

CBSE Class-12 Mathematics

NCERT solution

Chapter - 5

Continuity & Differentiability - Exercise 5.5

Differentiate the functions with respect to x in Exercise 1 to 5.

1. $\cos x \cos 2x \cos 3x$

Ans. Let $y = \cos x \cos 2x \cos 3x$ (i)

Taking logs on both sides, we have

$$\Rightarrow \log y = \log (\cos x \cos 2x \cos 3x)$$

$$\Rightarrow \log y = \log \cos x + \log \cos 2x + \log \cos 3x$$

$$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} \log \cos x + \frac{d}{dx} \log \cos 2x + \frac{d}{dx} \log \cos 3x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{\cos x} \frac{d}{dx} \cos x + \frac{1}{\cos 2x} \frac{d}{dx} \cos 2x + \frac{1}{\cos 3x} \frac{d}{dx} \cos 3x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{\cos x} (-\sin x) + \frac{1}{\cos 2x} (-\sin 2x) \frac{d}{dx} 2x + \frac{1}{\cos 3x} (-\sin 3x) \frac{d}{dx} 3x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\tan x - (\tan 2x) 2 - \tan 3x(3)$$

$$\Rightarrow \frac{dy}{dx} = -y(\tan x + 2 \tan 2x + 3 \tan 3x)$$

$$\Rightarrow \frac{dy}{dx} = -\cos x \cos 2x \cos 3x (\tan x + 2 \tan 2x + 3 \tan 3x)$$

2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Ans. Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \left(\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)} \right)^{\frac{1}{2}}$ (i)

Taking logs on both sides, we have

$$\Rightarrow \log y = \frac{1}{2} [\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \left[\frac{1}{x-1} \frac{d}{dx}(x-1) + \frac{1}{x-2} \frac{d}{dx}(x-2) - \frac{1}{x-3} \frac{d}{dx}(x-3) - \frac{1}{x-4} \frac{d}{dx}(x-4) - \frac{1}{x-5} \frac{d}{dx}(x-5) \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} y \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5} \right]$$

3. $(\log x)^{\cos x}$

Ans. Let $y = (\log x)^{\cos x}$ (i)

Taking logs on both sides, we have

$$\Rightarrow \log y = \log (\log x)^{\cos x} = \cos x \log (\log x)$$

$$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} [\cos x \log (\log x)]$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \cos x \frac{d}{dx} \log (\log x) + \log (\log x) \frac{d}{dx} \cos x \quad [\text{By Product rule}]$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \cos x \frac{1}{\log x} \frac{d}{dx} (\log x) + \log (\log x) (-\sin x)$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{\cos x}{\log x} \cdot \frac{1}{x} - \sin x \log (\log x)$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{\cos x}{x \log x} - \sin x \log (\log x) \right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x} \left[\frac{\cos x}{x \log x} - \sin x \log (\log x) \right]$$

4. $x^x - 2^{\sin x}$

Ans. Let $y = x^x - 2^{\sin x}$

Putting $u = x^x$ and $v = 2^{\sin x}$

$$\Rightarrow y = u - v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \dots\dots\dots(i)$$

Now, $u = x^x$

$$\Rightarrow \log u = \log x^x = x \log x$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} (x \log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{d}{dx} \log x + \log x \frac{d}{dx} x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = 1 + \log x$$

$$\Rightarrow \frac{du}{dx} = u(1 + \log x)$$

$$\Rightarrow \frac{du}{dx} = x^x (1 + \log x) \dots\dots\dots(ii)$$

Again, $v = 2^{\sin x}$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx} 2^{\sin x}$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \log 2 \frac{d}{dx} \sin x \left[\because \frac{d}{dx} a^{f(x)} = a^{f(x)} \log a \frac{d}{dx} f(x) \right]$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x} (\log 2) \cdot \cos x = \cos x \cdot 2^{\sin x} \log 2 \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i),

