

CBSE Class-12 Mathematics

NCERT solution

Chapter - 4

Determinants - Exercise 4.2

Using the properties of determinants and without expanding in Exercise 1 to 7, prove that:

$$1. \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Ans. Given:  $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

Operating  $C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0$$

$$\Rightarrow 0 = 0 \text{ [}\because C_1 \text{ and } C_2 \text{ are identical]}$$

$$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$$

$$2. \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Ans. 
$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Operating  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} = 0 = \text{R.H.S.}$$

[ $\therefore$  All entries of one column here first are zero]

3. 
$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Ans. 
$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix},$$

operating  $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix}$$

$$= 9 \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$

$= 9 \times 0 = 0$  [∵ two columns are identical] Proved.

$$4. \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

$$\text{Ans. } \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ba \\ 1 & ab & ca+cb \end{vmatrix}$$

Operating  $C_3 \rightarrow C_3 + C_2$

$$\begin{vmatrix} 1 & bc & ab+ab+ac \\ 1 & ca & ab+ab+ac \\ 1 & ab & ab+ab+ac \end{vmatrix}$$

$$= (ab+ab+ac) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$= (ab+ab+ac)(0) = 0$  [∵ two columns are identical] Proved.

$$5. \begin{vmatrix} (b+c) & q+r & y+z \\ (c+a) & r+p & z+x \\ (a+b) & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$\text{Ans. L.H.S.} = \begin{vmatrix} (b+c) & q+r & y+z \\ (c+a) & r+p & z+x \\ (a+b) & p+q & x+y \end{vmatrix}$$

operating  $R_1 \rightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} b+c+c+a+a+b & q+r+r+p+p+q & y+z+z+x+x+y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} (a+b+c) & (p+q+r) & (x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

[operating  $R_1 \rightarrow R_1 - R_2$ ]

$$= 2 \begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

[operating  $R_3 \rightarrow R_3 - R_1$ ]

$$= 2 \begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$$

[operating  $R_2 \rightarrow R_2 - R_3$ ]

$$= 2 \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$$

[Interchanging  $R_2$  and  $R_3$ ]

$$= -2 \begin{vmatrix} b & q & y \\ a & p & x \\ c & r & z \end{vmatrix}$$

[Interchanging  $R_2$  and  $R_3$ ]

$$= -(-2) \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$= 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{R.H.S.}$$

$$6. \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

$$\text{Ans. Let } \Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

[Taking  $(-1)$  common from each row]

$$\Rightarrow \Delta = (-1)^3 \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

Interchanging rows and columns in the determinants on R.H.S.,

$$\Delta = - \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = -\Delta$$

$$\Rightarrow \Delta + \Delta = 0$$

$$\Rightarrow 2\Delta = 0$$

$$\Rightarrow \Delta = 0 \text{ Proved.}$$

$$7. \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

$$\text{Ans. L.H.S.} = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

Taking common  $a, b, c$  from  $R_1, R_2, R_3$  respectively,

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & b \\ a & b & -c \end{vmatrix}$$

[operating  $R_1 \rightarrow R_1 + R_2$ ]

$$\begin{aligned}
 &= abc \begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix} \\
 &= abc \cdot 2c \begin{vmatrix} a & -b \\ a & b \end{vmatrix} \\
 &= abc \cdot 2c (ab + ab) \\
 &= abc \cdot 2c \cdot 2ab = 4a^2b^2c^2 = \text{R.H.S.}
 \end{aligned}$$

$$\text{8. (i)} \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$\text{(ii)} \quad \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

$$\text{Ans. (i) L.H.S.} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1,$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 1 & c-a & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & (b-a)(b+a) \\ 0 & c-a & (c-a)(c+a) \end{vmatrix}$$

Taking (b-a) and (c-a) common from  $R_2$  and  $R_3$  respectively.

