CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 8 Binomial Theorem

Exercise 8.2

Find the coefficient of:

1. x^5 in $(x+3)^8$

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1} = {}^nC_r x^{n-r} a^r$

$\therefore$ General term in the expansion of **$(x+3)^8$** is $T_{r+1} = {}^8C_r x^{8-r} (3)^r$ ....(i)

Comparing the indices of x in x^5 and in T_{r+1}

we get 8-r = 5

=> r = 3

Putting r = 3 in eq. (i), $T_4 = {}^8C_3 x^{8-3} (3)^3 = {}^8C_3 x^5 (3)^3$

Therefore, coefficient of x^5 on the expansion $(x+3)^8$ is ${}^8C_3 (3)^3$ = 56 x 27 = 1512.

2. a^5b^7 in $(a-2b)^{12}$

Ans. The general term in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

$\therefore$ General term in the expansion of $(a-2b)^{12}$ = $[a+(-2b)]^{12}$ is

$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r = {}^{12}C_r (-2)^r a^{12-r} b^r$ ....(i)

Comparing the indices of a and b in a^5b^7 and in T_{r+1} we get r = 7

Putting r= 7 in eq. (i), $T_8 = {}^{12}C_7 a^5 (-2b)^7 = {}^{12}C_7 (-2)^7 a^5 b^7$

Therefore, coefficient of a^5b^7 on the expansion $(a-2b)^{12}$ is ${}^{12}C_7(-2)^7$ = -101376

Write the general term in the expansion of

3. $(x^2-y)^6$

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1}={}^nC_rx^{n-r}a^r$

$\therefore$ General term in the expansion of $(x^2-y)^6$ = $[x^2+(-y)]^6$ is $T_{r+1}={}^6C_r(x^2)^{6-r}(-y)^r$

$\Rightarrow T_{r+1}=(-1)^r\ {}^6C_rx^{12-2r}y^r$

4. $(x^2-yx)^{12}, x\neq 0$

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1}={}^nC_rx^{n-r}a^r$

$\therefore$ General term in the expansion of $(x^2-yx)^{12}$ is $T_{r+1}={}^{12}C_r(x^2)^{12-r}(-yx)^r$

$\Rightarrow T_{r+1}=(-1)^r\ {}^{12}C_rx^{24-2r}y^rx^r=(-1)^r\ {}^{12}C_rx^{24-r}y^r$

5. Find the 4^th term in the expansion of $(x-2y)^{12}$.

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1}={}^nC_rx^{n-r}a^r$

$\therefore$ General term in the expansion of $(x-2y)^{12}$ is $T_{r+1}={}^{12}C_r(x)^{12-r}(-2y)^r$

$\Rightarrow T_{r+1}=(-1)^r\ {}^{12}C_r2^rx^{12-r}y^r$

Putting $r=3$, $T_4=(-1)^3\ {}^{12}C_3.2^3x^{12-3}y^3$

$\Rightarrow T_4 = -{}^{12}C_3.8x^9y^3 = -220\times 8x^9y^3 = -1760x^9y^3$

6. Find the 13th term in the expansion of $\left[9x-\frac{1}{3\sqrt{x}}\right]^{18}, x\neq 0.$

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1} = {}^nC_r x^{n-r}a^r$

$\therefore$ General term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$ is $T_{r+1} = {}^{18}C_r\,(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$ .....(i)

Putting $r=12$, $T_{13} = {}^{18}C_{12}\,(9x)^{18-12}\left(\frac{-1}{3\sqrt{x}}\right)^{12}$

$\Rightarrow T_{13} = {}^{18}C_{12}9^6x^6(-1)^{12}\left(\frac{1}{3^{12}x^6}\right)$

$= {}^{18}C_{12}\frac{9^6}{3^{12}} = {}^{18}C_{12} = 18564$ $\qquad \left[9^6 = (3^2)^6 = 3^{12}\right]$

Find the middle terms in the expansion of:

7. $\left(3-\frac{x^3}{6}\right)^7$

Ans. We have if n is odd, then n+1 is even, so there will be two middle terms in the expansion of $(a+b)^n$, namely $\left(\frac{n+1}{2}\right)$ th term and $\left(\frac{n+1}{2}+1\right)$ th term

Here $n=7$, which is an odd number.

Therefore, the middle terms are $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th}$ i.e., 4th and 5th terms .

The general term in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r}b^r$

$\therefore$ General term in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ is $T_{r+1} = {}^7C_r\,(3)^{7-r}\left(-\frac{x^3}{6}\right)^r$ .....(i)

Putting $r=3$ and $r=4$ in eq. (i),

$$T_4 = {}^7C_3\,(3)^{7-3}\left(-\frac{x^3}{6}\right)^3$$

$$= {}^7C_3\,(3)^4\,(-1)^3\,.\frac{x^9}{6^3}$$

$$= 35\times 81\times\frac{-x^9}{216} = \frac{-105}{8}x^9$$

$$T_5 = {}^7C_4\,(3)^{7-4}\left(-\frac{x^3}{6}\right)^4$$

$$= {}^7C_4\,(3)^3\,(-1)^4\,.\frac{x^{12}}{6^4}$$

$$= 35\times 27\times\frac{x^{12}}{1296} = \frac{35}{48}x^{12}$$

Hence the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ are $\frac{-105}{8}x^9$ and $\frac{35}{48}x^{12}$

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8. $\left(\frac{x}{3}+9y\right)^{10}$

Ans. We have if n is even, then n+1 is odd, so there will be a single middle term in the expansion of $(a+b)^n$,namely $\left(\frac{n}{2}+1\right)$ th term

Here $n=10$, which is an even number.

