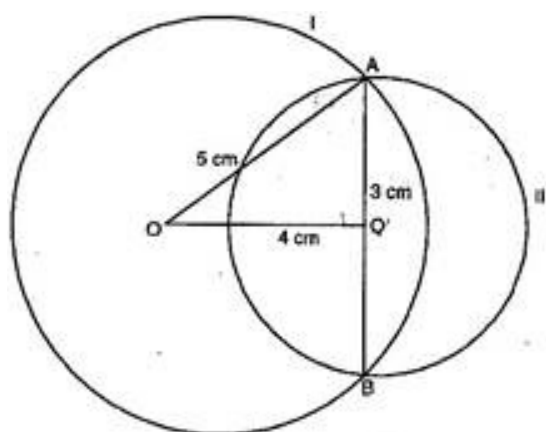


CBSE Class 9 Mathematics
NCERT Solutions
CHAPTER 10
Circles(Ex. 10.4)

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Ans. Let two circles with centres O and O' intersect each other at points A and B. On joining A and B, AB is a common chord.



Radius OA = 5 cm, Radius O'A = 3 cm,

Distance between their centers OO' = 4 cm

In triangle AOO',

$$5^2 = 4^2 + 3^2$$

$$\Rightarrow 25 = 16 + 9$$

$$\Rightarrow 25 = 25$$

Hence AOO' is a right triangle, right angled at O'.

Since, perpendicular drawn from the center of the circle bisects the chord.

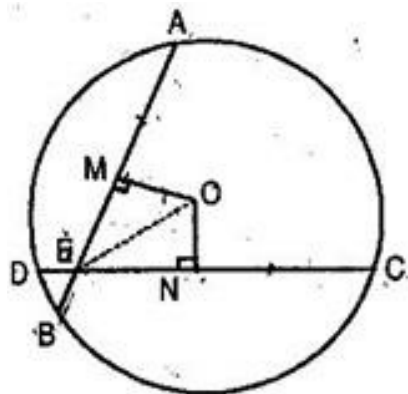
Hence O' is the mid-point of the chord AB. Also O' is the centre of the circle II.

Therefore length of chord AB = Diameter of circle II

∴ Length of chord AB = $2 \times 3 = 6$ cm.

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans. Given: Let AB and CD are two equal chords of a circle of centers O intersecting each other at point E within the circle.



To prove: (a) $AE = CE$

(b) $BE = DE$

Construction: Draw $OM \perp AB$, $ON \perp CD$. Also join OE.

Proof: In right triangles OME and ONE,

$$\angle OME = \angle ONE = 90^\circ \quad [\text{By construction}]$$

$$OM = ON$$

[Equal chords are equidistant from the centre]

$$OE = OE \quad [\text{Common}]$$

$$\therefore \triangle OME \cong \triangle ONE \quad [\text{RHS rule of congruency}]$$

$$\therefore ME = NE \quad [\text{By C.P.C.T.}] \dots\dots\dots(i)$$

Now, O is the centre of circle and $OM \perp AB$

$$\therefore AM = \frac{1}{2} AB$$

[Perpendicular from the centre bisects the chord](ii)

Similarly, $NC = \frac{1}{2} CD$ (iii)

But $AB = CD$ [Given]

From eq. (ii) and (iii), AM
 $= NC$ (iv)

Also $MB = DN$ (v)

Adding (i) and (iv), we get,

$$AM + ME = NC + NE$$

$$\Rightarrow AE = CE \text{ [Proved part (a)]}$$

Now $AB = CD$ [Given]

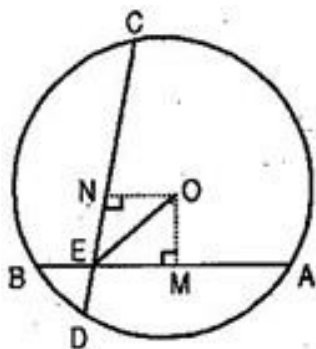
$AE = CE$ [Proved]

$$\Rightarrow AB - AE = CD - CE$$

$$\Rightarrow BE = DE \text{ [Proved part (b)]}$$

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chord.

Ans. Given: AB and CD be two equal chords of a circle with centre O intersecting each other within the circle at point E . OE is joined.



To prove: $\angle OEM = \angle OEN$

Construction: Draw $OM \perp AB$ and

$ON \perp CD$.

Proof: In right angled triangles OME and ONE ,

$$\angle OME = \angle ONE \text{ [Each } 90^\circ \text{] [By construction]}$$

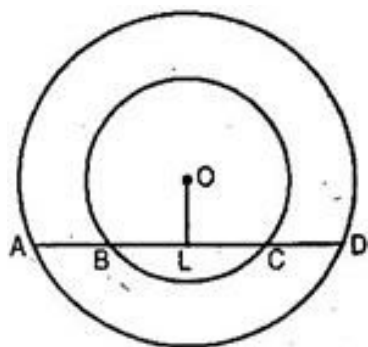
$OM = ON$ [Equal chords are equidistant from the centre]

$OE = OE$ [Common]

$$\therefore \triangle OME \cong \triangle ONE \text{ [RHS rule of congruency]}$$

$$\therefore \angle OEM = \angle OEN \text{ [By C.P.C.T.]}$$

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$. (See figure)



Ans. Given: Line l intersects two concentric circles with centre O at points A, B, C and D.

