CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 9 Sequences and Series

Exercise 9.2

1. Find the sum of odd integers from 1 to 2001.

Ans. Odd integers from 1 to 2001 are 1, 3, 5, 7, …….., 2001.

Here, $a=1, d=3-1=2$ and $a_n=2001$

$\therefore\ a_n=a+(n-1)d$

$\Rightarrow\ 2001=1+(n-1)\times 2$

$\Rightarrow\ 2001-1=(n-1)\times 2$

$\Rightarrow\ \frac{2000}{2}=(n-1)$

$\Rightarrow n=1000+1=1001$

Now, $S_n=\frac{n}{2}(a+l)$

$\Rightarrow S_{1001}=\frac{1001}{2}(1+2001)$

$\Rightarrow S_{1001}=\frac{1001}{2}\times 2002$ = 1002001

2. Find the sum of all natural numbers lying between 100 and 1000 which are multiples of 5.

Ans. According to question, series is 105, 110, 115, 120, ………, 995

Here $a = 105$, $d = 110 - 105 = 5$ and $a_n = 995$

$\therefore\ a_n = a + (n-1)d$

$\Rightarrow\ 995 = 105 + (n-1) \times 5$

$\Rightarrow\ 995 - 105 = (n-1) \times 5$

$\Rightarrow\ \frac{890}{5} = (n-1)$

$\Rightarrow n = 178 + 1 = 179$

Now, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{179} = \frac{179}{2}(105 + 995)$

$\Rightarrow S_{179} = \frac{179}{2} \times 1100$ = 98450

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3. In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20$^{\text{th}}$ term is -112.

Ans. According to question, a=2 and $S_5 = \frac{1}{4}[S_{10} - S_5]$

$\Rightarrow 4S_5 = S_{10} - S_5$

$\Rightarrow 5S_5 = S_{10}$

$\Rightarrow\ 5\left[\frac{5}{2}\{2 \times 2 + (5-1)d\}\right] = \frac{10}{2}[2 \times 2 + (10-1)d]$ since $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow\ \frac{25}{2}[4 + 4d] = 5[4 + 9d]$

$\Rightarrow\ 25[4+4d]=10[4+9d]$

$\Rightarrow 100+100d=40+90d$

$\Rightarrow 10d=-60$

$\Rightarrow d=-6$

Now, $a_n=a+(n-1)d$

$\Rightarrow\ a_{20}=2+(20-1)\times(-6)$

$\Rightarrow\ a_{20}=2-114=-112$

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4. How many terms of the A.P., $-6, \frac{-11}{2}, -5, \ldots\ldots$ are needed to give the sum -25 ?

Ans. Here, $a=-6, d=\frac{-11}{2}-(-6)=\frac{-11}{2}+6=\frac{1}{2}$

$\therefore\ S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow -25=\frac{n}{2}\left[2\times(-6)+(n-1)\times\frac{1}{2}\right]$

$\Rightarrow -50=n\left[-12+\frac{n-1}{2}\right]$

$\Rightarrow -50=n\left[\frac{-24+n-1}{2}\right]$

$\Rightarrow -100=n^2-25n$

$\Rightarrow n^2-25n+100=0$

$\Rightarrow (n-20)(n-5)=0$

$\Rightarrow n = 20$ or $n = 5$

5. In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.

Ans. Let a be the first term and d be the common difference of given A.P.

And $a_p = \frac{1}{q}$ and $a_q = \frac{1}{p}$

$\therefore\ a+(p-1)d = \frac{1}{q}$ and $a+(q-1)d = \frac{1}{p}$

$\Rightarrow\ a+pd-d = \frac{1}{q}$ .....(i) and $a+qd-d = \frac{1}{p}$ .....(ii)

Subtracting eq. (ii) from eq. (i), we get

$$a+pd-d-(a+qd-d) = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow a+pd-d-a-qd+d = \frac{p-q}{pq}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{p-q}{pq} \times \frac{1}{p-q} = \frac{1}{pq}$$

Putting value of d in eq. (i), we get

$$a + p\frac{1}{pq} - d = \frac{1}{q}$$

$$\Rightarrow a + \frac{1}{q} - d = \frac{1}{q}$$

$\Rightarrow a = \frac{1}{q} + d - \frac{1}{q} = d = \frac{1}{pq}$

Now, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$\Rightarrow S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1) \times \frac{1}{pq}\right]$$

$$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{2}{pq} + \frac{pq-1}{pq}\right]$$

$$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{2+pq-1}{pq}\right]$$

$$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{1+pq}{pq}\right] = \frac{pq+1}{2}$$

$$\Rightarrow S_{pq} = \frac{1}{2}(pq+1)$$

6. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116, find the last term.

