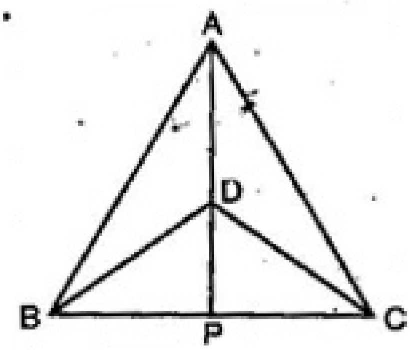
CBSE Class 9 Mathematics

NCERT Solutions

CHAPTER 7

Triangles(Ex. 7.3)

1. $\triangle$ABC and $\triangle$DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(i) $\triangle$ABD $\cong$ $\triangle$ACD

(ii) $\triangle$ABP $\cong$ $\triangle$ACP

(iii) AP bisects $\angle$A as well as $\angle$D.

(iv) AP is the perpendicular bisector of BC.

Ans. (i) $\triangle$ABC is an isosceles triangle.

$\therefore$ AB = AC

$\triangle$DBC is an isosceles triangle.

$\therefore$ BD = CD

Now in $\triangle$ABD and $\triangle$ACD,

AB = AC [Given]

BD = CD [Given]

AD = AD [Common]

$\therefore$ ΔABD $\cong$ Δ ACD [By SSS congruency]

$\Rightarrow$ $\angle$ BAD = $\angle$ CAD [By C.P.C.T.] ....(i)

(ii) Now in ΔABP and ΔACP,

AB = AC [Given]

$\angle$ BAD = $\angle$ CAD [From eq. (i)]

AP = AP [Common]

$\therefore$ ΔABP $\cong$ Δ ACP [By SAS congruency]

Also, BP = CP [By C.P.C.T.]............(ii)

(iii) Since ΔABP $\cong$ Δ ACP [From part (ii)]

$\Rightarrow$ $\angle$ BAP = $\angle$ CAP [By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle$ A.

In ΔBDP and ΔCDP,

BD = CD [Given]

DP = DP [Commmon]

BP = CP [From eqn (ii)]

Therefore, $\Delta BDP \cong \Delta CDP$ [By SSS Conruency]

=> $\angle BDP = \angle CDP$ [By C.P.C.T.].............(iii)

and $\angle BPD = \angle CPD$ [By C.P.C.T.] ....(iv)

Hence, AP bisects $\angle D$ from (iii)

(iv) Since=> $\angle BPD = \angle CPD$[By eqn (iv)]

Now $\angle$ BPD + $\angle$ CPD = 180° [Linear pair]

$\Rightarrow$ $\angle$BPD + $\angle$BPD = 180° [Using eq. (iii)]

$\Rightarrow$ 2$\angle$BPD = 180°

$\Rightarrow$ $\angle$BPD = 90°

$\Rightarrow$ AP $\perp$ BC ....(v)

From eq. (iv) and (v), we have AP = BP and AP $\perp$ BC. So, collectively AP is perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

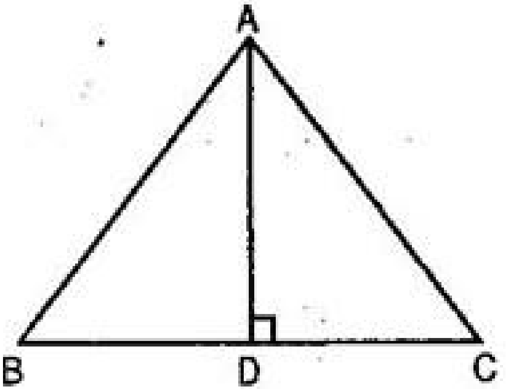
(i) AD bisects BC.

(ii) AD bisects $\angle$A.

Ans. In $\triangle$ABD and $\triangle$ACD,

AB = AC [Given]

$\angle$ADB = $\angle$ADC = 90° [AD $\perp$ BC]



