CBSE Class 9 Mathematics

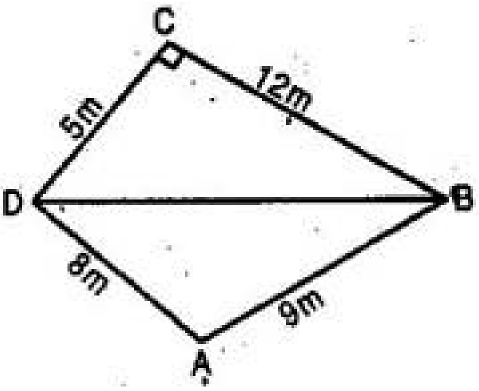
NCERT Solutions

CHAPTER 12

Herons Formula(Ex. 12.2)

1. A park, in the shape of a quadrilateral ABCD has $\angle$ C = 90, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Ans. Since BD divides quadrilateral ABCD in two triangles:



(i) Right triangle BCD and (ii) ΔABD.

In right triangle BCD, right angled at C,

therefore, Base = CD = 5 m and Altitude = BC = 12 m

$\therefore$ Area of ΔBCD = $\frac{1}{2}\times CD\times BC = \frac{1}{2}\times 5\times 12 = 30\ m^2$

In ΔABD, AB = 9 m, AD = 8 m

And BD = $\sqrt{CD^2+BC^2}$ [Using Pythagoras theorem]

$\Rightarrow$ BD = $\sqrt{(5)^2+(12)^2} = \sqrt{25+144} = \sqrt{169}$ = 13 m

Now, Semi=perimeter of ΔABD = $\frac{9+8+13}{2}$ = 15 m

Using Heron's formula,

Area of ΔABD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15(15-9)(15-8)(15-13)}$ = $\sqrt{15\times6\times7\times2}$

= $6\sqrt{35}$ = $6\times5.91\ m^2$ = 35.5 m^2(approx.)

$\therefore$ Area of quadrilateral ABCD = Area of ΔBCD + Area of ΔABD

= 30 + 35.5 = 65.5 m^2(approx.)

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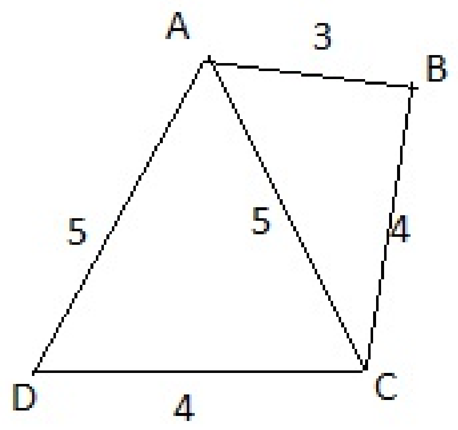
2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Ans. In quadrilateral ABCE, diagonal AC divides it in two triangles, ΔABC and ΔADC.



In ΔABC, Semi-perimeter of ΔABC = $\frac{3+4+5}{2}$ = 6 cm

Using Heron's formula,

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{6(6-3)(6-4)(6-5)}$ = $\sqrt{6\times3\times2\times1}$ = $6\ cm^2$

Again, In $\triangle$ADC, Semi-perimeter of $\triangle$ADC = $\frac{4+5+5}{2}$ = 7 cm

Using Heron's formula, Area of $\triangle$ADC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{7(7-4)(7-5)(7-5)}$ = $\sqrt{7\times3\times2\times2}$ = $2\sqrt{21}$

$= 2\times4.6 = 9.2\ cm^2$ (approx.)

Now area of quadrilateral ABCD = Area of $\triangle$ABC + Area of $\triangle$ADC

= 6 + 9.2 $= 15.2\ cm^2$ (approx.)

