CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 1 Sets

Miscellaneous Exercise

1. Decide among the following sets, which sets are subsets of each another:

A = {$x: x\in$ R and x satisfies $x^2-8x+12=0$}, B = {2, 4, 6}, C = {2, 4, ,6 , 8…….}, D = {6}

Ans. Given: A = {$x: x\in$ R and x satisfies $x^2-8x+12=0$}

= {$x: x\in$ R and x satisfies $(x-6)(x-2)=0$} = {2, 6}

B = {2, 4, 6}, C = {2, 4, 6, 8…….}, D = {6}

$\therefore$ A $\subset$ B, A $\subset$ C, B $\subset$ C, D $\subset$ A, D $\subset$ B and D $\subset$ C

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x\in$ A and A $\in$ B then $x\in$ B

(ii) If A $\subset$ B and B $\in$ C then A $\in$ C

(iii) If A $\subset$ B and B $\subset$ C then A $\subset$ C

(iv) If A $\not\subset$ B and B $\not\subset$ C then A $\not\subset$ C

(v) If $x\in$ A and A $\not\subset$ B then $x\in$ B

(vi) If A $\subset$ B and $x\notin$ C then $x\notin$ A

Ans. (i) The statement is false.

Let A = {1} and B = {{1}, 2}

Then 1 $\in$ A and A $\in$ B but 1 $\notin$ B

(ii) The statement is false.

Let A = {1} and B = {1, 2}, C {{1, 2}, 3}

Then $A \subset B$ and $B \in C$ but $A \notin C$

(iii) The statement is true.

Let $x \in A \Rightarrow x \in B$ ($\because A \subset B$)

$\Rightarrow x \in C$ ($\because B \subset C$)

$\therefore x \in A \Rightarrow x \in C \therefore A \subset C$

(iv) The statement is false.

Let A = {1, 2}, B = {2, 3}, C {1, 2, 5}

Then $A \not\subset B$ and $B \not\subset C$ but $A \subset C$

(v) The statement is false.

Let A = {1, 2} and B = {2, 3, 4, 5}

Then $1 \in A$ and $A \not\subset B$ but $1 \notin B$

(vi) The statement is true.

Let $x \in A \Rightarrow x \in B$ ($\because A \subset B$)

Now, $x \notin B \Rightarrow x \notin A$

3. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then show that B = C.

Ans. Since, $A = A \cap (A \cup B)$ and $A = A \cup (A \cap B)$

Now, it is given that $A \cup B = A \cup C$ and $A \cap B = A \cap C$

$\therefore B = B \cup (B \cap A)$

$= B \cup (A \cap B)$

$= B \cup (A \cap C)$

$= (B \cup A) \cap (B \cup C)$

$= (A \cup B) \cap (B \cup C)$

$= (A \cup C) \cap (B \cup C)$

$= (C \cup A) \cap (C \cup B)$

$= C \cup (A \cap B)$

$= C \cup (A \cap C)$

$= C \cup (C \cap A) = C$

$\therefore B = C$

4. Show that the following four conditions are equivalent:

(i) $A \subset B$

(ii) $A - B = \phi$

(iii) $A \cup B = B$

(iv) $A \cap B = A$

Ans. (i) $\Rightarrow$ (ii) $A - B = \{x : x \in A \text{ and } x \notin B\}$

Since $A \subset B$, Therefore $A - B = \phi$

(ii) $\Rightarrow$ (iii) $A - B = \phi$

$\Rightarrow A \subset B \Rightarrow A \cup B = B$

(iii) $\Rightarrow$ (iv) $A \cup B = B$

$\Rightarrow A \subset B \Rightarrow A \cap B = A$

(iv) $\Rightarrow$ (i) $A \cap B = A \Rightarrow A \subset B$

Therefore, (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (iv)

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Ans. Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$ [$\because A \subset B$]

$\therefore x \in C - A$

$\Rightarrow C - B \subset C - A$

6. Assume that P(A) = P(B), show that A = B

Ans. Let $x \in A$

$\Rightarrow \{x\} \in P(A)$

$\Rightarrow \{x\} \in P(B)$

$\Rightarrow x \in B$

$\therefore A \subset B$.........(i)

Let $x \in B$

$\Rightarrow \{x\} \in P(B)$

$\Rightarrow \{x\} \in P(A)$

$\Rightarrow x \in A$

$\therefore B \subset A$.........(ii)

From eq. (i) and (ii),

we have A = B

7. Is it true that for any set A and B, P(A) $\cup$ P(B) = P(A $\cup$ B)? Justify your answer.

Ans. No, it is not true.