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & (b+a) \\ 0 & 1 & (c+a) \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_3$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 0 & (b-c) \\ 0 & 1 & (c+a) \end{vmatrix}$$

[Expanding along 1<sup>st</sup> column]

$$= (b-a)(c-a) \begin{vmatrix} 0 & (b+a) \\ 1 & (c+a) \end{vmatrix}$$

$$= (b-a)(c-a)(0 - (b-c))$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a) = \text{R.H.S. Proved.}$$

$$\text{(ii) L.H.S.} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

operating  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} b-a & c-a \\ b^3-a^3 & c^3-a^3 \end{vmatrix}$$



$$\begin{aligned}
 &= 1 \begin{vmatrix} b-a & c-a \\ (b-a)(b^2+a^2+ab) & (c-a)(c^2+a^2+ac) \end{vmatrix} \\
 &= (b-a)(c-a) \begin{vmatrix} 1 & 1 \\ (b^2+a^2+ab) & (c^2+a^2+ac) \end{vmatrix} \\
 &= (b-a)(c-a)(c^2+a^2+ac-b^2-a^2-ab) \\
 &= (b-a)(c-a)(c^2-b^2+ac-ab) \\
 &= (b-a)(c-a)[(c-b)(c+b)+a(c-b)] \\
 &= (b-a)(c-a)(c-b)(c+b+a) \\
 &= -(a-b)(c-a)[-(b-c)(c+b+a)] \\
 &= (a-b)(b-c)(c-a)(a+b+c) = \text{R.H.S.}
 \end{aligned}$$

$$9. \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$\text{Ans. } \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$\text{L.H.S.} = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$$

[Multiplying  $R_1, R_2, R_3$  by  $x, y, z$  respectively]

$$= \frac{1}{xyz} \begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$

$$= \begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$

Taking xyz common from  $C_3$

$$= \frac{xyz}{xyz} \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

[operating  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ ]

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{vmatrix}$$

$$= 1 \begin{vmatrix} y^2 - x^2 & y^3 - x^3 \\ z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

$$= \begin{vmatrix} (y-x)(y+x) & (y-x)(y^2+x^2+yx) \\ (z-x)(z+x) & (z-x)(z^2+x^2+zx) \end{vmatrix}$$

$$\begin{aligned}
 &= (y-x)(z-x) \begin{vmatrix} y+x & y^2+x^2+yx \\ z+x & z^2+x^2+zx \end{vmatrix} \\
 &= (y-x)(z-x) \left[ (y+x)(z^2+x^2+zx) - (z+x)(y^2+x^2+xy) \right] \\
 &= (y-x)(z-x) \left[ yz^2+yx^2+xyz+xz^2+x^3+x^2z-zy^2-zx^2-xyz-xy^2-x^3-x^2y \right] \\
 &= (y-x)(z-x) \left[ yz^2-zy^2+xz^2-xy^2 \right] \\
 &= (y-x)(z-x) \left[ yz(z-y) + x(z^2-y^2) \right] \\
 &= (y-x)(z-x) \left[ yz(z-y) + x(z-y)(z+y) \right] \\
 &= (y-x)(z-x)(z-y) \left[ yz + x(z+y) \right] \\
 &= -(x-y)(z-x) \left[ -(y-z) \right] [yz+xz+xy] \\
 &= (x-y)(y-z)(z-x)(xy+yz+zx) = \text{R.H.S.}
 \end{aligned}$$

$$10. (i) \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

$$(ii) \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

$$\text{Ans. (i) L.H.S.} = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$[R_1 \rightarrow R_1 + R_2 + R_3]$$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Taking  $5x+4$  common from  $R_1$

$$= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

[operating  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$ ]

$$= (5x+4) \begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

$$= (5x+4) \cdot 1 \begin{vmatrix} 4-x & 0 \\ 0 & 4-x \end{vmatrix}$$

$$= (5x+4)(4-x)^2 = \text{R.H.S.}$$

$$\text{(ii) L.H.S.} = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & 2x & y+k \end{vmatrix}$$

$[C_1 \rightarrow C_1 + C_2 + C_3]$

$$= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

Taking  $3y+k$  common from  $C_1$

$$= (3y+k) \begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

[operating  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$ ]

$$= (3y+k) \begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

$$= (3y+k) \cdot 1 \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

$$= (3y+k)k^2 = k^2(3y+k)$$

= R.H.S. Proved.