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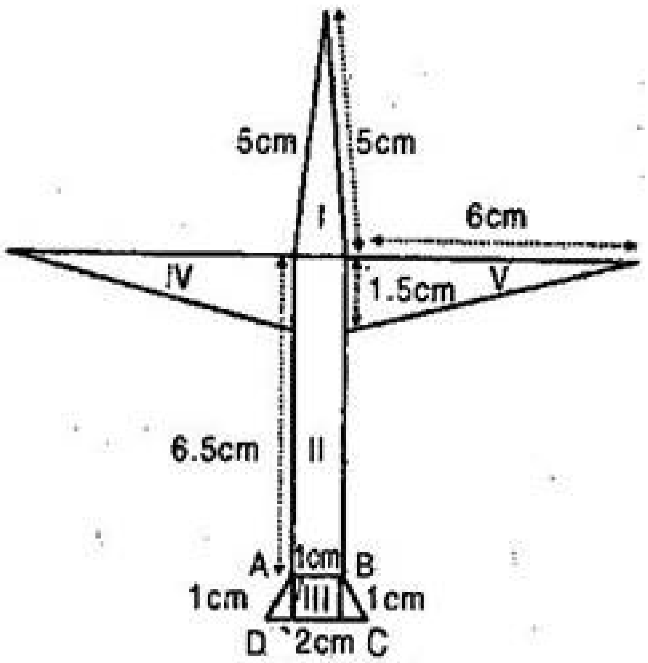
3. Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.



Ans. Area of triangular part I: Here, Semi-perimeter $(s) = \frac{5+5+1}{2}$ = 5.5 cm

Therefore, Area = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

= $\sqrt{5.5\times0.5\times0.5\times4.5}$ = $0.75\sqrt{11}$

$= 0.75\times3.31 = 2.4825\ cm^2$

Area of rectangular part II = Length x Breadth $= 6.5\times1 = 6.5\ cm^2$

Area of part III (trapezium): $\frac{1}{2}(AB + DC)\times AE$

= $\frac{1}{2}$ (AB + DC) $\times\sqrt{AD^2 - DE^2}$ = $\frac{1}{2}$ (1 + 2) $\times\sqrt{1-.025}$

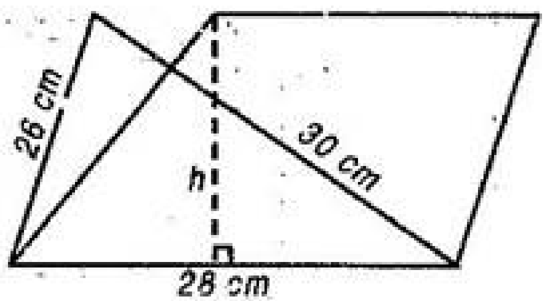
= $\frac{1}{2}\times3\times\frac{\sqrt{3}}{2}$ = $\frac{3\times1.732}{4} = 1.299\ cm^2$

Area of triangular parts IV & V: $2\left(\frac{1}{2}\times1.5\times6\right) = 9\ cm^2$

$\therefore$ **Total area** = 2.4825 + 6.5 + 1.299 + 9 $= 19.28\ cm^2$

4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 29 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Ans. Semi-perimeter of triangle (s) = $\frac{26+28+30}{2}$ = 42 cm



Using Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{42(42-26)(42-28)(42-30)}$

= $\sqrt{42 \times 16 \times 14 \times 12}$ = $336\ cm^2$

According to question, Area of parallelogram = Area of triangle

$\Rightarrow$ Base x Corresponding height = 336

$\Rightarrow$ $28 \times Height$ = 336

$\Rightarrow$ Height = 12 cm

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5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Ans. Here, AB = BC = CD = DA = 30 m and Diagonal AC = 48 m which divides the rhombus ABCD in two congruent triangle.

$\therefore$ Area of ΔABC = Area of ΔACD

Now, Semi-perimeter of ΔABC (s) = $\dfrac{30+30+48}{2}$ = 54 m

Now Area of rhombus ABCD = Area of ΔABC + Area of ΔACD

= 2 $\times$ Area of ΔABC [$\because$ Area of ΔABC = Area of ΔACD]

= $2\sqrt{s(s-a)(s-b)(s-c)}$ [Using Heron's formula]