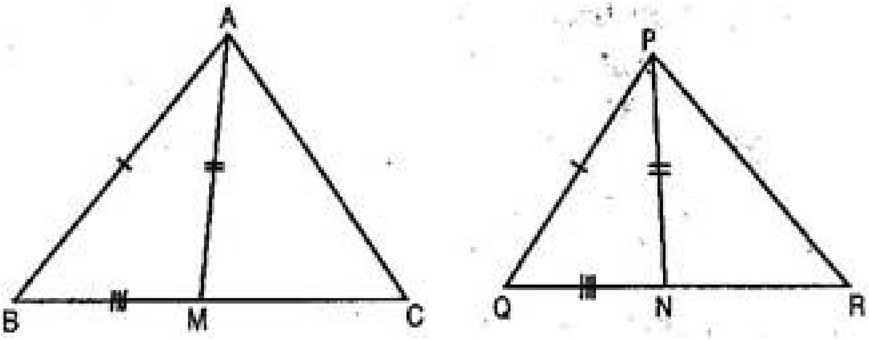
AD = AD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$ACD [RHS rule of congruency]

$\Rightarrow$ BD = DC [By C.P.C.T.]
$\Rightarrow$ AD bisects BC
Also $\angle$BAD = $\angle$CAD [By C.P.C.T.]

$\Rightarrow$ AD bisects $\angle$A. **Hence proved.**

3. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of $\triangle$PQR (See figure). Show that:



(i) $\triangle$ABM $\cong$ $\triangle$ PQN

(ii) $\triangle$ABC $\cong$ $\triangle$ PQR

Ans. AM is the median of $\triangle$ABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC ....(i)

PN is the median of $\triangle$PQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR ....(ii)

Now BC = QR [Given] $\Rightarrow$ $\frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\therefore$ BM = QN ....(iii)

(i) Now in $\triangle$ABM and $\triangle$PQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

$\therefore$ $\triangle$ABM $\cong$ $\triangle$PQN [By SSS congruency]

$\Rightarrow$ $\angle$B = $\angle$Q [By C.P.C.T.] ....(iv)

(ii) In $\triangle$ABC and $\triangle$PQR,

AB = PQ [Given]

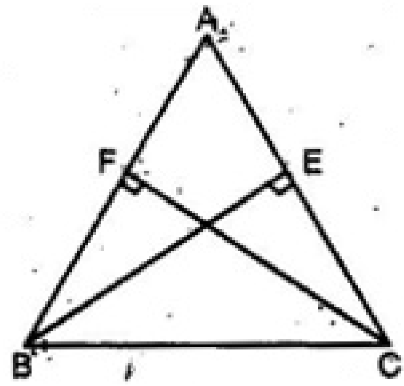
$\angle$B = $\angle$Q [Prove above]

BC = QR [Given]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$PQR [By SAS congruency]

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In $\triangle$BEC and $\triangle$CFB,



$\angle$BEC = $\angle$CFB [Each 90°]

BC = BC [Common]

BE = CF [Given]

$\therefore$ $\triangle$BEC $\cong$ $\triangle$CFB [RHS congruency]

$\Rightarrow$ EC = FB [By C.P.C.T.] .....(i)

Now In $\triangle$AEB and $\triangle$AFC

$\angle$AEB = $\angle$AFC [Each 90°]

$\angle$A = $\angle$A [Common]

BE = CF [Given]

$\therefore$ $\triangle$AEB $\cong$ $\triangle$AFC [AAS congruency]

$\Rightarrow$ AE = AF [By C.P.C.T.] ....(ii)

Adding eq. (i) and (ii), we get,

EC + AE = FB + AF

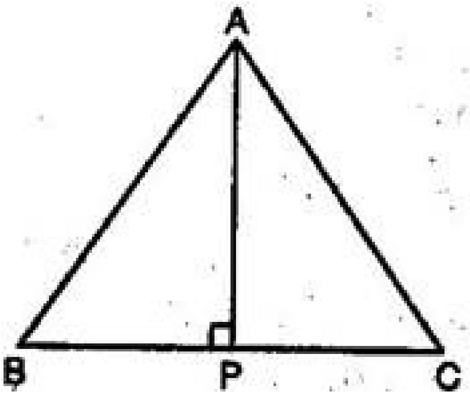
$\Rightarrow$ AB = AC

$\Rightarrow$ ABC is an isosceles triangle.

Hence proved.

5. ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$B = $\angle$C.

Ans. Given: ABC is an isosceles triangle in which AB = AC



To prove: $\angle$B = $\angle$C

Construction: Draw AP $\perp$ BC

Proof: In $\triangle$ABP and $\triangle$ACP

$\angle$APB = $\angle$APC = 90° [By construction]

AB = AC [Given]

AP = AP [Common]

$\therefore$ $\triangle$ABP $\cong$ $\triangle$ACP [RHS congruency]

$\Rightarrow$ $\angle$B = $\angle$C [By C.P.C.T.]

Hence proved.