Ans. Here $a = 25, d = 22 - 25 = -3$ and $S_n = 116$

We have $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$\Rightarrow 116 = \frac{n}{2}\left[2 \times 25 + (n-1) \times (-3)\right]$$

$$\Rightarrow 232 = n\left[50 - 3n + 3\right]$$

$$\Rightarrow 232 = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n + 232 = 0$$

$$\Rightarrow n=\frac{-(-53)\pm\sqrt{(-53)^2-4\times3\times232}}{2\times3}$$

$$\Rightarrow n=\frac{53\pm\sqrt{2809-2784}}{6}$$

$$\Rightarrow n=\frac{53\pm\sqrt{25}}{6}=\frac{53\pm5}{6}$$

$$\Rightarrow n=\frac{53+5}{6} \text{ or } n=\frac{53-5}{6}$$

$$\Rightarrow n=\frac{58}{6} \text{ or } n=\frac{48}{6}=8$$

But $n=\frac{58}{6}$ is not possible as $n\in N$. Therefore, $n=8$

Now, $a_n=a+(n-1)d$

$$\Rightarrow a_8=25+(8-1)\times(-3)$$

$$\Rightarrow a_8=25-21=4$$

7. Find the sum of n terms of an A.P. whose k^{th} term is $5k+1$.

Ans. Given: $a_k=5k+1$

Putting $k=1$ and k=n , we get

$a=5\times1+1=6$ and $a_n=5n+1$

$$\therefore\ S_n=\frac{n}{2}(a+l)$$

$$\Rightarrow\ S_n=\frac{n}{2}(6+5n+1)=\frac{n}{2}(5n+7)$$

8. If the sum of n terms of an A.P. is $(pn+qn^2)$, where p and q are constants, find the common difference.

Ans. Given: $S_n = pn + qn^2$

Put n=1 we get, $S_1 = p + q \Rightarrow a = p + q$....................(i)

Now $S_n = pn + qn^2$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = pn + qn^2$

$\Rightarrow \frac{n}{2}[2(p+q) + (n-1)d] = pn + qn^2$ (using .(i))

$\Rightarrow 2n(p+q) + n(n-1)d = 2pn + 2qn^2$

$\Rightarrow 2nq + n(n-1)d = 2qn^2$

$\Rightarrow n(n-1)d = 2qn(n-1)$

$\Rightarrow d = 2q$

9. The sums of n terms of two arithmetic progressions are on the ratio $5n+4:9n+6$. Find the ratio of their 18th terms.

Ans. Let a_1, a_2 and d_1, d_2 be the first terms and common differences of two A.P's respectively.

$$\therefore \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{5n+4}{9n+6}$$

Now, to get 18th term, $\frac{n-1}{2} = 17$

$$\Rightarrow n = 35$$

$$\therefore \frac{a_1 + 17d_1}{a_2 + 17d_2} = \frac{5\times 35+4}{9\times 35+6}$$

$$\Rightarrow \frac{a_1 + 17d_1}{a_2 + 17d_2} = \frac{179}{321}$$

Therefore, the ratio of 18th terms of two A.P.'s is 179: 321.

10. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.

Ans. Let a be the first term and d be the common difference of given A.P.

$\therefore$ $S_p = \frac{p}{2}\left[2a + (p-1)d\right]$ and $S_q = \frac{q}{2}\left[2a + (q-1)d\right]$

According to question, $S_p = S_q$

$$\Rightarrow \frac{p}{2}\left[2a + (p-1)d\right] = \frac{q}{2}\left[2a + (q-1)d\right]$$

$$\Rightarrow 2ap + p^2d - pd = 2aq + q^2d - qd$$

$$\Rightarrow 2ap - 2aq = q^2d - p^2d + pd - qd$$

$$\Rightarrow 2a(p-q) = \left[-(p^2 - q^2)d + (p-q)d\right]$$

$\Rightarrow 2a(p-q) = [-(p-q)(p+q)d + (p-q)d]$

$\Rightarrow 2a(p-q) = (p-q)[1-p-q]d$

$\Rightarrow a = \dfrac{(1-p-q)d}{2}$

Now $S_{p+q} = \dfrac{p+q}{2}\left[\dfrac{2(1-p-q)d}{2} + (p+q-1)d\right]$

$= \dfrac{p+q}{2}[d - pd - qd + pd + qd - d]$

$\Rightarrow S_{p+q} = \dfrac{p+q}{2} \times 0 = 0$

11. Sum of the first p, q and r terms of an A.P. are a, b and c respectively. Prove that

Ans. Let A be the first term and d be the common difference of given A.P.

