CBSE Class 9 Mathematics

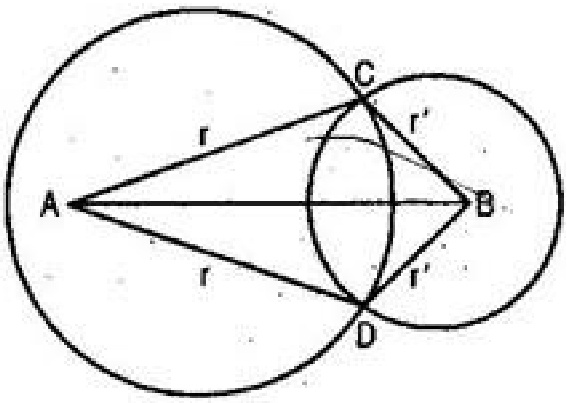
NCERT Solutions

CHAPTER 10

Circles(Ex. 10.6)

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Ans. Let two circles with respective centers A and B intersect each other at points C and D.



We have to prove $\angle$ACB = $\angle$ADB

Proof: In triangles ABC and ABD,

AC = AD = r

BC = BD = r'

AB = AB [Common]

$\therefore$ ΔABC $\cong$ ΔABD

[SSS rule of congruency]

$\Rightarrow$ $\angle$ACB = $\angle$ADB [By C.P.C.T.]

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD

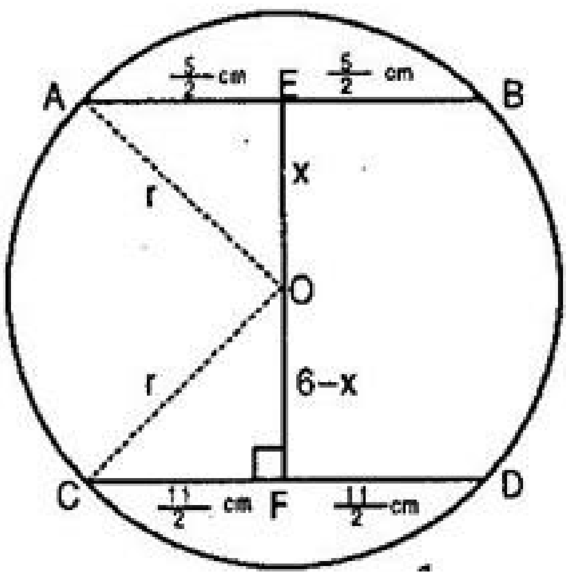
is 6 cm, find he radius of the circle.

Ans. Let O be the centre of the circle. Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2} \times AB = \frac{1}{2} \times 5 = \frac{5}{2}$ cm

And CF = FD = $\frac{1}{2} \times CD = \frac{1}{2} \times 11 = \frac{11}{2}$ cm



Let OE = x

$\therefore$ OF = 6 - x

Let radius of the circle be r.

In right angled triangle AEO,

$AO^2 = AE^2 + OE^2$

[Using Pythagoras theorem]

$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ....(i)

Again In right angled triangle CFO,

$OC^2 = CF^2 + OF^2$

[Using Pythagoras theorem]

$$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2 \text{....(ii)}$$

Equating eq. (i) and (ii),

$$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$$

$$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$$

$$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$$

$$\Rightarrow 12x = \frac{96}{4} + 36$$

$$\Rightarrow 12x = 24 + 36$$

$$\Rightarrow 12x = 60$$

$$\Rightarrow x = 5$$

Now from eq. (i),

$$r^2 = \frac{25}{4} + x^2$$

$$\Rightarrow r^2 = \frac{25}{4} + 5^2$$

$$\Rightarrow r^2 = \frac{125}{4}$$

$\Rightarrow r = \frac{5\sqrt{5}}{2}$ cm

Hence radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

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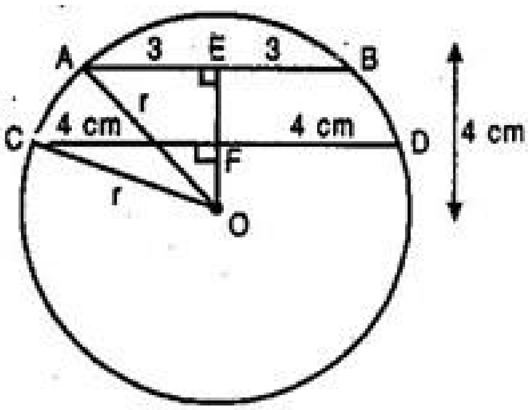
3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord form the centre?

