CBSE Class–11 Mathematics

NCERT Solutions

Chapter - 5 Complex Numbers and Quadratic Equations

Miscellaneous Exercise

1. Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$

Ans. Given: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$

= $\left[\left(i^{2}\right)^{9}+\frac{1}{\left(i^{2}\right)^{12}.i}\right]^{3}$

= $\left[(-1)^{9}+\frac{1}{(-1)^{12}.i}\right]^{3}$

= $\left[-1+\frac{1}{i}\right]^{3}$

= $(-1-i)^{3}$

= $-(1+i)^{3}$

= $-\left(1+i^{3}+3i+3i^{2}\right)$

= $-(1-i+3i-3)$

= $-(-2+2i)$

= $2-2i$

2. For any two complex numbers z_1 and z_2 prove that $\mathrm{Re}(z_1 z_2) = \mathrm{Re}z_1 \mathrm{Re}z_2 - \mathrm{Im}\, z_1\, \mathrm{Im}\, z_2$

Ans. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

Then $\mathrm{Re}(z_1) = a_1, \mathrm{Re}(z_2) = a_2, \mathrm{Im}(z_1) = b_1$ and $\mathrm{Im}(z_2) = b_2$

Now, $z_1 z_2 = (a_1 + ib_1)(a_2 + ib_2)$

= $a_1a_2 + ia_1b_2 + ia_2b_1 + i^2 b_1b_2$

= $(a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$

$\therefore \mathrm{Re}(z_1z_2) = a_1a_2 - b_1b_2$

= $\mathrm{Re}z_1 \mathrm{Re}z_2 - \mathrm{Im}\, z_1\, \mathrm{Im}\, z_2$

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3. Reduce $\left[\frac{1}{1-4i} - \frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]$ **to the standard form.**

Ans. Here: $\left[\frac{1}{1-4i} - \frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]$

= $\left[\frac{1+i-2+8i}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$

= $\left[\frac{-1+9i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]$

= $\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$

= $\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}$

= $\frac{33+31i}{28-10i} \times \frac{28+10i}{28+10i}$

= $\dfrac{924+330i+868i+310i^2}{(28)^2-(10i)^2}$

= $\dfrac{614+1198i}{784+100}$

= $\dfrac{2(307+599i)}{884}$

= $\dfrac{307+599i}{442}$

4. If $x-iy=\sqrt{\dfrac{a-ib}{c-id}}$ **prove that** $\left(x^2+y^2\right)^2=\dfrac{a^2+b^2}{c^2+d^2}$.

Ans. Given: $x-iy=\sqrt{\dfrac{a-ib}{c-id}}$

Squaring both sides, we get

=> $(x-iy)^2=\dfrac{a-ib}{c-id}$

$\Rightarrow \left|(x-iy)^2\right|=\left|\dfrac{a-ib}{c-id}\right|$

$\Rightarrow \left|(x-iy)\right|\left|(x-iy)\right|=\left|\dfrac{a-ib}{c-id}\right|$

$\Rightarrow \left(\sqrt{x^2+y^2}\right)\left(\sqrt{x^2+y^2}\right)=\dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$\Rightarrow \left(x^2+y^2\right)=\sqrt{\dfrac{a^2+b^2}{c^2+d^2}}$

Squaring both sides, we get

=> $\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$

5. Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$

(ii) $\frac{1+3i}{1-2i}$

Ans. (i) Here $\frac{1+7i}{(2-i)^2}$

= $\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}$

= $\frac{3+4i+21i+28i^2}{9-16i^2}=\frac{-25+25i}{25}=-1+i$

$\therefore z=r(\cos\theta+i\sin\theta)=-1+i$

$\therefore r\cos\theta=-1$ and $r\sin\theta=1$

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta+\sin^2\theta)=1+1$

$\Rightarrow r^2=2$

$\Rightarrow r=\sqrt{2}$

$\therefore \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}}$ and $\sin\theta=\frac{1}{\sqrt{2}}$

[θ lies in second quadrant]

$\therefore\ \theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Therefore, Polar form of z is $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$.

(ii) Here $\frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

= $\frac{1+2i+3i+6i^2}{1-4i^2} = \frac{-5+5i}{5} = -1+i$

$\therefore\ z = r(\cos\theta + i\sin\theta) = -1+i$

$\therefore\ r\cos\theta = -1$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$

$\therefore\ \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

[θ lies in second quadrant]

$\therefore\ \theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Therefore, Polar form of z is $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$.

