

CBSE Class 9 Mathematics
NCERT Solutions
CHAPTER 2
Polynomials(Ex. 2.5)

1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans. (i) $(x+4)(x+10)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$$\begin{aligned}(x+4)(x+10) &= x^2 + (4+10)x + (4 \times 10) \\ &= x^2 + 14x + 40\end{aligned}$$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2 + 14x + 40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$$\begin{aligned}(x+8)(x-10) &= x^2 + [8+(-10)]x + [8 \times (-10)] \\ &= x^2 - 2x - 80.\end{aligned}$$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2 - 2x - 80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$$\begin{aligned}(3x+4)(3x-5) &= (3x)^2 + [4+(-5)]3x + [4 \times (-5)] \\ &= 9x^2 - 3x - 20.\end{aligned}$$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2 - 3x - 20$.

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

We know that $(x+y)(x-y) = x^2 - y^2$.

We need to apply the above identity to find the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

$$\begin{aligned}\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) \\ = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}.\end{aligned}$$

Therefore, we conclude that the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ is $\left(y^4 - \frac{9}{4}\right)$.

(v) $(3+2x)(3-2x)$

We know that $(x+y)(x-y) = x^2 - y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$$\begin{aligned}(3+2x)(3-2x) &= (3)^2 - (2x)^2 \\ &= 9 - 4x^2.\end{aligned}$$

Therefore, we conclude that the product $(3+2x)(3-2x)$ is $(9 - 4x^2)$.

2. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Ans. (i) 103×107

103×107 can also be written as $(100+3)(100+7)$.

We can observe that, we can apply the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$(100+3)(100+7) = (100)^2 + (3+7)(100) + 3 \times 7$$

$$= 10000 + 1000 + 21$$

$$= 11021$$

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that, we can apply the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$(100-5)(100-4) = (100)^2 + [(-5) + (-4)](100) + (-5) \times (-4)$$

$$= 10000 - 900 + 20 = 9120$$

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y) = x^2 - y^2$ with respect to the expression $(100+4)(100-4)$, to get

$$(100+4)(100-4) = (100)^2 - (4)^2$$

$$= 10000 - 16$$

$$= 9984$$

Therefore, we conclude that the value of the product 104×96 is 9984.

3. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Ans. (i) $9x^2 + 6xy + y^2$

$$9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$$

We can observe that, we can apply the identity $(x+y)^2 = x^2 + 2xy + y^2$

$$\Rightarrow (3x)^2 + 2 \times 3x \times y + (y)^2 = (3x+y)^2.$$

$$\Rightarrow (3x+y)(3x+y)$$

(ii) $4y^2 - 4y + 1$

$$4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2$$

We can observe that, we can apply the identity $(x-y)^2 = x^2 - 2xy + y^2$

$$\Rightarrow (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y-1)^2.$$

$$\Rightarrow (2y-1)(2y-1)$$

(iii) $x^2 - \frac{y^2}{100}$

We can observe that, we can apply the identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$$\Rightarrow (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right).$$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

Ans. (i) $(x+2y+4z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(x + 2y + 4z)^2$.

$$\begin{aligned}(x + 2y + 4z)^2 &= (x)^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x \\ &= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx\end{aligned}$$

(ii) $(2x - y + z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(2x - y + z)^2$.

$$\begin{aligned}(2x - y + z)^2 &= [2x + (-y) + z]^2 \\ &= (2x)^2 + (-y)^2 + (z)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x \\ &= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx.\end{aligned}$$

(iii) $(-2x + 3y + 2z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 3y + 2z)^2$.

$$\begin{aligned}(-2x + 3y + 2z)^2 &= [(-2x) + 3y + 2z]^2 \\ &= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times 3y + 2 \times 3y \times 2z + 2 \times 2z \times (-2x) \\ &= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx\end{aligned}$$

(iv) $(3a - 7b - c)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(3a - 7b - c)^2$.

$$\begin{aligned}(3a - 7b - c)^2 &= [3a + (-7b) + (-c)]^2 \\ &= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times 3a \\ &= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.\end{aligned}$$

(v) $(-2x + 5y - 3z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 5y - 3z)^2$.

$$\begin{aligned} (-2x + 5y - 3z)^2 &= [(-2x) + 5y + (-3z)]^2 \\ &= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x) \times 5y + 2 \times 5y \times (-3z) + 2 \times (-3z) \times (-2x) \\ &= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx. \end{aligned}$$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

$$\begin{aligned} \left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 &= \left[\frac{a}{4} + \left(-\frac{b}{2}\right) + 1\right]^2 \\ &= \left(\frac{a}{4}\right)^2 + \left(-\frac{b}{2}\right)^2 + (1)^2 + 2 \times \frac{a}{4} \times \left(-\frac{b}{2}\right) + 2 \times \left(-\frac{b}{2}\right) \times 1 + 2 \times 1 \times \frac{a}{4} \\ &= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}. \end{aligned}$$

5. Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Ans. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

The expression $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ can also be written as

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x.$$

We can observe that, we can apply the identity

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \text{ with respect to the expression}$$

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x,$$

$$\text{We get } (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$$

Therefore, we conclude that after factorizing the expression

$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz, \text{ we get.}$$

$$(2x + 3y - 4z)(2x + 3y - 4z)$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

The expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$ can also be written as

$$(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x).$$

We can observe that, we can apply the identity

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \text{ with respect to the expression}$$

$$(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x), \text{ to get}$$

$$(-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

Therefore, we conclude that after factorizing the expression

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz,$$

$$\text{we get } (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans. (i) $(2x+1)^3$

$$\text{We know that } (x+y)^3 = x^3 + y^3 + 3xy(x+y).$$

$$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3 \times 2x \times 1(2x+1)$$

$$= 8x^3 + 1 + 6x(2x+1)$$

$$= 8x^3 + 12x^2 + 6x + 1.$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3 + 12x^2 + 6x + 1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\begin{aligned}\therefore (2a-3b)^3 &= (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a-3b) \\ &= 8a^3 - 27b^3 - 18ab(2a-3b) \\ &= 8a^3 - 36a^2b + 54ab^2 - 27b^3.\end{aligned}$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\begin{aligned}\left(\frac{3}{2}x+1\right)^3 &= \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \frac{3}{2}x \times 1 \left(\frac{3}{2}x+1\right) \\ &= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left(\frac{3}{2}x+1\right) \\ &= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.\end{aligned}$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\begin{aligned}\therefore \left(x - \frac{2}{3}y\right)^3 &= (x)^3 - \left(\frac{2}{3}y\right)^3 - 3 \times x \times \frac{2}{3}y \left(x - \frac{2}{3}y\right) \\ &= x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y\right) \\ &= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3.\end{aligned}$$

Therefore, the expansion of the expression $\left(x - \frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$.

7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans. (i) $(99)^3$

$(99)^3$ can also be written as $(100 - 1)^3$.

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$(100 - 1)^3 = (100)^3 - (1)^3 - 3 \times 100 \times 1(100 - 1)$$

$$= 1000000 - 1 - 300(99)$$

$$= 999999 - 29700$$

$$= 970299$$

(ii) $(102)^3$

$(102)^3$ can also be written as $(100 + 2)^3$.

Using identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$(100 + 2)^3 = (100)^3 + (2)^3 + 3 \times 100 \times 2(100 + 2)$$

$$= 1000000 + 8 + 600(102)$$

$$= 1000008 + 61200$$

$$=1061208$$

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$(1000-2)^3 = (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000-2)$$

$$=1000000000-8-6000(998)$$

$$=999999992-5988000$$

$$=994011992$$

8. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 + (b)^3 + 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b).$$

Using identity

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y) \text{ with respect to the expression}$$

$$(2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b), \text{ we get } (2a + b)^3 = (2a + b)(2a + b)(2a + b)$$

Therefore, after factorizing the expression

$$8a^3 + b^3 + 12a^2b + 6ab^2, \text{ we get } (2a + b)(2a + b)(2a + b).$$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression

$$(2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b), \text{ we get } (2a - b)^3 = (2a - b)(2a - b)(2a - b)$$

Therefore, after factorizing the expression

$$8a^3 - b^3 - 12a^2b + 6ab^2,$$

we get $(2a - b)(2a - b)(2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$$= (3)^3 - (5a)^3 - 3 \times 3 \times 3 \times 5a + 3 \times 3 \times 5a \times 5a$$

$$= (3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a).$$

Using identity

$$(x - y)^3 = x^3 - y^3 - 3xy(x - y) \text{ with respect to the expression}$$

$$(3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a), \text{ we get } (3 - 5a)^3.$$

Therefore, after factorizing the expression

$$27 - 125a^3 - 135a + 225a^2, \text{ we get } (3 - 5a)^3 = (3 - 5a)(3 - 5a)(3 - 5a)$$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$$

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

with respect to the expression

$$(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b),$$

we get $(4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b)$

Therefore, after factorizing the expression

$$64a^3 - 27b^3 - 144a^2b + 108ab^2$$

we get $(4a - 3b)(4a - 3b)(4a - 3b)$.

