CBSE Class–11 Mathematics
NCERT Solutions
Chapter - 3 Trigonometric Functions
Miscellaneous Exercise

Prove that:

1. $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Ans. L.H.S. = $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $cos\left(\frac{9\pi}{13}+\frac{\pi}{13}\right)+cos\left(\frac{9\pi}{13}-\frac{\pi}{13}\right)+cos\left(\frac{3\pi}{13}\right)+cos\left(\frac{5\pi}{13}\right)$

= $\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$ = 0 = R.H.S.

2. $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

Ans. L.H.S. = $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x$

= $\left[2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]\sin x+\left[-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]\cos x$

= [2sin2xcosx]sinx + [-2sin2xsinx]cosx

= 2sin2x.sinx.cosx - 2sin2x.sinx.cosx = 0 = RHS

3. $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Ans. L.H.S. = $(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$=\left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2+\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$

$=4\cos^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right)+4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$

$=4\cos^2\left(\frac{x+y}{2}\right)\left[\cos^2\left(\frac{x-y}{2}\right)+\sin^2\left(\frac{x-y}{2}\right)\right]$

$=4\cos^2\left(\frac{x+y}{2}\right)$ = R.H.S.

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4. $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\frac{x-y}{2}$

Ans. L.H.S. = $(\cos x-\cos y)^2+(\sin x-\sin y)^2$

$=\left[-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2+\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$

$=4\sin^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)+4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$

$=4\sin^2\left(\frac{x-y}{2}\right)\left[\sin^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x+y}{2}\right)\right]$

$=4\sin^2\left(\frac{x-y}{2}\right)$ = R.H.S.

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5. $\sin x+\sin 3x+\sin 5x+\sin 7x=4\cos x\cos 2x\sin 4x$

Ans. L.H.S. = $\sin x+\sin 3x+\sin 5x+\sin 7x$

= $(\sin 7x+\sin x)+(\sin 5x+\sin 3x)$

= $\left[2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)\right]+\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $[2\sin 4x\cos 3x]+[2\sin 4x\cos x]$

= $2\sin 4x[\cos 3x+\cos x]$

= $2\sin 4x\left[2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]$

= $2\sin 4x[2\cos 2x\cos x]$

= $4\cos x\cos 2x\sin 4x$ = R.H.S.

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6. $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

Ans. L.H.S. = $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

= $\dfrac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}$

= $\dfrac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$

= $\dfrac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)}$

= $\tan 6x$ = R.H.S.

7. $\sin 3x+\sin 2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Ans. L.H.S. = $\sin 3x+\sin 2x-\sin x$

= $(\sin 3x-\sin x)+\sin 2x$

= $\left[2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]+2\sin x\cos x$

= $2\cos 2x\sin x+2\sin x\cos x$

= $2\sin x[\cos 2x+\cos x]$

= $2\sin x\left[2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)\right]$

= $2\sin x\left[2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)\right]$

= $4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$ = R.H.S.

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following:

8. $\tan x=\frac{-4}{3}$, x **in quadrant II.**

Ans. Given: $\tan x=\frac{-4}{3}$, x in quadrant II.

$\therefore \sec^2 x=1+\tan^2 x$

$\Rightarrow \sec^2 x=1+\left(\frac{-4}{3}\right)^2$ = $1+\frac{16}{9}=\frac{25}{9}$

$\Rightarrow \sec x = \pm\frac{5}{3}$ $\cos x = \pm\frac{3}{5}$ = $\frac{-3}{5}$

[x lies in II quadrant]

Also $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \frac{x}{2}$ lies in first quadrant.

$\therefore \sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ are positive.

Now, $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2$

9. $\cos x = \frac{-1}{3}, x$ **in quadrant III.**

Ans. Given: $\cos x = \frac{-1}{3}, x$ in quadrant III.

Now, $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\pi}{2} < \frac{3\pi}{4}$

$\therefore \frac{x}{2}$ lies in second quadrant.

$\therefore \sin\frac{x}{2}$ are positive and $\cos\frac{x}{2}, \tan\frac{x}{2}$ are negative.

Now, $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{1}{3}}{2}} = \sqrt{\frac{2}{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{\sqrt{2}}{\sqrt{3}}}{-\frac{1}{\sqrt{3}}} = -\sqrt{2}$

10. $\sin x = \frac{1}{4}, x$ **in quadrant II.**

Ans. Given: $\sin x = \frac{1}{4}, x$ in quadrant II.

$\therefore \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \cos x = \pm\frac{\sqrt{15}}{4}$ $\cos x = -\frac{\sqrt{15}}{4} = \frac{-3}{5}$

[x lies in II quadrant]

Also $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \frac{x}{2}$ lies in first quadrant.

$\therefore \sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ are positive.

Now, $$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4-\sqrt{15}}{8}} = \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}} = \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}} \times \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4+\sqrt{15}}} = \frac{4+\sqrt{15}}{\sqrt{16-15}} = 4+\sqrt{15}$$