CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 9 Sequences and Series

Exercise 9.4

Find the sum to n terms in each of the series in Exercises 1 to 7.

1. $1\times 2+2\times 3+3\times 4+4\times 5+\ldots\ldots$

Ans. Given: $1\times 2+2\times 3+3\times 4+4\times 5+\ldots\ldots$ to n terms

$\therefore\ a_n$ = [n^{th} term of 1, 2, 3, ...][n^{th} term of 2, 3, 4, 5, ...]

= $\left[1+(n-1)\times 1\right]\left[2+(n-1)\times 1\right]$

= $n(n+1)=n^2+n$

$\therefore\ S_n=\sum_{k=1}^{n}a_k = \sum_{k=1}^{n}\left(k^2+k\right)$

= $\left[1^2+1\right]+\left[2^2+2\right]+\left[3^2+3\right]+\ldots\ldots\ldots\ldots\ldots+\left[n^2+n\right]$

= $\left[1^2+2^2+3^2+\ldots\ldots\ldots\ldots n^2\right]+\left[1+2+3+\ldots\ldots\ldots\ldots\ldots n\right]$

= $\dfrac{n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}$

= $\dfrac{n(n+1)}{2}\left[\dfrac{2n+1}{3}+1\right]$

= $\dfrac{n(n+1)}{2}\times\dfrac{(2n+4)}{3}$

= $\dfrac{n(n+1)(n+2)}{3}$

2. $1\times2\times3+2\times3\times4+3\times4\times5+\ldots.$

Ans. Given: $1\times2\times3+2\times3\times4+3\times4\times5+\ldots.$ to n terms

$\therefore$ a_n = [n^{th} term of 1, 2, 3, ...][n^{th} term of 2, 3, 4, 5, ...][n^{th} term of 3, 4, 5, ...]

= $\left[1+(n-1)\times1\right]\left[2+(n-1)\times1\right]\left[3+(n-1)\times1\right]$

= $n(n+1)(n+2)=n^3+3n^2+2n$

$\therefore$ $S_n=\sum_{k=1}^{n}a_k$ = $\sum_{k=1}^{n}\left(k^3+3k^2+2k\right)$

= $\left[1^3+3.1^2+2.1\right]+\left[2^3+3.2^2+2.2\right]+\left[3^3+3.3^2+2.3\right]+\ldots\ldots\ldots+\left[n^3+3n^2+2n\right]$

= $\left[1^3+2^3+3^3+\ldots\ldots\ldots\ldots n^3\right]+3\left[1^2+2^2+3^2+\ldots\ldots\ldots\ldots n^2\right]+2\left[1+2+3+\ldots\ldots\ldots\ldots\ldots n\right]$

$=\left[\frac{n(n+1)}{2}\right]^2+\frac{3n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}$

= $\frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{2}+n(n+1)$

= $n(n+1)\left[\frac{n(n+1)}{4}+\frac{2n+1}{2}+1\right]$

= $n(n+1)\left[\frac{n^2+n+4n+2+4}{4}\right]$

$=\frac{n(n+1)(n^2+5n+6)}{4}$

= $\frac{1}{4}n(n+1)(n+2)(n+3)$

3. $3\times1^2+5\times2^2+7\times3^2+\ldots..$

Ans. Given: $3\times1^2+5\times2^2+7\times3^2+\ldots$ to n terms

$\therefore$ a_n = [n^{th} term of 3, 5, 7, ...][n^{th} term of 1, 2, 3, 4, ...]^2

= $(2n+1)(n)^2$ = $2n^3+n^2$

$\therefore$ $S_n=\sum_{k=1}^{n}a_k$ = $\sum_{k=1}^{n}\left(2k^3+k^2\right)$

= $\left[2.1^3+1^2\right]+\left[2.2^3+2^2\right]+\left[2.3^3+3^2\right]+\ldots\ldots\ldots+\left[2n^3+n^2\right]$