$$\frac{dy}{dx} = x^x (1 + \log x) - \cos x \cdot 2^{\sin x} \log 2$$

5. $(x+3)^2 (x+4)^3 (x+5)^4$

Ans. Let $y = (x+3)^2 (x+4)^3 (x+5)^4 \dots\dots\dots(i)$

Taking logs on both sides, we have

$$\Rightarrow \log y = 2 \log(x+3) + 3 \log(x+4) + 4 \log(x+5)$$

$$\Rightarrow \frac{d}{dx} \log y = 2 \frac{d}{dx} \log(x+3) + 3 \frac{d}{dx} \log(x+4) + 4 \frac{d}{dx} \log(x+5)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = 2 \frac{1}{x+3} \frac{d}{dx} (x+3) + 3 \frac{1}{x+4} \frac{d}{dx} (x+4) + 4 \frac{1}{x+5} \frac{d}{dx} (x+5)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}$$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= y \left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5} \right) \\ \Rightarrow \frac{dy}{dx} &= (x+3)^2 (x+4)^3 (x+5)^4 \left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5} \right) \\ \Rightarrow \frac{dy}{dx} &= (x+3)^2 (x+4)^3 (x+5)^4 \left(\frac{2(x+4)(x+5) + 3(x+3)(x+5) + 4(x+3)(x+4)}{(x+3)(x+4)(x+5)} \right) \\ \Rightarrow \frac{dy}{dx} &= (x+3)^2 (x+4)^3 (x+5)^4 \left(\frac{2x^2 + 18x + 40 + 3x^2 + 24x + 45 + 4x^2 + 28x + 48}{(x+3)(x+4)(x+5)} \right) \\ \Rightarrow \frac{dy}{dx} &= (x+3)(x+4)^2 (x+5)^3 (9x^2 + 70x + 133)\end{aligned}$$

Differentiate the functions with respect to x in Exercise 6 to 11.

6. $\left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$

Ans. Let $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$

Putting $\left(x + \frac{1}{x}\right)^x = u$ and $x^{\left(1 + \frac{1}{x}\right)} = v$

$$y = u + v$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = \left(x + \frac{1}{x}\right)^x$

$$\Rightarrow \log u = \log \left(x + \frac{1}{x}\right)^x = x \log \left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \cdot \frac{1}{\left(x + \frac{1}{x}\right)} \frac{d}{dx} \left(x + \frac{1}{x}\right) + \log \left(x + \frac{1}{x}\right) \cdot 1$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\left(x + \frac{1}{x}\right)} \left(1 - \frac{1}{x^2}\right) + \log\left(x + \frac{1}{x}\right) \cdot 1$$

$$\Rightarrow \frac{du}{dx} = u \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right) \right]$$

$$= \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right) \right] \dots\dots\dots(ii)$$

Again $v = x^{\left(1 + \frac{1}{x}\right)}$

$$\Rightarrow \log v = \log x^{\left(1 + \frac{1}{x}\right)} = \left(1 + \frac{1}{x}\right) \log x$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \left(1 + \frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(\frac{-1}{x^2}\right)$$

$$\Rightarrow \frac{dv}{dx} = v \left[\left(1 + \frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(\frac{-1}{x^2}\right) \right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1 + \frac{1}{x}\right)} \left[\frac{1}{x} \left(1 + \frac{1}{x}\right) - \frac{1}{x^2} \log x \right] \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i),

$$\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right) \right] + x^{\left(1 + \frac{1}{x}\right)} \left[\frac{1}{x} \left(1 + \frac{1}{x}\right) - \frac{1}{x^2} \log x \right]$$

7. $(\log x)^x + x^{\log x}$

Ans. Let $y = (\log x)^x + x^{\log x} = u + v$

where $u = (\log x)^x$ and $v = x^{\log x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = (\log x)^x$

$$\Rightarrow \log u = \log (\log x)^x = x \log (\log x)$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} [x \log (\log x)]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{d}{dx} [\log (\log x)] + \log (\log x) \frac{d}{dx} x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{1}{\log x} \frac{d}{dx} \log x + \log (\log x) \cdot 1$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{1}{\log x} \frac{1}{x} + \log (\log x)$$

$$\Rightarrow \frac{du}{dx} = u \left[\frac{1}{\log x} + \log (\log x) \right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log (\log x) \right] \dots\dots\dots(ii)$$

Again $v = x^{\log x}$

$$\Rightarrow \log v = \log x^{\log x} = \log x \cdot \log x = (\log x)^2$$

$$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} (\log x)^2$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = 2 \log x \frac{d}{dx} (\log x)$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = 2 \log x \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = v \left(\frac{2}{x} \log x \right) = x^{\log x} \cdot \frac{2}{x} \log x$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x - 1} \log x \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i),

$$\frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + 2x^{\log x - 1} \log x$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^x \left[\frac{1 + \log x \log(\log x)}{\log x} \right] + 2x^{\log x - 1} \log x$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{x-1} (1 + \log x \log(\log x)) + 2x^{\log x - 1} \log x$$

