

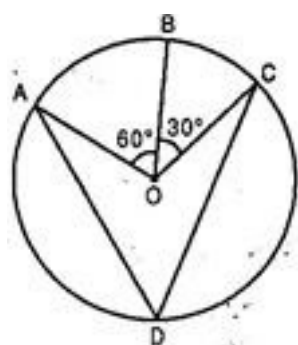
CBSE Class 9 Mathematics

NCERT Solutions

CHAPTER 10

Circles(Ex. 10.5)

1. In figure, A, B, C are three points on a circle with centre O such that $\angle BOC = 30^\circ$, $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$



Ans. $\angle AOC = \angle AOB + \angle BOC$

$$\Rightarrow \angle AOC = 60^\circ + 30^\circ = 90^\circ$$

Now $\angle AOC = 2 \angle ADC$

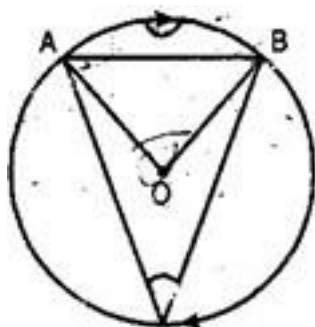
[\because Angle subtended by an arc, at the centre of the circle is double the angle subtended by the same arc at any point in the remaining part of the circle]

$$\Rightarrow \angle ADC = \frac{1}{2} \angle AOC$$

$$\Rightarrow \angle ADC = \frac{1}{2} \times 90^\circ = 45^\circ$$

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord on a point on the minor arc and also at a point on the major arc.

Ans. Let AB be the minor arc of circle.



\therefore Chord AB = Radius OA = Radius OB

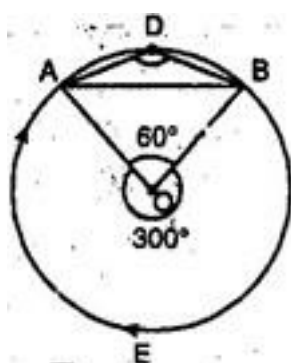
$\therefore \triangle AOB$ is an equilateral triangle.

$$\Rightarrow \angle AOB = 60^\circ$$

$$\text{Now } m\widehat{AB} + m\widehat{BA} = 360^\circ$$

$$\Rightarrow \angle AOB + \angle BOA = 360^\circ$$

$$\Rightarrow 60^\circ + \angle BOA = 360^\circ$$



$$\Rightarrow \angle BOA = 360^\circ - 60^\circ = 300^\circ$$

D is a point in the minor arc.

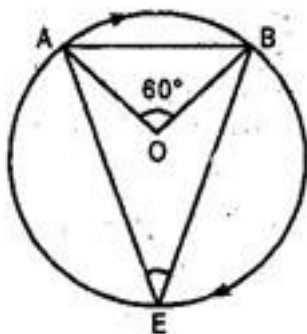
$$\therefore m\widehat{BA} = 2 \angle BDA$$

$$\Rightarrow \angle BOA = 2 \angle BDA$$

$$\Rightarrow \angle BDA = \frac{1}{2} \angle BOA = \frac{1}{2} \times 300^\circ$$

$$\Rightarrow \angle BDA = 150^\circ$$

Thus angle subtended by major arc, \widehat{BA} at any point D in the minor arc is 150° .



Let E be a point in the major arc \widehat{BA} .

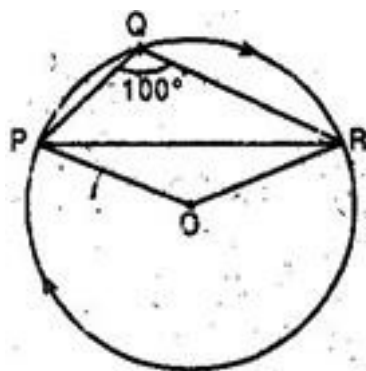
$$\therefore m\widehat{AB} = 2 \angle AEB$$

$$\Rightarrow \angle AOB = 2 \angle AEB$$

$$\Rightarrow \angle AEB = \frac{1}{2} \angle AOB$$

$$\Rightarrow \angle AEB = \frac{1}{2} \times 60^\circ = 30^\circ$$

3. In figure, $\angle PQR = 100^\circ$, where P, Q, R are points on a circle with centre O. Find $\angle OPR$.



Ans. In the figure, Q is a point in the minor arc \widehat{PR} .

$$\therefore m\widehat{RP} = 2 \angle PQR$$

$$\Rightarrow \angle ROP = 2 \angle PQR$$

$$\Rightarrow \angle ROP = 2 \times 100^\circ = 200^\circ$$

$$\text{Now } m\widehat{PR} + m\widehat{RP} = 360^\circ$$

$$\Rightarrow \angle POR + \angle ROP = 360^\circ$$

$$\Rightarrow \angle POR + 200^\circ = 360^\circ$$

$$\Rightarrow \angle POR = 360^\circ - 200^\circ = 160^\circ \dots (i)$$

Now $\triangle OPR$ is an isosceles triangle.