Taking A = {1, 2} and B = {2, 3}

Then A $\cup$ B = {1, 2, 3}

P (A) = $\{\phi, \{1\}, \{2\}, \{1, 2\}\}$ and

P (B) = $\{\phi, \{2\}, \{3\}, \{2, 3\}\}$

$\therefore$ P (A) $\cup$ P (B) = $\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}\}$ ....(i)

And P (A $\cup$ B) = $\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$ ....(ii)

From eq. (i) and (ii), P (A) $\cup$ P (B) $\neq$ P (A $\cup$ B)

8. Show that for any sets A and B, A = (A $\cap$ B) $\cup$ (A – B) and A $\cup$ (B – A) = (A $\cup$ B)

Ans. Since (A $\cap$ B) $\cup$ (A – B) = (A $\cap$ B) $\cup$ (A $\cap$ B')

$\Rightarrow$ (A $\cap$ B) $\cup$ (A – B) = A $\cap$ B $\cup$ B' = A $\cap$ U = A

Therefore, A = (A $\cap$ B) $\cup$ (A – B)

Also A $\cup$ (B – A) = A $\cup$ (B $\cap$ A') = (A $\cup$ B) $\cap$ (A $\cup$ A') = (A $\cup$ B) $\cap$ U = A $\cup$ B

Therefore, A $\cup$ (B – A) = A $\cup$ B

9. Using properties of sets, show that:

(i) A $\cup$ (A $\cap$ B) = A

(ii) A $\cap$ (A $\cup$ B) = A

Ans. (i) If $A \subset B$, then $A \cap B = B$

Also $A \cap B \subset A$

$\therefore A \cup (A \cap B) = A$

(ii) If $A \subset B$, then $A \cap B = A$

Also $A \subset A \cup B$

$\therefore A \cap (A \cup B) = A$

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10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Ans. Let A = {1, 2, 3, 4}, B = {2, 3, 4, 5, 6} and C = {2, 3, 4, 9, 10}

$\therefore A \cap B = \{2, 3, 4\}$

And $A \cap C = \{2, 3, 4\}$

Therefore, we have $A \cap B = A \cap C$

But $B \neq C$

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11. Let A and B sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X. Show that A = B.

[Hint: $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law]

Ans. Given: $A \cup X = B \cup X$ for some set X.

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)$

$\Rightarrow A = (A \cap B) \cup \phi$

$\Rightarrow A = (A \cap B)$

$\Rightarrow A \subset B$..........(i)

Also $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B$

$\Rightarrow (B \cap A) \cup \phi = B$

$\Rightarrow B \cap A = B \Rightarrow B \subset A$ ....(ii)

From eq. (i) and (ii), we have A = B

12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.

Ans. Let A = {1, 2}, B = {1, 4} and C = {2, 4}

$\therefore A \cap B = \{1\} \neq \phi$ $B \cap C = \{4\} \neq \phi$

And $A \cap C = \{2\} \neq \phi$

But $A \cap B \cap C = \phi$

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee.

Ans. Let T be the set of students who like tea and C be the set of students who like coffee.

$\therefore n(C) = 225, n(T) = 150$ and $n(C \cap T) = 100$

$\because n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore n(C \cup T) = 150 + 225 - 100 = 275$

$\therefore$ Number of students taking either tea or coffee = 275

$\therefore$ Number of students taking neither tea nor coffee = 600 – 275 = 325

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Ans. Let H be the set of students who know Hindi and E be the set of students who know English.