8. $(\sin x)^x + \sin^{-1} \sqrt{x}$

Ans. Let $y = (\sin x)^x + \sin^{-1} \sqrt{x} = u + v$

where $u = (\sin x)^x$ and $v = \sin^{-1} \sqrt{x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = (\sin x)^x$

$$\Rightarrow \log u = \log (\sin x)^x = x \log (\sin x)$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} [x \log (\sin x)]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{d}{dx} [\log (\sin x)] + \log (\sin x) \frac{d}{dx} x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \frac{1}{\sin x} \frac{d}{dx} \sin x + \log (\sin x) \cdot 1$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\sin x} \cos x + \log(\sin x) = x \cot x + \log \sin x$$

$$\Rightarrow \frac{du}{dx} = u [x \cot x + \log \sin x]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x] \dots\dots\dots(ii)$$

Again $v = \sin^{-1} \sqrt{x}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx} \sqrt{x} \left[\because \frac{d}{dx} \sin^{-1} f(x) = \frac{1}{\sqrt{1-(f(x))^2}} \frac{d}{dx} f(x) \right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{x-x^2}} \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i),

$$\frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x-x^2}}$$

9. $x^{\sin x} + (\sin x)^{\cos x}$

Ans. Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Putting $u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$, we get $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = x^{\sin x}$

$$\Rightarrow \log u = \log x^{\sin x} = \sin x \log x$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} (\sin x \log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \sin x \frac{d}{dx} \log x + \log x \frac{d}{dx} \sin x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \sin x \frac{1}{x} + \log x (\cos x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \frac{\sin x}{x} + \cos x \log x$$

$$\Rightarrow \frac{du}{dx} = u \left(\frac{\sin x}{x} + \cos x \log x \right)$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \log x \right) \dots (ii)$$

Again $v = (\sin x)^{\cos x}$

$$\Rightarrow \log v = \log (\sin x)^{\cos x} = \cos x \log \sin x$$

$$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} [\cos x \log (\sin x)]$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \cos x$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \cos x \frac{1}{\sin x} \frac{d}{dx} \sin x + \log \sin x (-\sin x)$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \cot x \cdot \cos x - \sin x \log \sin x$$

$$\Rightarrow \frac{dv}{dx} = v(\cot x \cos x - \sin x \log \sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} (\cot x \cos x - \sin x \log \sin x) \dots\dots\dots(iii)$$

Putting values from eq. (ii) and (iii) in eq. (i),

$$\frac{dy}{dx} = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \log x \right) + (\sin x)^{\cos x} (\cot x \cos x - \sin x \log \sin x)$$

10. $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$

Ans. Let $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$

Putting $u = x^{x \cos x}$ and $v = \frac{x^2 + 1}{x^2 - 1}$,

we have $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = x^{x \cos x}$

$$\Rightarrow \log u = \log x^{x \cos x} = x \cos x \log x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \frac{d}{dx} (x \cos x \log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \frac{d}{dx} (x) \cdot \cos x \log x + x \frac{d}{dx} (\cos x) \log x + x \cos x \frac{d}{dx} (\log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = 1 \cdot \cos x \log x + x(-\sin x) \log x + x \cos x \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = u (\cos x \log x - x \sin x \log x + \cos x)$$

$$\Rightarrow \frac{du}{dx} = x^{x \cos x} (\cos x \log x - x \sin x \log x + \cos x) \dots\dots\dots(ii)$$

Again $v = \frac{x^2 + 1}{x^2 - 1}$

$$\Rightarrow \frac{dv}{dx} = \frac{(x^2 - 1) \frac{d}{dx}(x^2 + 1) - (x^2 + 1) \frac{d}{dx}(x^2 - 1)}{(x^2 - 1)^2}$$

$$\Rightarrow \frac{dv}{dx} = \frac{(x^2 - 1) 2x - (x^2 + 1) 2x}{(x^2 - 1)^2}$$

$$\Rightarrow \frac{dv}{dx} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2}$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{(x^2 - 1)^2} \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i),

$$\Rightarrow \frac{dy}{dx} = x^{x \cos x} (\cos x \log x - x \sin x \log x + \cos x) + \frac{-4x}{(x^2 - 1)^2}$$