= $2\left(1^3+2^3+3^3+\ldots\ldots\ldots n^3\right)+\left(1^2+2^2+3^2+\ldots\ldots\ldots n^2\right)$

= $2\left[\frac{n(n+1)}{2}\right]^2+\frac{n(n+1)(2n+1)}{6}$

$=2\frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{6}$

= $\frac{n(n+1)}{2}\left[n(n+1)+\frac{2n+1}{3}\right]$

= $\frac{n(n+1)}{2}\left[\frac{3n^2+3n+2n+1}{3}\right]$

= $\frac{n(n+1)\left(3n^2+3n+2n+1\right)}{6}$

= $\frac{n(n+1)\left(3n^2+5n+1\right)}{6}$

4. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\ldots\ldots$

Ans. Given: $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+$.................. to n terms

$$\therefore\ a_n=\frac{1}{(n^{th}\text{ term of }1,2,3,\ldots\ldots\ldots)(n^{th}\text{ term of }2,3,4,\ldots\ldots\ldots)}$$

$$=\frac{1}{[1+(n-1)\times 1][2+(n-1)\times 1]}$$

Let $\frac{1}{n(n+1)}=\frac{A}{n}+\frac{B}{n+1}$ [By partial fraction]

Then 1=A(n+1) + Bn

Put n=0 then A=1

Put n=-1 then B=-1

$$\therefore\ \frac{1}{n(n+1)}=\frac{1}{n}+\frac{-1}{n+1}$$

$$\therefore\ a_1=\frac{1}{1}-\frac{1}{2}\quad a_2=\frac{1}{2}-\frac{1}{3}\quad a_3=\frac{1}{3}-\frac{1}{4}\quad\ldots\ldots\ldots$$

And $S_n=a_1+a_2+a_3+\ldots\ldots\ldots+a_n$

$$=\frac{1}{1}-\frac{1}{n+1}=\frac{n}{n+1}$$

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5. $5^2+6^2+7^2+\ldots\ldots+20^2$

Ans. Given: $5^2+6^2+7^2+\ldots\ldots+20^2$

$$=\left(1^2+2^2+3^2+\ldots\ldots\ldots+20^2\right)-\left(1^2+2^2+3^2+4^2\right)$$

$$=\sum_{n=1}^{20}n^2-\sum_{n=1}^{4}n^2$$

= $\frac{20(20+1)(40+1)}{6}-\frac{4(4+1)(8+1)}{6}$

= $\frac{20\times21\times41}{6}-\frac{20\times9}{6}$

= $\frac{20}{6}(861-9)$ = 2840

6. $\mathbf{3\times8+6\times11+9\times14}$ + ...

Ans. Given: $\mathbf{3\times8+6\times11+9\times14}$ + ... to n terms

$\therefore\ a_n$ = [n^{th} term of 3, 6, 9, ...][n^{th} term of 8, 11, 14, ...]

= $\left[3+(n-1)\times3\right]\left[8+(n-1)\times3\right]$

= $3n(3n+5)$ = $9n^2+15n$

$\therefore\ S_n=\sum_{k=1}^{n}a_k$ = $\sum_{k=1}^{n}\left(9k^2+15k\right)$

= $\left[9.1^2+15.1\right]+\left[9.2^2+15.2\right]+\left[9.3^2+15.3\right]..........+\left[9n^2+15n\right]$

= $9\left(1^2+2^2+3^2+............n^2\right)+15\left(1+2+3+............n\right)$

$=9\frac{n(n+1)(2n+1)}{6}+15\frac{n(n+1)}{2}$

= $\frac{n(n+1)}{2}(6n+3+15)$

= $\frac{n(n+1)}{2}(6n+18)$

= $3n(n+1)(n+3)$

7. $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+\ldots\ldots$

Ans. Given: $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+\ldots\ldots$ to n terms

$\therefore\ a_n = \left(1^2+2^2+3^2+\ldots\ldots\ldots\ldots n^2\right)$

$= \dfrac{n(n+1)(2n+1)}{6}$

$= \dfrac{1}{6}\left(2n^3+3n^2+n\right)$

$\therefore\ S_n=\sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\dfrac{1}{6}\left(2k^3+3k^2+k\right)$