$$11. (i) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$(ii) \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

$$\text{Ans. (i) L.H.S.} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$[R_1 \rightarrow R_1 + R_2 + R_3]$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking  $a+b+c$  common from  $R_1$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$[C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c) \cdot 1 \cdot \begin{vmatrix} -b-c-a & 0 \\ 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c) \{-(b+c+a)\} \{-(c+a+b)\}$$

$$= (a+b+c)^3 = \text{R.H.S. Proved.}$$

$$\text{(ii) L.H.S.} = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

$[C_1 \rightarrow C_1 + C_2 + C_3]$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

Taking  $2(x+y+z)$  common from  $C_1$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

$[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z) \cdot 1 \cdot \begin{vmatrix} x+y+z & 0 \\ 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z) [(x+y+z)^2 - 0]$$

$$= 2(x+y+z)^3 = \text{R.H.S. Proved.}$$

$$12. \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

$$\text{Ans. L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$[R_1 \rightarrow R_1 + R_2 + R_3]$

$$= \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}^T$$

Taking  $1+x+x^2$  common from  $R_1$

$$= (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$[C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$

$$= (1+x+x^2) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2) \cdot 1 \cdot \begin{vmatrix} 1-x^2 & x-x^2 \\ x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2) \cdot 1 \cdot \begin{vmatrix} (1-x)(1+x) & x(1-x) \\ -x(1-x) & 1-x \end{vmatrix}$$

$$= (1+x+x^2) [(1-x)^2(1+x) + x^2(1-x)^2]$$

$$= (1+x+x^2)(1-x)^2(1+x+x^2)$$

$$= (1+x+x^2)^2(1-x)^2$$

$$= [(1+x+x^2)(1-x)]^2$$

$$= (1-x+x-x^2+x^2-x^3)^2$$



$$= (1-x^3)^2 = \text{R.H.S.} \quad \text{Proved.}$$

$$13. \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

$$\text{Ans. L.H.S.} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$[C_1 \rightarrow C_1 - b C_3 \text{ and } C_2 \rightarrow C_2 + a C_3]$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

$$[R_3 \rightarrow R_3 - b R_1]$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 2a \\ -a & 1-a^2+b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 (1-a^2+b^2+2a^2)$$

$$= (1 + a^2 + b^2)^3 = \text{R.H.S.}$$

$$14. \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

$$\text{Ans. L.H.S.} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Multiplying  $C_1, C_2, C_3$  by  $a, b, c$  respectively and then dividing the determinant by  $abc$ ,

$$= \frac{1}{abc} \begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix}$$

Taking  $a, b$ , and  $c$  common from  $R_1, R_2$  and  $R_3$  respectively.

$$= \frac{abc}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

$$[C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= \frac{abc}{abc} \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2 + 1 & c^2 \\ 1 & b^2 & c^2 + 1 \end{vmatrix}$$

$$[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (1 + a^2 + b^2 + c^2)(1)(1 - 0)$$

$$= 1 + a^2 + b^2 + c^2 = \text{R.H.S.} \quad \text{Proved.}$$

Choose the correct answer in Exercises 15 and 16.

15. Let A be a square matrix of order  $3 \times 3$ , then  $|kA|$  is equal to:

(A)  $k|A|$

(B)  $k^2|A|$

(C)  $k^3|A|$

(D)  $3k|A|$

**Ans.** Let  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$  be a square matrix of order  $3 \times 3$ . .....(i)

$$\therefore kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

$$\Rightarrow |kA| = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$$

$$\Rightarrow |kA| = k^3 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= k^3 |A| \text{ [From eq. (i)]}$$

Therefore, option (C) is correct.

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**16. Which is the following is correct:**

- (A) Determinant is a square matrix.**
- (B) Determinant is a number associated to a matrix.**
- (C) Determinant is a number associated to a square matrix.**
- (D) None of these.**

**Ans.** Since, Determinant is a number associated to a square matrix.

Therefore, option (C) is correct.