11. $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

Ans. Let $y = (x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

Putting $u = (x \cos x)^x$ and $v = (x \sin x)^{\frac{1}{x}}$,

we have $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(i)$$

Now $u = (x \cos x)^x$

$$\Rightarrow \log u = \log (x \cos x)^x = x \log (x \cos x)$$

$$\Rightarrow \log u = x(\log x + \log \cos x)$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} \{x(\log x + \log \cos x)\}$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \left[\frac{1}{x} + \frac{1}{\cos x} \cdot (-\sin x) \right] + (\log x + \log \cos x) \cdot 1$$

$$\Rightarrow \frac{du}{dx} = u [1 - x \tan x + \log (x \cos x)]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x [1 - x \tan x + \log (x \cos x)] \dots\dots\dots(ii)$$

Again $v = (x \sin x)^{\frac{1}{x}}$

$$\Rightarrow \log v = \log (x \sin x)^{\frac{1}{x}} = \frac{1}{x} \log (x \sin x)$$

$$\Rightarrow \log v = \frac{1}{x} (\log x + \log \sin x)$$

$$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} \left\{ \frac{1}{x} (\log x + \log \sin x) \right\}$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \frac{1}{x} \left[\frac{1}{x} + \frac{1}{\sin x} \cdot \cos x \right] + (\log x + \log \sin x) \left(\frac{-1}{x^2} \right)$$

$$\Rightarrow \frac{dv}{dx} = v \left[\frac{1}{x^2} + \frac{\cot x}{x} - \frac{\log(x \sin x)}{x^2} \right]$$

$$\Rightarrow \frac{dv}{dx} = (x \sin x)^{\frac{1}{x}} \left[\frac{1}{x^2} + \frac{\cot x}{x} - \frac{\log(x \sin x)}{x^2} \right] \dots\dots\dots(iii)$$

Putting the values from eq. (ii) and (iii) in eq. (i)

$$\frac{dy}{dx} = (x \cos x)^x [1 - x \tan x + \log(x \cos x)] + (x \sin x)^{\frac{1}{x}} \left[\frac{1}{x^2} + \frac{\cot x}{x} - \frac{\log(x \sin x)}{x^2} \right]$$

Find $\frac{dy}{dx}$ in the following Exercise 12 to 15

12. $x^y + y^x = 1$

Ans. Given: $x^y + y^x = 1$

$$\Rightarrow u + v = 1,$$

where $u = x^y$ and $v = y^x$

$$\therefore \frac{d}{dx} u + \frac{d}{dx} v = \frac{d}{dx} 1$$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0 \dots\dots\dots(i)$$

Now $u = x^y$

$$\Rightarrow \log u = \log x^y = y \log x$$

$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} (y \log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = y \frac{d}{dx} \log x + \log x \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = u \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{du}{dx} = x^y \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) = x^y \frac{y}{x} + x^y \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = x^{y-1} y + x^y \log x \cdot \frac{dy}{dx} \dots\dots\dots(ii)$$

Again $v = y^x$

$$\Rightarrow \log v = \log y^x = x \log y$$

$$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} (x \log y)$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = x \frac{d}{dx} \log y + \log y \frac{d}{dx} x$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1$$

$$\Rightarrow \frac{dv}{dx} = v \left(\frac{x}{y} \frac{dy}{dx} + \log y \right)$$

$$\Rightarrow \frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = y^x \frac{x}{y} \frac{dy}{dx} + y^x \log y$$

$$\Rightarrow \frac{dv}{dx} = y^{x-1} x \frac{dy}{dx} + y^x \log y \dots\dots\dots(iii)$$

Putting values from eq. (ii) and (iii) in eq. (i),

$$\Rightarrow x^{y-1} y + x^y \log x \cdot \frac{dy}{dx} + y^{x-1} x \frac{dy}{dx} + y^x \log y = 0$$

$$\Rightarrow \frac{dy}{dx} (x^y \log x + y^{x-1} x) = -x^{y-1} y - y^x \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x^{y-1} y - y^x \log y)}{x^y \log x + y^{x-1} x}$$