$\therefore\ S_p = \dfrac{p}{2}[2A + (p-1)d] = a$

$\Rightarrow A + \dfrac{p-1}{2}d = \dfrac{a}{p}$ ....(i)

$S_q = \dfrac{q}{2}[2A + (q-1)d] = b$

$\Rightarrow A + \dfrac{q-1}{2}d = \dfrac{b}{q}$ ....(ii)

$S_r = \dfrac{r}{2}[2A + (r-1)d] = c$

$\Rightarrow \mathrm{A}+\frac{r-1}{2}d=\frac{c}{r}$ ....(iii)

Now $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Putting the values of $\frac{a}{p}$, $\frac{b}{q}$ and $\frac{c}{r}$ from eq. (i), (ii) and (iii), we get

$$\left[\mathrm{A}+\frac{p-1}{2}d\right](q-r)+\left[\mathrm{A}+\frac{q-1}{2}d\right](r-p)+\left[\mathrm{A}+\frac{r-1}{2}d\right](p-q)=0$$

$$\Rightarrow \mathrm{A}(q-r+r-p+p-q)+d\left[\frac{p-1}{2}(q-r)+\frac{q-1}{2}(r-p)+\frac{r-1}{2}(p-q)\right]=0$$

$$\Rightarrow \mathrm{A}(0)+d\left[\frac{pq-pr-q+r}{2}+\frac{qr-pq-r+p}{2}+\frac{pr-qr-p+q}{2}\right]=0$$

$$\Rightarrow 0+d\left[\frac{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}{2}\right]=0$$

$$\Rightarrow 0+d\left[\frac{0}{2}\right]=0$$

$$\Rightarrow 0+0=0$$

$$\Rightarrow 0=0$$

$\Rightarrow$ L.H.S. = R.H.S. Proved.

12. The ratio of the sum of m and n terms of an A.P. is $m^2:n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m-1):(2n-1)$.

Ans. Let a be the first term and d be the common difference of given A.P.

$\therefore\ \mathrm{S}_m=\frac{m}{2}\left[2a+(m-1)d\right]$ and $\mathrm{S}_n=\frac{n}{2}\left[2a+(n-1)d\right]$

According to question, $\frac{S_m}{S_n} = \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{2} \times \frac{2}{m}$

$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

$\Rightarrow 2an + n(m-1)d = 2am + m(n-1)d$

$\Rightarrow 2an - 2am = (mn-m)d - (mn-n)d$

$\Rightarrow 2a(n-m) = (mn - m - mn + n)d$

$\Rightarrow 2a(n-m) = (n-m)d$

$\Rightarrow d = 2a$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a}$

$= \frac{a(1+2m-2)}{a(1+2n-2)} = \frac{2m-1}{2n-1}$

$\therefore\ a_m : a_n = (2m-1) : (2n-1)$

13. If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Ans. Given: $S_n = 3n^2 + 5n$ and $a_m = 164$

Put n=1 and n=2 in S_n we get, $S_1 = 3 + 5 = 8 \Rightarrow a = 8$ ....(i)

and $S_2 = 3(4) + 5(2) = 12 + 10 = 22$

We have $a_n = S_n - S_{n-1}$

$\therefore\ a_2 = S_2 - S_1$

$\Rightarrow a_2 = 22 - 8 = 14$

Now $d = a_2 - a_1 = 14 - 8 = 6$

$\therefore\ a_m = 164 \Rightarrow a + (m-1)d = 164$

$\Rightarrow 8 + (m-1)6 = 164$

$\Rightarrow 8 + 6m - 6 = 164$

$\Rightarrow 6m = 162$

$\Rightarrow m = \frac{162}{6} = 27$

14. Insert five numbers between 8 and 26 so that the resulting sequence is an A.P.

Ans. Let A_1, A_2, A_3, A_4 and A_5 be five numbers between 8 and 26 such that

8, A_1, A_2, A_3, A_4, A_5, 26 are in A.P

Here, $a = 8$ and $a_7 = 26$ and let d be the common difference.