Ans. Let AB = 6 cm and CD = 8 cm are the chords of circle with centre O. Join OA and OC.



Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2}$ x 6 = 3 cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2}$ x 8 = 4 cm

Perpendicular distance of chord AB from the centre O is OE.

$\therefore$ OE = 4 cm

Now in right angled triangle AOE,

$OA^2 = AE^2 + OE^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 3^2 + 4^2$

$\Rightarrow r^2 = 9 + 16 = 25$

$\Rightarrow r = 5$ cm

Perpendicular distance of chord CD from the center O is OF.

Now in right angled triangle OFC,

$OC^2 = CF^2 + OF^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 4^2 + (\mathrm{OF})^2$

$\Rightarrow 5^2 = 4^2 + (\mathrm{OF})^2$

$\Rightarrow 25 = 16 + (\mathrm{OF})^2$

$\Rightarrow OF^2 = 9$

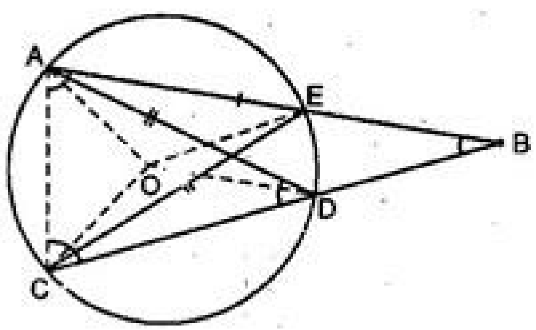
$\Rightarrow OF = 3$cm

Hence distance of other chord from the centre is 3 cm.

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect chords AD and CE with the circle. Prove that $\angle$ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Ans. Vertex B of $\angle$ABC is located outside the circle with centre O.

Side AB intersects chord CE at point E and side BC intersects chord AD at point D with the circle.



We have to prove that

$\angle ABC = \frac{1}{2}[\angle AOC - \angle DOE]$

Join OA, OC, OE and OD.

Now $\angle AOC = 2\angle AEC$

[Angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point in the alternate segment of the circle]

$\Rightarrow \frac{1}{2}\angle AOC = \angle AEC$ ...(i)

Similarly $\frac{1}{2}\angle DOE = \angle DCE$ ....(ii)

Subtracting eq. (ii) from eq. (i),

$\frac{1}{2}[\angle AOC - \angle DOE] = \angle AEC - \angle DCE$ ....(iii)

Now $\angle AEC = \angle ADC$

[Angles in same segment in circle] ....(iv)

Also $\angle DCE = \angle DAE$

[Angles in same segment in circle] ....(v)

Using eq. (iv) and (v) in eq. (iii),

$\frac{1}{2}[\angle AOC - \angle DOE]$

$= \angle DAE + \angle ABD - \angle DAE$

$\Rightarrow \frac{1}{2}[\angle AOC - \angle DOE] = \angle ABD$

Or $\frac{1}{2}[\angle AOC - \angle DOE] = \angle ABC$

Hence proved.

5. Prove that the circle drawn with any drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.

Ans. Let ABCD be a rhombus in which diagonals AC and BD intersect each other at point O.

As we know that diagonals of a rhombus bisect and perpendicular to each other.

$\therefore \angle AOB = 90^\circ$

And if we draw a circle with side AB as diameter, it will definitely **pass through point O** (the point intersection of diagonals) because then $\angle AOB = 90^\circ$ will be the angle in a semi-circle.

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced it necessary) at E. Prove that AE = AD.

Ans. In figure (a),

ABCD is a parallelogram.

$\Rightarrow \angle 1 = \angle 3$ ....(i)

ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(ii)

And $\angle 5 + \angle 6 = 180^\circ$ ....(iii)

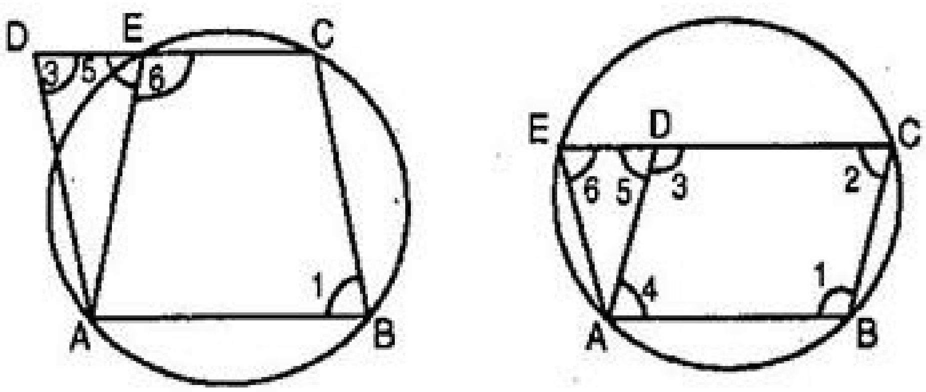
[Linear pair]