$$\therefore OP = OR \text{ [radii of the circle]}$$

$$\Rightarrow \angle OPR = \angle ORP \text{ [angles opposite to equal sides are equal]} \dots (ii)$$

Now in isosceles triangle OPR ,

$$\angle OPR + \angle ORP + \angle POR = 180^\circ$$

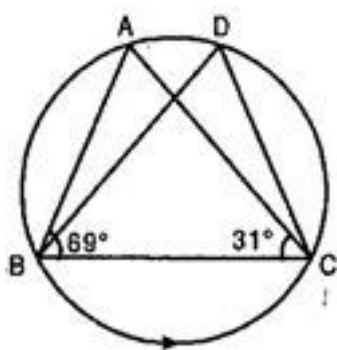
$$\Rightarrow \angle OPR + \angle ORP + 160^\circ = 180^\circ$$

$$\Rightarrow 2 \angle OPR = 180^\circ - 160^\circ \text{ [Using (i) \& (ii)]}$$

$$\Rightarrow 2 \angle OPR = 20^\circ$$

$$\Rightarrow \angle OPR = 10^\circ$$

4. In figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.



Ans. In triangle ABC,

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$\Rightarrow \angle BAC + 69^\circ + 31^\circ = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 69^\circ - 31^\circ$$

$$\Rightarrow \angle BAC = 80^\circ \dots\dots(i)$$

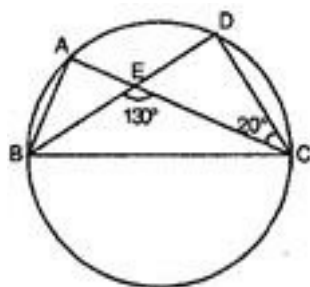
Since, A and D are the points in the same segment of the circle.

$$\therefore \angle BDC = \angle BAC$$

[Angles subtended by the same arc at any points in the alternate segment of a circle are equal]

$$\Rightarrow \angle BDC = 80^\circ \text{ [Using (i)]}$$

5. In figure, A, B, C, D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Ans. Given: $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$

$$\angle DEC = 180^\circ - \angle BEC = 180^\circ - 130^\circ = 50^\circ \text{ [Linear pair]}$$

Now in $\triangle DEC$,

$$\angle DEC + \angle DCE + \angle EDC = 180^\circ \text{ [Angle sum property]}$$

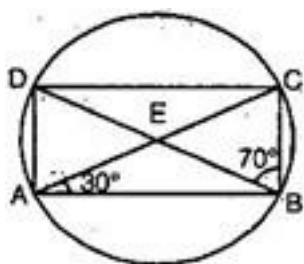
$$\Rightarrow 50^\circ + 20^\circ + \angle EDC = 180^\circ$$

$$\Rightarrow \angle EDC = 110^\circ$$

$$\Rightarrow \angle BAC = \angle EDC = 110^\circ \text{ [Angles in same segment]}$$

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. $\angle DBC = 70^\circ$, $\angle BAC$ is 30° find $\angle BCD$. Further if $AB = BC$, find $\angle ECD$.

Ans. For chord CD



$$\angle CBD = \angle CAD \text{ (Angles in same segment)}$$

$$\angle CAD = 70^\circ \quad [\text{Given: } \angle CBD = 70^\circ]$$

$$\angle BAD = \angle BAC + \angle CAD$$

$$= 30^\circ + 70^\circ = 100^\circ$$

$$\angle BCD + \angle BAD = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral are supplementary)}$$

$$\angle BCD + 100 = 180^\circ$$

$$\angle BCD = 80^\circ$$

In $\triangle ABC$

$$AB = BC \text{ (given)}$$

$$\therefore \angle BCA = \angle CAB \text{ (Angles opposite to equal sides of a triangle are equal)}$$

$$\angle BCA = 30^\circ$$

$$\text{We have } \angle BCD = 80^\circ \Rightarrow \angle BCA + \angle ACD = 80^\circ$$

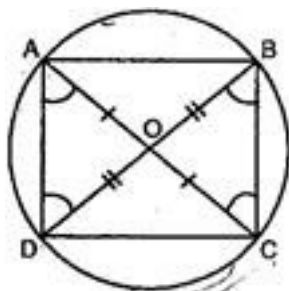
$$30^\circ + \angle ACD = 80^\circ$$

$$\angle ACD = 50^\circ$$

$$\Rightarrow \angle ECD = 50^\circ$$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Ans. Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.



$$\angle BAD = \frac{1}{2} \angle BOD = \frac{180^\circ}{2} = 90^\circ$$

(Consider BD as a chord)

$$\angle BCD + \angle BAD = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral are supplementary)}$$

$$\angle BCD = 180^\circ - 90^\circ = 90^\circ$$

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{180^\circ}{2} = 90^\circ$$

(Considering AC as a chord)

$$\angle ADC + \angle ABC = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral are supplementary)}$$

$$90^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 90^\circ$$

Here, each interior angle of cyclic quadrilateral is of 90° and its diagonals are equal (given). Hence it is a rectangle.