13. $y^x = x^y$

Ans. Given: $y^x = x^y$

$$\Rightarrow x^y = y^x$$

$$\Rightarrow \log x^y = \log y^x$$

$$\Rightarrow y \log x = x \log y$$

$$\Rightarrow \frac{d}{dx} (y \log x) = \frac{d}{dx} (x \log y)$$

$$\Rightarrow y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$$

$$\Rightarrow \left(\log x - \frac{x}{y} \right) \frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\Rightarrow \left(\frac{y \log x - x}{y} \right) \frac{dy}{dx} = \frac{x \log y - y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$$

14. $(\cos x)^y = (\cos y)^x$

Ans. Given: $(\cos x)^y = (\cos y)^x$

$$\Rightarrow \log (\cos x)^y = \log (\cos y)^x$$

$$\Rightarrow y \log \cos x = x \log \cos y$$

$$\Rightarrow \frac{d}{dx}(y \log \cos x) = \frac{d}{dx}(x \log \cos y)$$

$$\Rightarrow y \frac{d}{dx} \log \cos x + \log \cos x \frac{dy}{dx} = x \frac{d}{dx} \log \cos y + \log \cos y \frac{d}{dx} x$$

$$\Rightarrow y \frac{1}{\cos x} \frac{d}{dx} \cos x + \log \cos x \frac{dy}{dx} = x \frac{1}{\cos y} \frac{d}{dx} \cos y + \log \cos y$$

$$\Rightarrow y \frac{1}{\cos x} (-\sin x) + \log \cos x \frac{dy}{dx} = x \frac{1}{\cos y} \left(-\sin y \frac{dy}{dx} \right) + \log \cos y$$

$$\Rightarrow -y \tan x + \log \cos x \frac{dy}{dx} = -x \tan y \frac{dy}{dx} + \log \cos y$$

$$\Rightarrow x \tan y \frac{dy}{dx} + \log \cos x \frac{dy}{dx} = y \tan x + \log \cos y$$

$$\Rightarrow \frac{dy}{dx} (x \tan y + \log \cos x) = y \tan x + \log \cos y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$$

15. $xy = e^{x-y}$

Ans. Given: $xy = e^{x-y}$

$$\Rightarrow \log xy = \log e^{x-y}$$

$$\Rightarrow \log x + \log y = (x-y) \log e$$

$$\Rightarrow \log x + \log y = (x-y) \quad [\because \log e = 1]$$

$$\Rightarrow \frac{d}{dx} \log x + \frac{d}{dx} \log y = \frac{d}{dx} (x - y)$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{1}{y} + 1 \right) = \frac{x-1}{x}$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{1+y}{y} \right) = \frac{x-1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$$

16. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence $f'(1)$.

Ans. Given: $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ (i)

$$\Rightarrow \log f(x) = \log \{(1+x)(1+x^2)(1+x^4)(1+x^8)\}$$

$$\Rightarrow \log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

\Rightarrow

$$\frac{1}{f(x)} \frac{d}{dx} f(x) = \frac{1}{1+x} \frac{d}{dx} (1+x) + \frac{1}{1+x^2} \frac{d}{dx} (1+x^2) + \frac{1}{1+x^4} \frac{d}{dx} (1+x^4) + \frac{1}{1+x^8} \frac{d}{dx} (1+x^8)$$

$$\Rightarrow \frac{1}{f(x)} f'(x) = \frac{1}{1+x} \cdot 1 + \frac{1}{1+x^2} \cdot 2x + \frac{1}{1+x^4} \cdot 4x^3 + \frac{1}{1+x^8} \cdot 8x^7$$

$$\Rightarrow f'(x) = f(x) \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \right]$$

Putting the value of $f(x)$ from eq. (i),

$$f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8) \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \right]$$

$$\Rightarrow f'(1) = (1+1)(1+1^2)(1+1^4)(1+1^8) \left[\frac{1}{1+1} + \frac{2 \times 1}{1+1^2} + \frac{4 \times 1^3}{1+1^4} + \frac{8 \times 1^7}{1+1^8} \right]$$

$$\Rightarrow f'(1) = (2)(2)(2)(2) \left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2} \right]$$

$$\Rightarrow f'(1) = 16 \left[\frac{15}{2} \right]$$

$$\Rightarrow 8 \times 15 = 120$$

17. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:

(i) by using product rule.