From eq. (ii) and (iii), $\angle 1 = \angle 5$ ....(iv)

Now, from eq. (i) and (iv),

$\angle 3 = \angle 5$

$\Rightarrow$ AE = AD [Sides opposite to equal angles are equal]



(a) (b)

In figure (b),

ABCD is a parallelogram.

$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Also AB $||$ CD and BC meets them.

$\therefore \angle 1 + \angle 2 = 180^\circ$ ....(i) [Sum of interior angles]

And AD $||$ BC and EC meets them.

$\therefore \angle 5 = \angle 2$ ....(ii) [Corresponding angles]

Since ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(iii)

From eq. (i) and (iii),

$\angle 1 + \angle 2 = \angle 1 + \angle 6$

$\Rightarrow \angle 2 = \angle 6$

But from eq. (ii), $\angle 2 = \angle 5$

$\therefore \angle 5 = \angle 6$

Now in triangle AED,

$\angle 5 = \angle 6$

$\Rightarrow$ AE = AD [Sides opposite to equal angles]

Hence in both the cases, AE = AD

7. AC and BD are chords of a circle which bisect each other. Prove that:

(i) AC and BD are diameters.

(ii) ABCD is a rectangle.

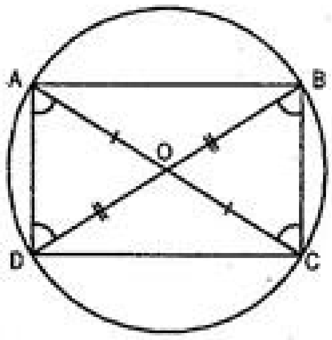
Ans. Given: AC and BD of a circle bisect each other at O.

Then OA = OC and OB = OD

To prove: (i) AC and BD are the diameters. In other words, O is the centre of the circle.

(ii) ABCD is a rectangle.

Proof: (i) In triangles AOD and BOC,



AO = OC [given]

$\angle$ AOD = $\angle$ BOC [Vertically opp.]

OD = OB [given]

$\therefore \triangle AOD \cong \triangle COB$ [SAS congruency]

$\Rightarrow$ AD = CB [By C.P.C.T.]

Similarly $\Delta AOB \cong \Delta COD$

$\Rightarrow$ AB = CD

$\Rightarrow \overset{\frown}{AB} \cong \overset{\frown}{CD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{CD} + \overset{\frown}{BC}$

$\Rightarrow \overset{\frown}{ABC} \cong \overset{\frown}{BCD}$

$\Rightarrow$ AC = BD [Chords opposites to equal arcs]

$\therefore$ AC and BD are the diameters as only diameters can bisects each other as the chords of the circle.

(ii) AC is the diameter. [Proved in (i)]

$\therefore \angle B = \angle D = 90^\circ$ ....(i) [Angle in semi-circle]

Similarly BD is the diameter.

$\therefore \angle A = \angle C = 90^\circ$ ...(ii) [Angle in semi-circle]

Now diameters AC = BD

$\Rightarrow \overset{\frown}{AC} \cong \overset{\frown}{BD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AC} - \overset{\frown}{DC} \cong \overset{\frown}{BD} - \overset{\frown}{DC}$

$\Rightarrow \overset{\frown}{AD} \cong \overset{\frown}{BC}$

$\Rightarrow$ AD = BC [Chords corresponding to the equal arcs] ....(iii)

Similarly AB = DC ....(iv)

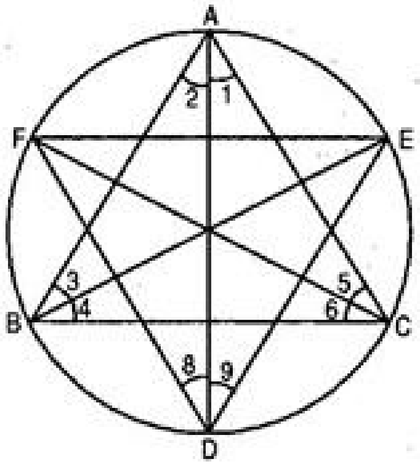
From eq. (i), (ii), (iii) and (iv), we observe that each angle of the quadrilateral is 90° and opposite sides are equal.