Therefore, the middle term is $\left(\frac{10}{2}+1\right)^{th}$ term i.e., 6th term.

The general term in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

$\therefore$ General term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{10-r} (9y)^r$ .....(i)

Putting $r=5$ in eq. (i),

$T_6 = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$

= ${}^{10}C_5 \frac{x^5}{3^5} 9^5 y^5$

= ${}^{10}C_5 3^5 x^5 y^5 \quad \left[9^5 = (3^2)^5 = 3^{10}\right]$

= $252 \times 243 x^5 y^5$

= $61236 x^5 y^5$

Hence the middle terms in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $61236x^5y^5$

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9. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Ans. We know that,

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^nC_n x^n$ and general term of this expansion is $T_{r+1} = {}^nC_r x^r$

$\therefore$ General term in the expansion of $(1+a)^{m+n}$ will be $T_{r+1} = {}^{m+n}C_r a^r$

Now, Coefficient of a^m in the expansion of $(1+a)^{m+n} = {}^{m+n}C_m$ and

Coefficient of a^n in the expansion of $(1+a)^{m+n} = {}^{m+n}C_n$

But we have ${}^nC_r = {}^nC_{n-r}$

$\therefore$ Coefficient of a^m = ${}^{m+n}C_m = {}^{m+n}C_{(m+n)-m} = {}^{m+n}C_n$= Coefficient of a^n

Hence proved

10. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

Ans. We know that,

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^nC_nx^n$ and general term of this expansion is $T_{r+1} = {}^nC_rx^r$

Therefore the coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms are ${}^nC_{r-2}, {}^nC_{r-1}$ and ${}^nC_r$ respectively.

Given: ${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r = 1:3:5$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{3}$ and $\frac{{}^nC_{r-1}}{{}^nC_{r-2}} = \frac{3}{1}$

$\Rightarrow \frac{n!}{r!(n-r)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{5}{3}$ and $\frac{n!}{(r-1)!(n-r+1)!} \times \frac{(r-2)!(n-r+2)!}{n!} = \frac{3}{1}$

$\Rightarrow \frac{(r-1)!(n-r+1)(n-r)!}{r(r-1)!(n-r)!} = \frac{5}{3}$ and $\frac{(r-2)!(n-r+2)(n-r+1)!}{(r-1)(r-2)!(n-r+1)!} = \frac{3}{1}$

$\Rightarrow \frac{n-r+1}{r} = \frac{5}{3}$ and $\frac{n-r+2}{r-1} = \frac{3}{1}$

$\Rightarrow 3n - 3r + 3 = 5r$ and $n - r + 2 = 3r - 3$

$\Rightarrow 3n - 8r + 3 = 0$ ....(i) and $n - 4r + 5 = 0$ ....(ii)

Multiplying equation (ii) by 2 and subtracting it from equation (i),we get n-7=0 $\Rightarrow n = 7$

Now substituting $n = 7$ in equation (i) we get 24-8r=0 $\Rightarrow r = 3$

Therefore we have $n=7$ and $r=3$

11. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

Ans. We know that general term in the expansion of $(1+x)^n$ is $T_{r+1}={}^nC_r x^r$

Therefore Coefficient of x^n in the expansion of $(1+x)^{2n}$ is

$$= {}^{2n}C_n=\frac{(2n)!}{n!n!}=\frac{2n(2n-1)!}{n(n-1)!n!}$$

$$= 2\frac{(2n-1)!}{(n-1)!n!} \text{ ....(i)}$$

Also Coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is ${}^{2n-1}C_n=\frac{(2n-1)!}{(n-1)!n!}$ ....(ii)

From eq. (i) and eq. (ii), it is clear that Coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the Coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

Ans. The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1}={}^nC_r x^{n-r}a^r$

Assuming that x^2 occur in the $(r+1)^{th}$ term of expression $(1+x)^m$, we obtain

$T_{r+1}={}^mC_r(1)^{m-r}(x)^r={}^mC_r(x)^r$

Comparing the indices of x^2 in term we get r = 2

so, cofficient of x^2 is ${}^mC_2$. So

$=> {}^mC_2 = 6$

$=> \frac{m!}{2!(m-2)!} = 6$

$=> \frac{m(m-1)(m-2)!}{2!(m-2)!} = 6$

$=> \frac{m(m-1)}{2} = 6$

=> m^2 - m = 12

=> m^2-m -12=0

=> (m-4)(m+3)= 0

So, m = 4 or -3

Thus the positive value of m for which coefficient of x^2 in expression $(1+x)^m$ is 6 is 4.