

CBSE Class-11 Mathematics

NCERT Solutions

Chapter - 5 Complex Numbers and Quadratic Equations

Exercise 5.1

Express each of the complex numbers given in the exercises 1 to 10 in the form $a+ib$:

1. $(5i)\left(\frac{-3}{5}i\right)$

Ans. Here $(5i)\left(\frac{-3}{5}i\right) = -3i^2$

$$= -3 \times -1$$

$$= 3$$

2. $i^9 + i^{19}$

Ans. Here $i^9 + i^{19}$

$$= (i^2)^4 \cdot i + (i^2)^9 \cdot i = (-1)^4 \cdot i + (-1)^9 \cdot i = i - i = 0$$

3. i^{-39}

Ans. $i^{-39} = \frac{1}{i^{39}} = \frac{1}{(i^2)^{19} \cdot i} = \frac{1}{(-1)^{19} \cdot i} = \frac{1}{-i}$

$$= \frac{-1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$$

4. $3(7+i7) + i(7+i7)$

Ans. Here $3(7+i7) + i(7+i7)$

$$= 21 + 21i + 7i + 7i^2 = 21 + 28i - 7 = 14 + 28i$$

5. $(1-i) - (-1+i6)$

Ans. Here $(1-i) - (-1+i6)$

$$= 1 - i + 1 - i6 = 2 - 7i$$

6. $\left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}i\right)$

Ans. Here $\left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}i\right)$

$$= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i$$

$$= \left(\frac{1}{5} - 4\right) + \left(\frac{2}{5} - \frac{5}{2}\right)i = \frac{-19}{5} - \frac{21}{10}i$$

7. $\left[\left(\frac{1}{3} + \frac{7}{3}i\right) + \left(4 + \frac{1}{3}i\right)\right] - \left[\frac{-4}{3} + i\right]$

Ans. Here $\left[\left(\frac{1}{3} + \frac{7}{3}i\right) + \left(4 + \frac{1}{3}i\right)\right] - \left[\frac{-4}{3} + i\right]$

$$= \left[\left(\frac{1}{3} + 4\right) + \left(\frac{7}{3} + \frac{1}{3}i\right)\right] - \left[\frac{-4}{3} + i\right]$$

$$= \frac{13}{3} + \frac{8}{3}i + \frac{4}{3} - i = \left(\frac{13}{3} + \frac{4}{3}\right) + \left(\frac{8}{3} - 1\right)i = \frac{17}{3} + \frac{5}{3}i$$

8. $(1-i)^4$

Ans. Here $(1-i)^4 = \left[(1-i)^2\right]^2$

$$= [1 + i^2 - 2i]^2 = (1 - 1 - 2i)^2 = (-2i)^2$$
$$= 4i^2 = -4$$

9. $\left(\frac{1}{3} + 3i\right)^3$

Ans. Here $\left(\frac{1}{3} + 3i\right)^3$

$$= \left(\frac{1}{3}\right)^3 + (3i)^3 + 3 \times \left(\frac{1}{3}\right)^2 \times 3i + 3 \times \frac{1}{3} \times (3i)^2$$

$$= \frac{1}{27} + 27i^3 + i + 9i^2$$

$$= \frac{1}{27} - 27i + i - 9$$

$$= \frac{1}{27} - 9 - 26i$$

$$= \frac{-242}{27} - 26i$$

10. $\left(-2 - \frac{1}{3}i\right)^3$

Ans. Here $\left(-2 - \frac{1}{3}i\right)^3 = -\left(2 + \frac{1}{3}i\right)^3$

$$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times (2)^2 \times \frac{1}{3}i + 3 \times 2 \times \left(\frac{1}{3}i\right)^2\right]$$

$$= -\left[8 + \frac{1}{27}i^3 + 4i + \frac{2}{3}i^2\right]$$

$$= -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right]$$
$$= -\left[8 - \frac{2}{3} - \frac{1}{27}i + 4i\right] = \frac{-22}{3} - \frac{107}{27}i$$

Find the multiplicative inverse of each of the complex numbers given in the exercises 11 to 13.

11. $4 - 3i$

Ans. Multiplicative Inverse of $4 - 3i$

$$= \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$
$$= \frac{4 + 3i}{(4)^2 - (3i)^2}$$
$$= \frac{4 + 3i}{16 - 9i^2}$$
$$= \frac{4 + 3i}{16 + 9} = \frac{1}{25}(4 + 3i)$$
$$= \frac{4}{25} + i\frac{3}{25}$$

12. $\sqrt{5} + 3i$

Ans. Multiplicative Inverse of $\sqrt{5} + 3i$

$$= \frac{1}{\sqrt{5} + 3i} \times \frac{\sqrt{5} - 3i}{\sqrt{5} - 3i}$$
$$= \frac{\sqrt{5} - 3i}{(\sqrt{5})^2 - (3i)^2}$$

$$\begin{aligned}
 &= \frac{\sqrt{5}-3i}{5-9i^2} \\
 &= \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}(\sqrt{5}-3i) \\
 &= \frac{\sqrt{5}}{14} - i\frac{3}{14}
 \end{aligned}$$

13. $-i$

Ans. Multiplicative Inverse of $-i$

$$\begin{aligned}
 &= \frac{1}{-i} \times \frac{i}{i} \\
 &= \frac{i}{-i^2} \\
 &= \frac{i}{-(-1)} = i
 \end{aligned}$$

14. Express the following expression in the form of $a+ib$: $\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(3+\sqrt{2}i)-(3-\sqrt{2}i)}$

Ans. Here $\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(3+\sqrt{2}i)-(3-\sqrt{2}i)}$

$$\begin{aligned}
 &= \frac{(3)^2 - (\sqrt{5}i)^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \\
 &= \frac{9-5i^2}{2\sqrt{2}i} \\
 &= \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i} \\
 &= \frac{7}{\sqrt{2}i} = \frac{7}{\sqrt{2}i} \times \frac{i}{i} = \frac{7i}{\sqrt{2}i^2} = \frac{-7i}{\sqrt{2}} = \frac{-7\sqrt{2}i}{2}
 \end{aligned}$$