$\therefore n(\mathrm{H}) = 100, n(\mathrm{E}) = 50$ and $n(\mathrm{H} \cap \mathrm{E}) = 25$

$\because n(\mathrm{H} \cup \mathrm{E}) = n(\mathrm{H}) + n(\mathrm{E}) - n(\mathrm{H} \cap \mathrm{E})$

$\therefore n(\mathrm{H} \cup \mathrm{E}) = 100 + 50 - 25 = 125$

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15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 real all three newspapers. Find:

(i) the number of people who read at least one of the newspaper.

(ii) the number of people who read exactly one newspaper.

Ans. Given: $n(\mathrm{U}) = a + b + c + d + e + f + g + h = 60$ .....(i)

$n(\mathrm{H}) = a + b + c + d = 25$ .....(ii)

$n(\mathrm{T}) = b + c + f + g = 26$ .....(iii)

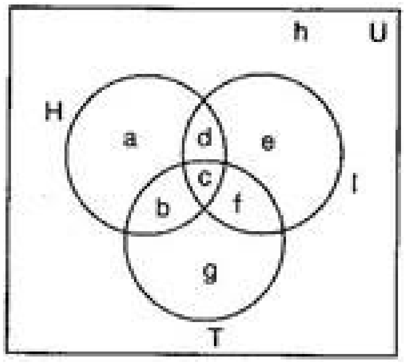
$n(\mathrm{I}) = c + d + e + f = 26$ .....(iv)

$n(\mathrm{H} \cap \mathrm{I}) = c + d = 9$ .....(v)

$n(\mathrm{H} \cap \mathrm{T}) = b + c = 11$ .....(vi)

$n(\mathrm{T} \cap \mathrm{I}) = c + f = 8$ .....(vii)

$n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I}) = c = 3$ .....(viii)



Putting value of c in eq. (vii), $3+f=8 \Rightarrow f=5$

Putting value of c in eq. (vi), $3+b=11 \Rightarrow b=8$

Putting value of c in eq. (v), $3+d=9 \Rightarrow d=6$

Putting value of c, d, f in eq. (iv), $3+6+e+5=26 \Rightarrow e=12$

Putting value of c, d, f in eq. (iii), $8+3+5+g=26 \Rightarrow g=10$

Putting value of b, c, d in eq. (ii), $a+8+3+6=25 \Rightarrow a=8$

(i) Number of people who read at least one of the three newspapers

= $a+b+c+d+e+f+g+h$

= 8 + 8 + 3 + 6 + 12 + 5 + 10 = 52

(ii) Number of people who read exactly one newspaper

= $a+e+g=8+12+10=30$

16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B,12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Ans. Given: $n(\mathrm{A})=a+b+c+d=21$ .....(i)

$n(\mathrm{B})=b+c+f+g=26$ .....(ii)

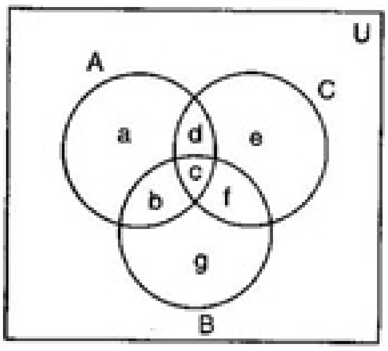
$n(C) = c + d + e + f = 29$ .....(iii)

$n(A \cap B) = b + c = 14$ .....(iv)

$n(C \cap A) = c + d = 12$ .....(v)

$n(B \cap C) = c + f = 14$ .....(vi)

$n(A \cap B \cap C) = c = 8$ .....(vii)



Putting value of c in eq. (iv), $b + 8 = 14 \Rightarrow b = 6$

Putting value of c in eq. (v), $8 + d = 12 \Rightarrow d = 4$

Putting value of c in eq. (vi), $8 + f = 14 \Rightarrow f = 6$

Putting value of c, d, f in eq. (iii), $8 + 4 + e + 6 = 29 \Rightarrow e = 11$

Number of people who like product C only = 11.