Hence ABCD is a rectangle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that angles of the triangle are $\left(90^\circ - \frac{A}{2}\right)$, $\left(90^\circ - \frac{B}{2}\right)$ and $\left(90^\circ - \frac{C}{2}\right)$ respectively.

Ans. According to question, AD is bisector of $\angle$ A.

$\therefore \angle 1 = \angle 2 = \frac{A}{2}$



And BE is the bisector of $\angle$ B.

$\therefore \angle 3 = \angle 4 = \frac{B}{2}$

Also CF is the bisector of $\angle$ C.

$\therefore \angle 5 = \angle 6 = \frac{C}{2}$

Since the angles in the same segment of a circle are equal.

$\therefore \angle 9 = \angle 3$ [angles subtended by $\overset{\frown}{AE}$] ....(i)

And $\angle 8 = \angle 5$ [angles subtended by $\overset{\frown}{FA}$] ....(ii)

Adding both equations,

$\angle 9 + \angle 8 = \angle 3 + \angle 5$

$\Rightarrow \angle D = \frac{B}{2} + \frac{C}{2}$

Similarly $\angle E = \frac{A}{2} + \frac{C}{2}$

And $\angle F = \frac{A}{2} + \frac{B}{2}$

In triangle DEF,

$\angle D + \angle E + \angle F = 180^\circ$ [Angle sum property]

$\Rightarrow \angle D = 180^\circ - (\angle E + \angle F)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{C}{2} + \frac{A}{2} + \frac{B}{2}\right)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) - \frac{A}{2}$

$\Rightarrow \angle D = 180^\circ - 90^\circ - \frac{A}{2}$ [$\because \angle A + \angle B + \angle C = 180^\circ$]

$\Rightarrow \angle D = 90^\circ - \frac{A}{2}$

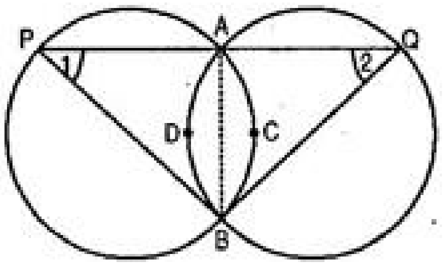
Similarly, we can prove that

$\angle E = 90^\circ - \frac{B}{2}$ and $\angle F = 90^\circ - \frac{C}{2}$

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Ans. Given: Two equal circles intersect in A and B.

A straight line through A meets the circles in P and Q.



To prove: BP = BQ

Construction: Join A and B.

Proof: AB is a common chord and the circles are equal.

$\therefore$ Arc about the common chord are equal, i.e.,

$\overset{\frown}{ACB} = \overset{\frown}{ADB}$

Since equal arcs of two equal circles subtend equal angles at any point on the remaining part of the circle, then we have,

$\angle 1 = \angle 2$

In triangle PBQ,

$\angle 1 = \angle 2$ [proved]

$\therefore$ Sides opposite to equal angles of a triangle are equal.

Then we have, BP = BQ

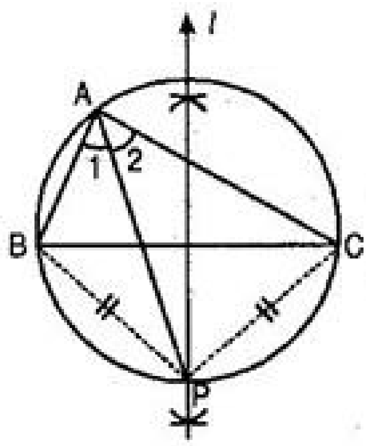
10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.

Ans. Given: ABC is a triangle and a circle passes through its vertices.

Angle bisector of $\angle$ A and the perpendicular bisector (say l) of its opposite side BC intersect each other at a point P.

To prove: Circumcircle of triangle ABC also passes through point P.

Proof: Since any point on the perpendicular bisector is equidistant from the end points of the corresponding side,



$\therefore$ BP = PC ....(i)

Also we have $\angle 1 = \angle 2$ [$\because$ AP is the bisector of $\angle$A (given)] ....(ii)

From eq. (i) and (ii) we observe that equal line segments are subtending equal angles in the same segment i.e., at point A of circumcircle of $\triangle$ABC. Therefore BP and PC acts as chords of circumcircle of $\triangle$ABC and the corresponding congruent arcs $\overparen{BP}$ and $\overparen{PC}$ acts as parts of circumcircle. Hence point P lies on the circumcircle. In other words, points A, B, P and C are concyclic (proved).