Solve each of the equations in exercises 6 to 9:

6. $3x^2-4x+\frac{20}{3}=0$

Ans. Given: $3x^2-4x+\frac{20}{3}=0$

Comparing with $ax^2+bx+c=0$,

$a=3, b=-4$ and $c=\frac{20}{3}$

$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

= $\frac{-(-4)\pm\sqrt{(-4)^2-4\times3\times\frac{20}{3}}}{2\times3}$

= $\frac{4\pm\sqrt{16-80}}{6}$

= $\frac{4\pm\sqrt{64i}}{6}$

= $\frac{4\pm8i}{6}$ = $\frac{4}{6}\pm i\frac{8}{6}=\frac{2}{3}\pm i\frac{4}{3}$

7. $x^2-2x+\frac{3}{2}=0$

Ans. Given: $x^2-2x+\frac{3}{2}=0$

Comparing with $ax^2+bx+c=0$,

$a=1, b=-2$ and $c=\frac{3}{2}$

$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-2)\pm\sqrt{(-2)^2-4\times1\times\frac{3}{2}}}{2\times1}$

$=\frac{2\pm\sqrt{-2}}{2}$

$=\frac{2\pm\sqrt{2}i}{2}$

$=1\pm i\frac{\sqrt{2}}{2}$

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8. $27x^2-10x+1=0$

Ans. Given: $27x^2-10x+1=0$

Comparing with $ax^2+bx+c=0$,

$a=27, b=-10$ and $c=1$

$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-10)\pm\sqrt{(-10)^2-4\times27\times1}}{2\times27}$

$=\frac{10\pm\sqrt{-8}}{54}$

= $\frac{10 \pm 2\sqrt{2}i}{54}$

$= \frac{10}{54} \pm i\frac{2\sqrt{2}}{54}$

= $= \frac{5}{27} \pm i\frac{\sqrt{2}}{27}$

9. $21x^2 - 28x + 10 = 0$

Ans. Given: $21x^2 - 28x + 10 = 0$

Comparing with $ax^2 + bx + c = 0$, $a = 21, b = -28$ and $c = 10$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(-28) \pm \sqrt{(-28)^2 - 4 \times 21 \times 10}}{2 \times 21}$

= $\frac{28 \pm \sqrt{-56}}{42}$

= $\frac{28 \pm \sqrt{56}i}{42}$

$= \frac{28}{42} \pm i\frac{2\sqrt{14}}{42}$

= $\frac{2}{3} \pm i\frac{\sqrt{14}}{21}$

10. If $z_1 = 2 - i, z_2 = 1 + i,$ **find** $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

Ans. Here $z_1 = 2 - i$ and $z_2 = 1 + i$

= $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

= $\left|\frac{2-i+1+i+1}{2-i-1-i+i}\right|$

= $\left|\frac{4}{1-i}\right|$

= $\frac{|4|}{|1-i|} = \frac{4}{\sqrt{1^2+1^2}}$

= $\frac{4}{\sqrt{2}} = 2\sqrt{2}$

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11. If $a+ib = \frac{(x+i)^2}{2x^2+1}$, **prove that** $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Ans. Here $a+ib = \frac{(x+i)^2}{2x^2+1}$

= $\frac{x^2+i^2+2ix}{2x^2+1} = \frac{x^2-1}{2x^2+1} + i\frac{2x}{2x^2+1}$

Comparing both sides, we have

$a = \frac{x^2-1}{2x^2+1}$ and $b = \frac{2x}{2x^2+1}$

$\therefore a^2+b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$

= $\frac{(x^2-1)^2+(2x)^2}{(2x^2+1)^2}$

= $\frac{x^4+1-2x^2+4x^2}{(2x^2+1)^2}$

$\Rightarrow a^2+b^2 = \frac{x^4+1+2x^2}{(2x^2+1)^2}$

$= \dfrac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2}$ Proved.

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12. Let $z_1 = 2 - i, z_2 = -2 + i$ find:

(i) $\mathrm{Re}\left(\dfrac{z_1 z_2}{\overline{z_1}}\right)$

(ii) $\mathrm{I}m\left(\dfrac{1}{z_1 \overline{z_1}}\right)$

Ans. Here $z_1 = 2 - i$ and $z_2 = -2 + i$

$\therefore \overline{z_1} = 2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -3 + 4i$

$\therefore \dfrac{z_1 z_2}{\overline{z_1}} = \dfrac{-3+4i}{2+i} \times \dfrac{2-i}{2-i} = \dfrac{-6+3i+8i-4i^2}{4-i^2} = \dfrac{-2+11i}{5} = \dfrac{-2}{5} + \dfrac{11}{5}i$

$\therefore \mathrm{Re}\left(\dfrac{z_1 z_2}{\overline{z_1}}\right) = \dfrac{-2}{5}$

(ii) $\dfrac{1}{z_1 \overline{z_1}} = \dfrac{1}{(2-i)(2+i)} = \dfrac{1}{4-i^1} = \dfrac{1}{5}$

$\therefore \mathrm{I}m\left(\dfrac{1}{z_1 \overline{z_1}}\right) = 0$

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13. Find the modulus and argument of the complex number $\dfrac{1+2i}{1-3i}$.