= $2 \times \sqrt{54(54-30)(54-30)(54-48)}$

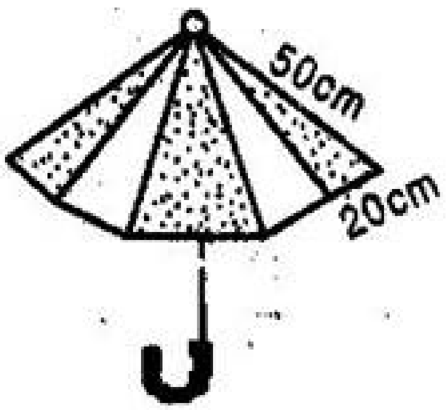
= $2 \times \sqrt{54 \times 24 \times 24 \times 6}$ = 2 x 3 x 6 x 24

$= 864\ m^2$

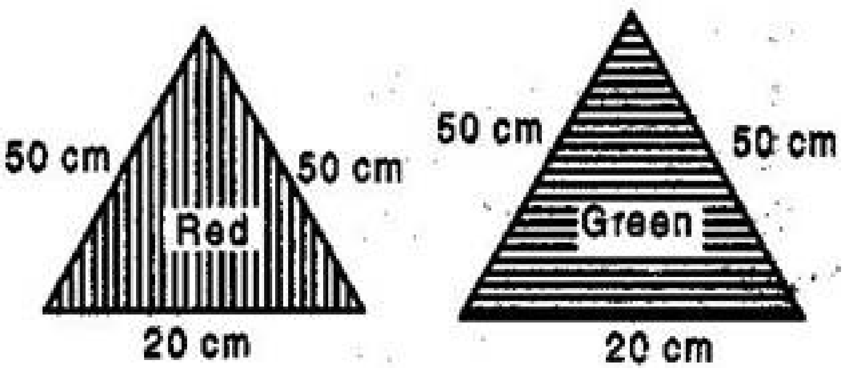
$\because$ Field available for 18 cows to graze the grass $= 864\ m^2$

$\therefore$ Field available for 1 cow to graze the grass = $\frac{864}{18} = 48\, m^2$

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?



Ans. Here, sides of each of 10 triangular pieces of two different colours are 20 cm, 50 cm and 50 cm.



Semi-perimeter of each triangle (s) = $\frac{20+50+50}{2}$ = 60 cm

Now, Area of each triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{60(60-20)(60-50)(60-50)}$

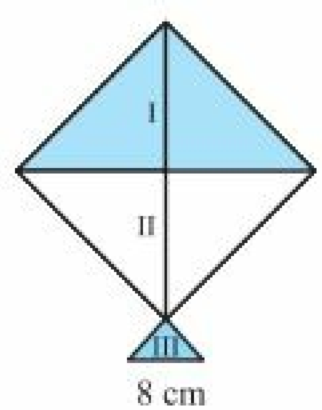
= $\sqrt{60\times40\times10\times10}$ = $200\sqrt{6}cm^2$

According to question, there are 5 pieces of red colour and 5 pieces of green colour.

$\therefore$ Cloth required for 5 red pieces = $5\times 200\sqrt{6}$ = $1000\sqrt{6}cm^2$

And Cloth required to 5 green pieces = $5\times 200\sqrt{6}$ = $1000\sqrt{6}cm^2$

7. A kite is in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure.



How much paper of each side has been used in it?

Ans. Let ABCD is a square of side a cm and diagonals AC = BD = 32 cm

In right triangle ABC, $AB^2 + BC^2 = AC^2$ [Using Pythagoras theorem]

$\Rightarrow a^2 + a^2 = (32)^2$

$\Rightarrow 2a^2 = 32\times 32$

$\Rightarrow a^2 = \frac{32\times 32}{2}$ = 512

$\Rightarrow$ Area of square $= 512\ cm^2$ [Area of square = $side\times side$]

Diagonal BD divides the square in two equal triangular parts I and II.