(ii) by expanding the product to obtain a single polynomial

(iii) by logarithmic differentiation.

Do they all give the same answer?

Ans. Let $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$ (i)

$$(i) \frac{dy}{dx} = (x^2 - 5x + 8) \frac{d}{dx}(x^3 + 7x + 9) + (x^3 + 7x + 9) \frac{d}{dx}(x^2 - 5x + 8)$$

$$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(3x^2 + 7) + (x^3 + 7x + 9)(2x - 5)$$

$$\Rightarrow \frac{dy}{dx} = 3x^4 + 7x^2 - 15x^3 - 35x + 24x^2 + 56 + 2x^4 - 5x^3 + 14x^2 - 35x + 18x - 45$$

$$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \dots\dots\dots(ii)$$

$$(ii) \ y = (x^2 - 5x + 8)(x^3 + 7x + 9)$$

$$\Rightarrow y = x^5 + 7x^3 + 9x^2 - 5x^4 - 35x^2 - 45x + 8x^3 + 56x + 72$$

$$\Rightarrow y = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$$

$$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \dots\dots\dots(iii)$$

$$(iii) \ y = (x^2 - 5x + 8)(x^3 + 7x + 9)$$

$$\Rightarrow \log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$$

$$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} \log(x^2 - 5x + 8) + \frac{d}{dx} \log(x^3 + 7x + 9)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2 - 5x + 8} \frac{d}{dx} (x^2 - 5x + 8) + \frac{1}{x^3 + 7x + 9} \frac{d}{dx} (x^3 + 7x + 9)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2 - 5x + 8} (2x - 5) + \frac{1}{x^3 + 7x + 9} (3x^2 + 7)$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{2x - 5}{x^2 - 5x + 8} + \frac{3x^2 + 7}{x^3 + 7x + 9} \right]$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{2x - 5}{x^2 - 5x + 8} + \frac{3x^2 + 7}{x^3 + 7x + 9} \right]$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{(2x - 5)(x^3 + 7x + 9) + (3x^2 + 7)(x^2 - 5x + 8)}{(x^2 - 5x + 8)(x^3 + 7x + 9)} \right]$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{2x^4 + 14x^2 + 18x - 5x^3 - 35x - 45 + 3x^4 - 15x^3 + 24x^2 + 7x^2 - 35x + 56}{(x^2 - 5x + 8)(x^3 + 7x + 9)} \right]$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{5x^4 - 20x^3 + 45x^2 - 52x + 11}{(x^2 - 5x + 8)(x^3 + 7x + 9)} \right]$$

$$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9) \left[\frac{5x^4 - 20x^3 + 45x^2 - 52x + 11}{(x^2 - 5x + 8)(x^3 + 7x + 9)} \right] \text{ [From eq. (i)]}$$

$$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \dots\dots\dots(\text{iv})$$

From eq. (ii), (iii) and (iv), we can say that value of $\frac{dy}{dx}$ is same obtained by three different methods.

18. If u, v and w are functions of x , then show that

$\frac{d}{dx}(u.v.w) = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$ **in two ways – first by repeated application of product rule, second by logarithmic differentiation.**

Ans. Given: u, v and w are functions of x .

To prove: $\frac{d}{dx}(u.v.w) = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

(i) By repeated application of product rule:

$$\text{L.H.S.} = \frac{d}{dx}(u.v.w)$$

$$= \frac{d}{dx}[(uv).w]$$

$$= uv \frac{d}{dx} w + w \frac{d}{dx}(uv)$$

$$\begin{aligned}
 &= uv \frac{dw}{dx} + w \left[u \frac{dv}{dx} + v \frac{du}{dx} \right] \\
 &= uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx} \\
 &= \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}
 \end{aligned}$$

= R.H.S Hence proved.

(ii) By Logarithmic differentiation:

Let $y = uvw$

$$\Rightarrow \log y = \log (u \cdot v \cdot w)$$

$$\Rightarrow \log y = \log u + \log v + \log w$$

$$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} \log u + \frac{d}{dx} \log v + \frac{d}{dx} \log w$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx}$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right]$$

Putting $y = uvw$, we get

$$\frac{d}{dx} (u \cdot v \cdot w) = uvw \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right]$$

$$\Rightarrow \frac{d}{dx} (u \cdot v \cdot w) = \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx} \text{ Hence proved.}$$