Ans. Let $z=\frac{1+2i}{1-3i}\times\frac{1+3i}{1+3i}=\frac{1+3i+2i+6i^2}{1-9i^2}=\frac{-5+5i}{10}=\frac{-1+i}{2}$

=> $\frac{-1}{2}+\frac{i}{2}=r\left(\cos\theta+i\sin\theta\right)$

$\therefore r\cos\theta=\frac{-1}{2}$ and $r\sin\theta=\frac{1}{2}$

Squaring both sides and adding both the equations, we get

$r^2\left(\cos^2\theta+\sin^2\theta\right)=\frac{1}{4}+\frac{1}{4}$

$\Rightarrow r^2=\frac{1}{2}$

$\Rightarrow r=\frac{1}{\sqrt{2}}$

$\therefore \frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2}$ and $\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$

$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}}$ and $\sin\theta=\frac{1}{\sqrt{2}}$

[θ lies in second quadrant]

$\therefore \theta=\left(\pi-\frac{\pi}{4}\right)=\frac{3\pi}{4}$

Therefore, $|z|=\frac{1}{\sqrt{2}}$ and $\arg(z)=\frac{3\pi}{4}$

14. Find the real numbers x and y if (x-iy)(3+5i) is the conjugate of -6-24i.

Ans. Here $\overline{-6-24i}=-6+24i$

Now $(x-iy)(3+5i)=-6+24i$

$\Rightarrow 3x+5xi-3yi-5yi^2=-6+24i$

$\Rightarrow (3x+5y)+(5x-3y)i=-6+24i$

Comparing both sides, we have $3x+5y=-6$ and $5x-3y=24$

Solving both equations, we have $x=3$ and $y=-3$

15. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Ans. Here $\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|$ = $\left|\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}\right|$

= $\left|\frac{1+i^2+2i-1-i^2+2i}{1-i^2}\right|$

= $\left|\frac{4i}{2}\right|=|2i|=\sqrt{4}=2$

16. If $(x+iy)^3=u+iv$, **then show that** $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$.

Ans. Given: $(x+iy)^3=u+iv$

$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$

$\Rightarrow (x^3-3xy^2)+(3x^2y-y^3)i=u+iv$

Comparing both sides, we have

$u=x(x^2-3y^2)$ and $v=y(3x^2-y^2)$

Now, $\frac{u}{x}+\frac{v}{y}=\frac{x\left(x^2-3y^2\right)}{x}+\frac{y\left(3x^2-y^2\right)}{y}$

= $x^2-3y^2+3x^2-y^2=4x^2-4y^2=4\left(x^2-y^2\right)$

17. If α and β are different complex numbers with $|\beta|=1$ then find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$.

Ans. Here $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|^2$ = $\left[\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right]\left[\overline{\frac{\beta-\alpha}{1-\overline{\alpha}\beta}}\right]=\left[\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right]\left[\frac{\overline{\beta}-\overline{\alpha}}{1-\alpha\overline{\beta}}\right]$

= $\frac{\beta\overline{\beta}-\beta\overline{\alpha}-\alpha\overline{\beta}+\alpha\overline{\alpha}}{1-\overline{\alpha}\beta-\alpha\overline{\beta}+|\overline{\alpha}|^2|\beta|^2}$

= $\frac{|\beta|^2-\overline{\alpha}\beta-\alpha\overline{\beta}+|\alpha|^2}{1-\overline{\alpha}\beta-\alpha\overline{\beta}+|\alpha|^2|\beta|^2}$

= $\frac{1-\overline{\alpha}\beta-\alpha\overline{\beta}+|\alpha|^2}{1-\overline{\alpha}\beta-\alpha\overline{\beta}+|\alpha|^2}=1$

$\therefore \left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|=1$

18. Find the number of non-zero integral solutions of the equation $|1-i|^x=2^x$.

Ans. Here $|1-i|^x=2^x$

$\Rightarrow |\sqrt{1^2+(-1)^2}|^x=2^x$

$\Rightarrow \left[\sqrt{2}\right]^x=2^x$

$\Rightarrow 2^{\frac{x}{2}}=2^x$

$\Rightarrow \frac{x}{2} = x$

$\Rightarrow \frac{x}{2} - x = 0$

$\Rightarrow \frac{-x}{2} = 0$

$\Rightarrow x = 0$

19. If $(a+ib)(c+id)(e+if)(g+ih) = A + iB$ **then show that:**

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2 + B^2$.

Ans. Given: $(a+ib)(c+id)(e+if)(g+ih) = A + iB$

Taking modulus on both sides,

$|(a+ib)(c+id)(e+if)(g+ih)| = |A + iB|$

$\Rightarrow |(a+ib)||(c+id)||(e+if)||(g+ih)| = |A + iB|$

$\Rightarrow \left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(\sqrt{g^2+h^2}\right) = \left(\sqrt{A^2+B^2}\right)$

Squaring both sides, we get

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2 + B^2$

20. If $\left[\frac{1+i}{1-i}\right]^m = 1$, **then find the least positive integral value of** m.

Ans. Given: $\left[\frac{1+i}{1-i}\right]^m = 1$

$\Rightarrow \left[\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right]^{m}=1$

$\Rightarrow \left[\frac{(1+i)^{2}}{1-i^{2}}\right]^{m}=1$

$\Rightarrow \left[\frac{1+i^{2}+2i}{1+1}\right]^{m}=1$

$\Rightarrow \left[\frac{1-1+2i}{2}\right]^{m}=1$

$\Rightarrow \left[\frac{2i}{2}\right]^{m}=1$

$\Rightarrow [i]^{m}=1$

$\Rightarrow [i]^{m}=1^{4}$

$\Rightarrow m=4$