$\therefore\ a_7 = a + (7-1)d = 26$

$\Rightarrow\ 8 + 6d = 26$

$\Rightarrow 6d = 18$

$\Rightarrow d = 3$

Now, $A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a+4d = 8+4\times 3 = 8+12 = 20$

$A_5 = a+5d = 8+5\times 3 = 8+15 = 23$

Hence the five numbers are 11,14,17,20 and 23

15. If $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between 'a' and 'b' then find the value of n.

Ans. Since, A.M. between a and b is $\dfrac{a+b}{2}$.

$\therefore \dfrac{a^n+b^n}{a^{n-1}+b^{n-1}} = \dfrac{a+b}{2}$

$\Rightarrow 2a^n + 2b^n = a^n + ab^{n-1} + b^n + ba^{n-1}$

$\Rightarrow a^n + b^n = ab^{n-1} + ba^{n-1}$

$\Rightarrow a^n - ba^{n-1} = ab^{n-1} - b^n$

$\Rightarrow a^{n-1}(a-b) = b^{n-1}(a-b)$

$\Rightarrow a^{n-1} = b^{n-1}$

$\Rightarrow \dfrac{a^{n-1}}{b^{n-1}} = 1$

$\Rightarrow \left(\dfrac{a}{b}\right)^{n-1} = \left(\dfrac{a}{b}\right)^0$

$\Rightarrow n-1 = 0$

$\Rightarrow n = 1$

16. Between 1 and 31, m numbers have been inserted in such a way that resulting

sequence is an A.P. and the ratio of 7th and $(m-1)^{th}$ numbers is 5: 9. Find the value of m.

Ans. Let $A_1, A_2, A_3, A_4, \ldots\ldots, A_m$ be m numbers between 1 and 31.

Here, $a=1$ and let the common difference be d.

$\therefore\ a_{m+2}=31$

$\Rightarrow a+(m+2-1)d=31$

$\Rightarrow 1+(m+1)d=31$

$\Rightarrow d=\dfrac{30}{m+1}$

Now, $A_7=a+7d=1+7\times\left(\dfrac{30}{m+1}\right)=\dfrac{m+1+210}{m+1}=\dfrac{m+211}{m+1}$

And $A_{m-1}=a+(m-1)d=1+(m-1)\times\left(\dfrac{30}{m+1}\right)=\dfrac{m+1+30m-30}{m+1}=\dfrac{31m-29}{m+1}$

According to question, $\dfrac{A_7}{A_{m-1}}=\dfrac{5}{9}$

$\Rightarrow \dfrac{\dfrac{m+211}{m+1}}{\dfrac{31m-29}{m+1}}=\dfrac{5}{9}$

$\Rightarrow \dfrac{m+211}{31m-29}=\dfrac{5}{9}$

$\Rightarrow 9m+1899=155m-145$

$\Rightarrow 146m=2044$

$\Rightarrow m=14$

17. A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs. 5 every month, what amount he will pay in the 30th installment?

Ans. Amount of 1st installment = Rs. 100 and Amount of 2nd installment = Rs. 105

The monthly installments 100,105,110, ………….. form an A.P

$\therefore\ a = 100, d = 105 - 100 = 5$ and $n = 30$

Now $a_n = a + (n-1)d$

$\Rightarrow a_{30} = 100 + (30-1)\times 5$

$\Rightarrow a_{30} = 100 + 29\times 5$

$\Rightarrow a_{30} = 100 + 145$ = Rs. 245

Therefore, the amount of 30th installment is Rs. 245.

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18. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

Ans. Let the number of sides of polygon be n. The interior angles of the polygon form an A.P.

Here, $a = 120^\circ$ and $d = 5^\circ$

Since Sum of interior angles of a polygon with n sides is $(n-2)\times 180^\circ$

$\therefore\ S_n = (n-2)\times 180^\circ$

$\Rightarrow \frac{n}{2}\left[2\times 120 + (n-1)\times 5\right] = 180n - 360$

$\Rightarrow 120n + \frac{5n^2 - 5n}{2} = 180n - 360$

$\Rightarrow 240n + 5n^2 - 5n = 360n - 720$

$\Rightarrow 5n^2 - 125n + 720 = 0$

divide by 5, we get

$=> n^2 - 25n + 144 = 0$

$\Rightarrow (n-16)(n-9) = 0$

$\Rightarrow n = 16$ or $n = 9$

But $n = 16$ not possible because $a_{16} = a + 15d = 120 + 15 \times 5 = 195^\circ > 180^\circ$

Therefore, number of sides of the polygon are 9.