$

25. $(x+\cos x)(x-\tan x)$

Ans. Given: $f(x)=(x+\cos x)(x-\tan x)$

$\therefore\ f'(x)=\frac{d}{dx}\left[(x+\cos x)(x-\tan x)\right]$ [product rule]

$\Rightarrow\quad f'(x)=(x+\cos x)\frac{d}{dx}(x-\tan x)+(x-\tan x)\frac{d}{dx}(x+\cos x)$

$[\frac{d}{dx}(\tan x)=\sec^2 x]$

$\Rightarrow\ f'(x)=(x+\cos x)(1-\sec^2 x)+(x-\tan x)(1-\sin x)$

$[\frac{d}{dx}(\cos x)=-\sin x]$

$\Rightarrow\ f'(x)=-\tan^2 x(x+\cos x)+(x-\tan x)(1-\sin x)$ $[\sec^2\theta-\tan^2\theta=1]$

26. $\frac{4x+5\sin x}{3x+7\cos x}$

Ans. Given: $f(x)=\frac{4x+5\sin x}{3x+7\cos x}$

$\therefore\ f'(x)=\frac{d}{dx}\left(\frac{4x+5\sin x}{3x+7\cos x}\right)$

$\Rightarrow\ f'(x)=\frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$ [Quotient rule]

$\Rightarrow\ f'(x)=\frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$

$\Rightarrow$

$f'(x)=\frac{12x+15x\cos x+28\cos x+35\cos^2 x-12x+28x\sin x+35\sin^2 x-15\sin x}{(3x+7\cos x)^2}$

$$\Rightarrow \qquad f'(x) = \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35\left(\cos^2 x + \sin^2 x\right)}{\left(3x + 7\cos x\right)^2}$$

$[\sin^2 x + \cos^2 x = 1]$

$$\Rightarrow f'(x) = \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{\left(3x + 7\cos x\right)^2}$$

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27. $\dfrac{x^2\cos\left(\dfrac{\pi}{4}\right)}{\sin x}$

Ans. Given: $f(x) = \dfrac{x^2\cos\left(\dfrac{\pi}{4}\right)}{\sin x}$

$$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}\right] \qquad \text{[Quotient rule]}$$

$$\Rightarrow f'(x) = \frac{\sin x\frac{d}{dx}\left[x^2\cos\left(\frac{\pi}{4}\right)\right] - x^2\cos\left(\frac{\pi}{4}\right)\frac{d}{dx}(\sin x)}{\sin^2 x} \qquad \left[\frac{d}{dx}(\sin x) = \cos x\right]$$

$$\Rightarrow f'(x) = \frac{\sin x\left[2x\cos\left(\frac{\pi}{4}\right)\right] - x^2\cos\left(\frac{\pi}{4}\right)(\cos x)}{\sin^2 x} \qquad \left[\frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$$

$$\Rightarrow f'(x) = \frac{2x\sin x\cos\left(\frac{\pi}{4}\right) - x^2\cos x\cos\left(\frac{\pi}{4}\right)}{\sin^2 x}$$

= $f'(x) = \dfrac{x\cos\left(\frac{\pi}{4}\right)[2\sin x - x\cos x]}{\sin^2 x}$

28. $\dfrac{x}{1+\tan x}$

Ans. Given: $f(x) = \dfrac{x}{1+\tan x}$

$\therefore\ f'(x) = \dfrac{d}{dx}\left(\dfrac{x}{1+\tan x}\right)$

$\Rightarrow\ f'(x) = \dfrac{(1+\tan x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$ $[\frac{d}{dx}(\tan x) = \sec^2 x]$

= $\dfrac{(1+\tan x)(1) - x(\sec^2 x)}{(1+\tan x)^2}$

$\Rightarrow\ f'(x) = \dfrac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

29. $(x+\sec x)(x-\tan x)$

Ans. Given: $f(x) = (x+\sec x)(x-\tan x)$

$\therefore\ f'(x) = \dfrac{d}{dx}[(x+\sec x)(x-\tan x)]$ [product rule]

$\Rightarrow\ f'(x) = (x+\sec x)\dfrac{d}{dx}(x-\tan x) + (x-\tan x)\dfrac{d}{dx}(x+\sec x)$

$[\frac{d}{dx}(\tan x) = \sec^2 x]$

= $(x+\sec x)(1-\sec^2 x) + (x-\tan x)(1+\sec x\tan x)$

$[\frac{d}{dx}(\sec x) = \sec x\tan x]$

30. $\frac{x}{\sin^n x}$

Ans. Given: $f(x) = \frac{x}{\sin^n x}$

$\therefore\ f'(x) = \frac{d}{dx}\left(\frac{x}{\sin^n x}\right)$

$\Rightarrow\ f'(x) = \frac{\sin^n x \frac{d}{dx}(x) - x\frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$

$= \frac{\sin^n x.1 - x.n.\sin^{n-1} x.\frac{d}{dx}(\sin x)}{(\sin^n x)^2}$ $[\frac{d}{dx}(\sin x) = \cos x]$

$\Rightarrow\ f'(x) = \frac{\sin^n x - nx\sin^{n-1} x\cos x}{(\sin^n x)^2}$

$= \frac{\sin^{n-1} x(\sin x - nx\cos x) - nx\sin^{n-1} x\cos x}{\sin^{2n} x}$

$= \frac{\sin^{n-1} x(\sin x - nx\cos x)}{\sin^{2n} x}$ = $\sin^{n-1-2n} x(\sin x - nx\cos x)$ = $\sin^{-(n+1)} x(\sin x - nx\cos x)$

$= \frac{\sin x - nx\cos x}{\sin^{n+1} x}$