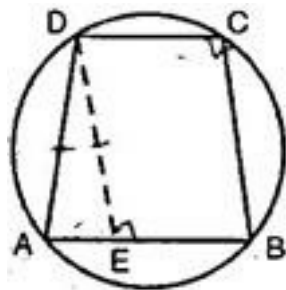
8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Ans. Given: A trapezium ABCD in which $AB \parallel CD$ and $AD = BC$.

To prove: The points A, B, C, D are concyclic.

Construction: Draw $DE \parallel CB$.

Proof: Since $DE \parallel CB$ and $EB \parallel DC$.



\therefore EBCD is a parallelogram.

$\therefore DE = CB$ and $\angle DEB = \angle DCB$

Now $AD = BC$ and $DA = DE$

$\Rightarrow \angle DAE = \angle DEB$

But $\angle DEA + \angle DEB = 180^\circ$

$\Rightarrow \angle DAE + \angle DCB = 180^\circ$

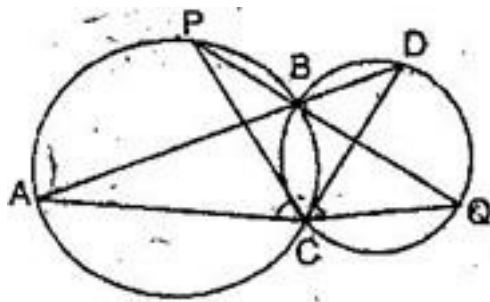
$[\because \angle DEA = \angle DAE \text{ and } \angle DEB = \angle DCB]$

$\Rightarrow \angle DAB + \angle DCB = 180^\circ$

$\Rightarrow \angle A + \angle C = 180^\circ$

Hence, ABCD is a cyclic trapezium.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.



Ans. In triangles ACD and QCP,

$$\angle A = \angle P \text{ and } \angle Q = \angle D \text{ [Angles in same segment]}$$

$$\therefore \angle ACD = \angle QCP \text{ [Third angles of two triangles](i)}$$

Subtracting $\angle PCD$ from both the sides of eq. (i), we get,

$$\angle ACD - \angle PCD = \angle QCP - \angle PCD$$

$$\Rightarrow \angle ACP = \angle QCD$$

Hence proved.

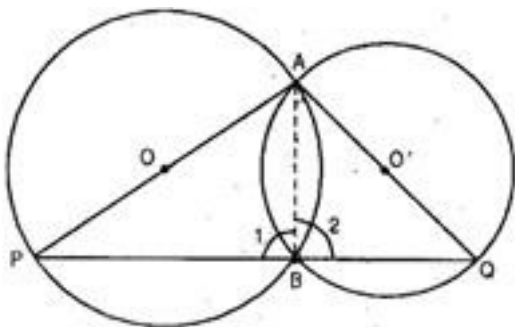
10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans. Given: Two circles intersect each other at points A and B. AP and AQ be their respective diameters.

To prove: Point B lies on the third side PQ.

Construction: Join A and B.

Proof: AP is a diameter.



$$\therefore \angle 1 = 90^\circ$$

[Angle in semicircle]

Also AQ is a diameter.

$$\therefore \angle 2 = 90^\circ$$

[Angle in semicircle]

$$\angle 1 + \angle 2 = 90^\circ + 90^\circ$$

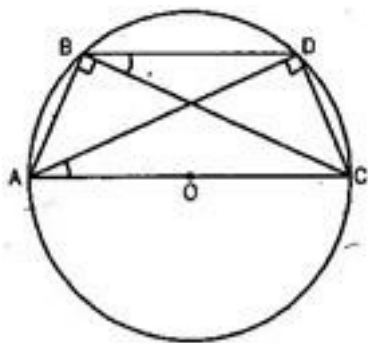
$$\Rightarrow \angle PBQ = 180^\circ$$

\Rightarrow PBQ is a straight line.

Thus point B, i.e. point of intersection of these circles lies on the third side i.e., on PQ.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Ans. We have ABC and ADC two right triangles, right angled at B and D respectively.



$$\Rightarrow \angle ABC = \angle ADC \text{ [Each } 90^\circ \text{]}$$

If we draw a circle with AC (the common hypotenuse) as diameter, this circle will definitely pass through of an arc AC, Because B and D are the points in the alternate segment of an arc AC.

Now we have \widehat{CD} subtending $\angle CBD$ and $\angle CAD$ in the same segment.

$$\therefore \angle CAD = \angle CBD$$

Hence proved.

12. Prove that a cyclic parallelogram is a rectangle.

Ans. Given: A cyclic parallelogram ABCD.

To prove: ABCD is a rectangle.

Proof: $\angle B = \angle D$ [Opposite angles of a parallelogram are equal]

But, $\angle B + \angle D = 180^\circ$ [Sum of opposite angles of a parallelogram]

$$\Rightarrow 2\angle B = 180^\circ$$

$$\Rightarrow \angle B = 90^\circ \quad \text{and} \quad \angle D = 90^\circ$$

Similarly,

$$\angle A = 90^\circ \quad \text{and} \quad \angle C = 90^\circ$$

Hence, ABCD is a rectangle.