$$(v) \ 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} + 3 \times 3p \times \frac{1}{6} \times \frac{1}{6}$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

with respect to the expression $(3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right)$,

$$\text{to get } \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right) \left(3p - \frac{1}{6}\right) \left(3p - \frac{1}{6}\right)$$

Therefore, after factorizing the expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$,

$$\text{we get } \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right) \left(3p - \frac{1}{6}\right) \left(3p - \frac{1}{6}\right)$$

9. Verify:

$$(i) \ x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$(ii) \ x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\text{Ans. (i) } x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$.

$$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$= (x + y) \left[(x + y)^2 - 3xy \right]$$

$$\because \text{We know that } (x + y)^2 = x^2 + 2xy + y^2$$

$$\therefore x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$$

$$= (x+y)(x^2 - xy + y^2)$$

Therefore, the desired result has been verified.

$$(ii) x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$= (x-y)[(x-y)^2 + 3xy]$$

* We know that $(x-y)^2 = x^2 - 2xy + y^2$

$$\therefore x^3 - y^3 = (x-y)(x^2 - 2xy + y^2 + 3xy)$$

$$= (x-y)(x^2 + xy + y^2)$$

Therefore, the desired result has been verified.

10. Factorize:

$$(i) 27y^3 + 125z^3$$

$$(ii) 64m^3 - 343n^3$$

$$\text{Ans. (i) } 27y^3 + 125z^3$$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$.

$$\begin{aligned} (3y)^3 + (5z)^3 &= (3y+5z)[(3y)^2 - 3y \times 5z + (5z)^2] \\ &= (3y+5z)(9y^2 - 15yz + 25z^2). \end{aligned}$$

$$(ii) 64m^3 - 343n^3$$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.

$$\begin{aligned} (4m)^3 - (7n)^3 &= (4m-7n)[(4m)^2 + 4m \times 7n + (7n)^2] \\ &= (4m-7n)(16m^2 + 28mn + 49n^2) \end{aligned}$$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m - 7n)(16m^2 + 28mn + 49n^2)$.

11. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Ans. The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z.$$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

$$\begin{aligned} \therefore (3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z &= (3x + y + z) \left[(3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x \right] \\ &= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz). \end{aligned}$$

Therefore, we conclude that after factorizing the expression $27x^3 + y^3 + z^3 - 9xyz$, we get $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$.

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right]$

Ans.

LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

And also, we know that $(x - y)^2 = x^2 - 2xy + y^2$.

$$\begin{aligned} &\frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right] \\ &\frac{1}{2}(x + y + z) \left[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2) \right] \\ &\frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx) \\ &(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx). \end{aligned}$$

Therefore, we can conclude that the desired result is verified.

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Ans. We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

We need to substitute $x + y + z = 0$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$, to get

$$x^3 + y^3 + z^3 - 3xyz = (0) \times (x^2 + y^2 + z^2 - xy - yz - zx)$$

$$x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz.$$

Therefore, the desired result is verified.

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Ans. (i) $(-12)^3 + (7)^3 + (5)^3$

Let $a = -12$, $b = 7$ and $c = 5$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = -12 + 7 + 5 = 0$

$$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$$

$$= -1260$$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$, $b = -15$ and $c = -13$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = 28 - 15 - 13 = 0$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380$$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

Ans. (i) Area : $25a^2 - 35a + 12$

The expression $25a^2 - 35a + 12$ can also be written as $25a^2 - 15a - 20a + 12$.

$$25a^2 - 15a - 20a + 12 = 5a(5a - 3) - 4(5a - 3)$$
$$= (5a - 4)(5a - 3).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2 - 35a + 12$ is Length = $(5a - 4)$ and Breadth = $(5a - 3)$.

(ii) Area : $35y^2 + 13y - 12$

The expression $35y^2 + 13y - 12$ can also be written as $35y^2 + 28y - 15y - 12$.

$$35y^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4)$$
$$= (7y - 3)(5y + 4).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13y - 12$ is Length = $(7y - 3)$ and Breadth = $(5y + 4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i) *Volume*: $3x^2 - 12x$

(ii) *Volume*: $12ky^2 + 8ky - 20k$

Ans. (i) *Volume* : $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x - 4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is 3, x and $(x - 4)$. so we get length=3, breadth= x , height= $(x-4)$

(ii) *Volume* : $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$$k(12y^2 + 8y - 20) = k(12y^2 - 12y + 20y - 20)$$

$$= k[12y(y - 1) + 20(y - 1)]$$

$$= k(12y + 20)(y - 1)$$

$$= 4k \times (3y + 5) \times (y - 1).$$

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k$, $(3y + 5)$ and $(y - 1)$.

so we get length= $4k$, breadth= $3y+5$, height= $y-1$