To prove: $AB = CD$

Construction: Draw $OL \perp l$

Proof: AD is a chord of outer circle and $OL \perp AD$.

$$\therefore AL = LD \text{(i) [Perpendicular drawn from the centre bisects the chord]}$$

Now, BC is a chord of inner circle and

$$OL \perp BC$$

$$\therefore BL = LC \text{ ...(ii) [Perpendicular drawn from the centre bisects the chord]}$$

Subtracting (ii) from (i), we get,

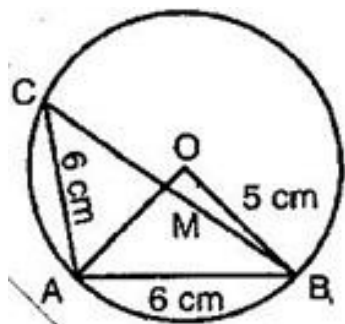
$$AL - BL = LD - LC$$

$$\Rightarrow AB = CD$$

5. Three girls Reshma, Salma and Mandip are standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Ans. Let Reshma, Salma and Mandip takes the position C, A and B on the circle. Given: $AB = AC$

The centre lies on the bisector of $\angle BAC$.



Let M be the point of intersection of BC and OA.

Again, since $AB = AC$ and AM bisects

$\angle CAB$.

$\therefore AM \perp CB$ and M is the mid-point of CB.

Let $OM = x$, then $MA = 5 - x$

From right angled triangle OMB,

$$OB^2 = OM^2 + MB^2$$

$$\Rightarrow 5^2 = x^2 + MB^2 \dots\dots\dots(i)$$

Again, in right angled triangle AMB,

$$AB^2 = AM^2 + MB^2$$

$$\Rightarrow 6^2 = (5 - x)^2 + MB^2 \dots\dots\dots(ii)$$

Equating the value of MB^2 from eq. (i) and (ii),

$$5^2 - x^2 = 6^2 - (5 - x)^2$$

$$\Rightarrow (5 - x)^2 - x^2 = 6^2 - 5^2$$

$$\Rightarrow 25 - 10x + x^2 - x^2 = 36 - 25$$

$$\Rightarrow 10x = 25 - 11$$

$$\Rightarrow 10x = 14$$

$$\Rightarrow x = \frac{14}{10}$$

Hence, from eq. (i),

$$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2$$

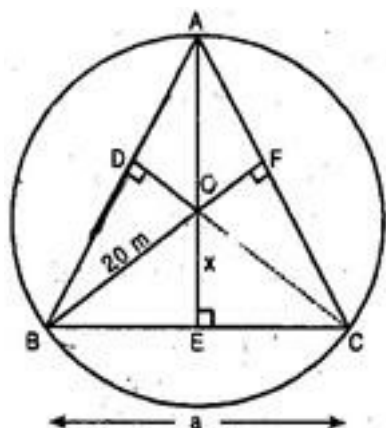
$$= \left(5 + \frac{14}{10}\right) \left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$$

$$\Rightarrow MB = \frac{8 \times 6}{10} = 4.8 \text{ cm}$$

$$\therefore BC = 2MB = 2 \times 4.8 = 9.6 \text{ cm}$$

6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Ans. Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.



$$AB = BC = CA = a \text{ [say]}$$

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

$$\therefore OD = OE = OF = x \text{ cm [say]}$$

Join OA, OB and OC.

$$\Rightarrow \text{Area of } \triangle AOB$$

$$= \text{Area of } \triangle BOC = \text{Area of } \triangle AOC$$

And Area of $\triangle ABC$

$$= \text{Area of } \triangle AOB + \text{Area of } \triangle BOC + \text{Area of } \triangle AOC$$

$$\Rightarrow \text{And Area of } \triangle ABC = 3 \times \text{Area of } \triangle BOC$$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 3 \left(\frac{1}{2} BC \times OE \right)$$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 3 \left(\frac{1}{2} \times a \times x \right)$$

$$\Rightarrow \frac{a^2}{a} = 3 \times \frac{1}{2} \times \frac{4}{\sqrt{3}} \times x$$

$$\Rightarrow a = 2\sqrt{3}x \dots\dots\dots(i)$$

Now, $CE \perp BC$

$$\therefore BE = EC = \frac{1}{2} BC \quad [\because \text{Perpendicular drawn from the centre bisects the chord}]$$

$$\Rightarrow BE = EC = \frac{1}{2} a$$

$$\Rightarrow BE = EC = \frac{1}{2} (2\sqrt{3}x) \quad [\text{Using eq. (i)}]$$

$$\Rightarrow BE = EC = \sqrt{3}x$$

Now in right angled triangle BEO,

$$OE^2 + BE^2 = OB^2 \quad [\text{Using Pythagoras theorem}]$$

$$\Rightarrow x^2 + (\sqrt{3}x)^2 = (20)^2$$

$$\Rightarrow x^2 + 3x^2 = 400$$

$$\Rightarrow 4x^2 = 400$$

$$\Rightarrow x^2 = 100$$

$$\Rightarrow x = 10 \text{ m}$$

$$\text{And } a = 2\sqrt{3}x = 2\sqrt{3} \times 10 = 20\sqrt{3} \text{ m}$$

Thus distance between any two boys is $20\sqrt{3}$ m.