$\therefore$ Area of shaded part I = Area of shaded part II = $\frac{1}{2}\times 512 = 256\ cm^2$

Now, semi-perimeter of shaded part III $(s) = \frac{6+6+8}{2}$ = 10 cm

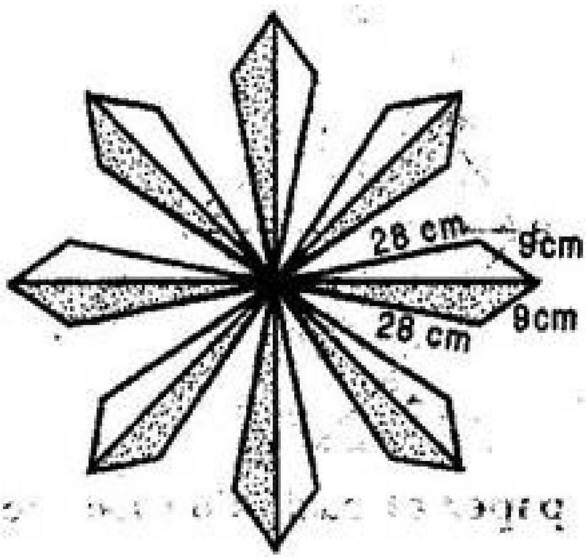
Area of shaded part III = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{10(10-6)(10-6)(10-8)}$

= $\sqrt{10\times4\times4\times2}$ = $8\sqrt{5}$

$= 8\times2.236 = 17.88\ cm^2$

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 paise per cm^2.



Ans. Here, Sides of a triangular shaped tile area 9 cm, 28 cm and 35 cm.

Semi-perimeter of tile (s) = $\frac{9+28+35}{2}$ = 36 cm

Area of triangular shaped tile = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{36(36-9)(36-28)(36-35)}$

= $\sqrt{36\times27\times8\times1}$ = $36\sqrt{6}$

$= 36\times2.45 = 88.2\ cm^2$ (approx.)

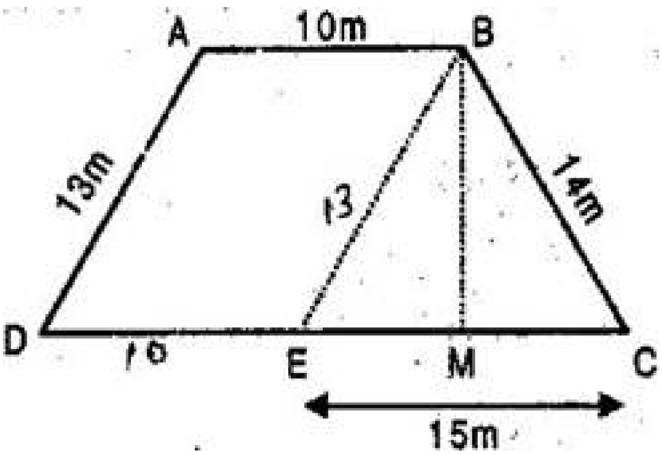
$\therefore$ Area of 16 such tiles $= 16\times88.2 = 1411.2\ cm^2$ (Approx.)

$\because$ Cost of polishing $1\ cm^2$ of tile = Rs. 0.50

$\therefore$ Cost of polishing $1411.2\ cm^2$ of tile = $Rs.\ 0.50\times1411.2$ = Rs. 705.60 (Approx.)

9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Ans. Let ABCD be a field in the shape of trapezium and parallel side AB = 10 m & CD = 25 m



And Non-parallel sides AD = 13 m and BC = 14 m

Draw BM $\perp$ DC and BE $\|$ AD so that ABED is a parallelogram.

$\therefore$ BE = AD = 13 m and DE = AB = 10 m

Now in ΔBEC, Semi-perimeter $(s) = \dfrac{13+14+15}{2}$ = 21 m

Area of ΔBEC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{21(21-13)(21-14)(21-15)}$

= $\sqrt{21\times8\times7\times6}$ = $84\ m^2$

And Area of ΔBEC = $84\ m^2$

$\Rightarrow \frac{1}{2}\times EC\times BM$ = 84

$\Rightarrow \frac{1}{2}\times15\times BM$ = 84

$\Rightarrow$ BM = $\frac{84\times2}{15}$ = 11.2 m

Now area of trapezium ABCD = $\frac{1}{2}(AB + CD)\times BM$

= $\frac{1}{2}(10 + 25)\times11.2 = 196\ m^2$