$= \dfrac{1}{6}\left(2.1^3+3.1^2+1\right)+\dfrac{1}{6}\left(2.2^3+3.2^2+2\right)+\ldots\ldots\ldots+\dfrac{1}{6}\left(2.n^3+3n^2+n\right)$

$= \dfrac{1}{6}\left[2\left(1^3+2^3+\ldots\ldots+n^3\right)+3\left(1^2+2^2+\ldots\ldots+n^2\right)+\left(1+2+3+\ldots\ldots+n\right)\right]$

$= \dfrac{1}{6}\left[2\left\{\dfrac{n(n+1)}{2}\right\}^2+\dfrac{3n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}\right]$

$= \dfrac{n(n+1)}{12}\left[\dfrac{n(n+1)+2n+1+1}{1}\right]$

$= \dfrac{n(n+1)}{12}\left(n^2+3n+2\right)$

$= \dfrac{n(n+1)(n+1)(n+2)}{12}$

$= \dfrac{n(n+1)^2(n+2)}{12}$

Find the sum to n terms in each of the series in Exercises 8 to 10 whose n^{th} terms is given by

8. $n(n+1)(n+4)$

Ans. Given: $a_n = n(n+1)(n+4) = n^3 + 5n^2 + 4n$

$\therefore \ S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^3 + 5k^2 + 4k$

$= (1^3 + 5.1^2 + 4.1) + (2^3 + 5.2^2 + 4.2) + \ldots\ldots\ldots\ldots (n^3 + 5.n^2 + 4.n)$

$= (1^3 + 2^3 + \ldots\ldots\ldots\ldots + n^3) + 5(1^2 + 2^2 + \ldots\ldots\ldots\ldots + n^2) + 4(1 + 2 + 3 + \ldots\ldots\ldots\ldots + n)$

$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$

$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1)$

$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{5(2n+1)}{6} + 2\right]$

$= n(n+1)\left[\frac{3n^2+3n+20n+10+24}{12}\right]$

$= \frac{n(n+1)(3n^2+23n+34)}{12}$

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9. $n^2 + 2^n$

Ans. Given: $a_n = n^2 + 2^n$

$\therefore \ S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2 + 2^k$

$= (1^2 + 2^1) + (2^2 + 2^2) + (3^2 + 2^3) \ldots\ldots\ldots\ldots (n^2 + 2^n)$

= $\left(1^2+2^2+3^2+\ldots\ldots\ldots\ldots+n^2\right)+\left(2^1+2^2+2^3+\ldots\ldots\ldots+2^n\right)$

= $\dfrac{n(n+1)(2n+1)}{6}+\dfrac{2\left(2^n-1\right)}{2-1}$

= $\dfrac{n(n+1)(2n+1)}{6}+2\left(2^n-1\right)$

10. $(2n-1)^2$

Ans. Given: $a_n=(2n-1)^2=4n^2-4n+1$

$\therefore\ S_n=\sum\limits_{k=1}^{n}a_k$ = $\sum\limits_{k=1}^{n}4k^2-4k+1$

= $\left(4.1^2-4.1+1\right)+\left(4.2^2-4.2+1\right)+\ldots\ldots\ldots\ldots\ldots\left(4.n^2-4n+1\right)$

$= 4(1^2+2^2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+n^2)-4(1+2+3+\ldots\ldots\ldots+n)+(1+1+\ldots\ldots\ldots\ldots$

$= \frac{4n(n+1)(2n+1)}{6}-\frac{4n(n+1)}{2}+n$

$= n\left(\frac{2(n+1)(2n+1)}{3}-2(n+1)+1\right)$

= $n\left(\dfrac{4n^2+6n+2-6n-6+3}{3}\right)$

= $n\left(\dfrac{4n^2-1}{3}\right)$

= $\dfrac{n(2n+1)(2n-1)}{3}$