CBSE Class 9 Mathematics

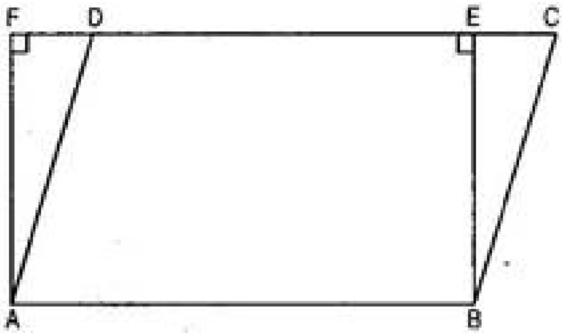
NCERT Solutions

CHAPTER 9

Areas of Parallelograms and Triangles(Ex. 9.4)

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Ans. Given: Parallelogram ABCD and rectangle ABEF are on same base AB and between the same parallel lines AB and CF.



$\therefore$ ar ($||gmABCD$) = ar (rect. ABEF)

To prove: AB + BC + CD + AD > AB + BE + EF + AF

Proof: AB = CD [$\because$ opposites sides of a parallelogram are always equal]

AB = EF [$\because$ opposites sides of a rectangle are always equal]

$\therefore$ CD = EF

Adding AB both sides,

AB + CD = AB + EF ....(i)

Now since a perpendicular is the shortest line segment from a point to a given line,

$\therefore$ BC > BE

and AD > AF

$\therefore$ BC + AD > BE + AF..........(ii)

From eq. (i) and (ii),

AB + BC + CD + AD > AB + BE + EF + AF

Hence proved.

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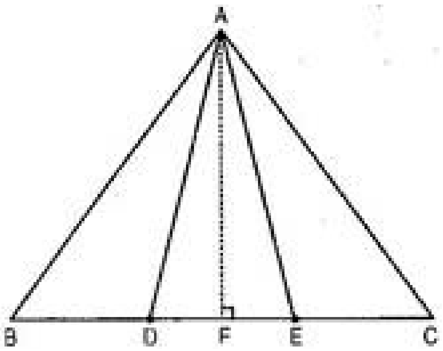
2. In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC). Can you know answer the question that you have left in the 'introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



Ans. Let AF be Perpendicular on BC **.** So AF is the height of $\triangle$'s ABD,ADE and AEC

there fore ar($\triangle$ ABD) = $\frac{1}{2}$ x BD x AF .... (1)

ar($\triangle$ADE) = $\frac{1}{2}$ x DE x AF .... (2)

ar($\triangle$AEC) = $\frac{1}{2}$ x EC x AF ....(3)

Since BD = DE = EC ....(4)

From eq(1), eq(2),eq(3) and eq(4)

we can say that

ar($\triangle$ABD) = ar($\triangle$ADE) = ar($\triangle$ AEC)

Yes the altitude of all the triange are same the Budhia has use the result for this question in dividing the land into three parts of equal area

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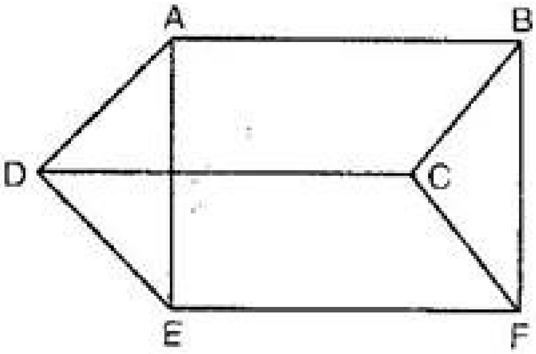
3. In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Ans. Given :- ABCD, DCFE and ABFE are parallelograms

To Prove :- ar ($\triangle$ADE) = ar ($\triangle$BCF)

Proof:-

Since opposite sides of a parallelogram are always equal.

$\therefore$ AD = BC (Opposite sides of parallelogram ABCD)

DE = CF (Opposite sides of parallelogram DEFC)

AE = BF (Opposite sides of parallelogram ABFE)

Now in $\triangle$ ADE and $\triangle$ BCF

AE = BF [Opposite sides of parallelogram ABFE]

DE = CF [Opposite sides of parallelogram DCFE]

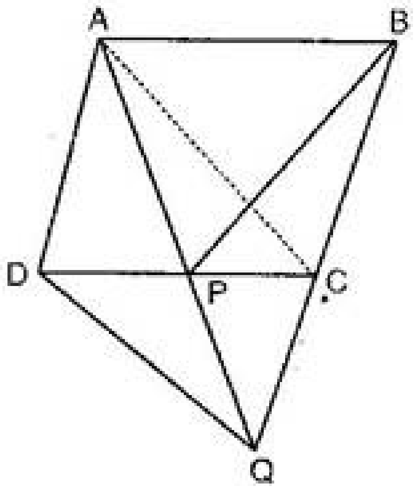
And AD = BC [Opposite sides of parallelogram ABCD]

$\therefore$ $\triangle$ADE $\cong$ $\triangle$BCF [By SSS congruency]

$\therefore$ ar ($\triangle$ADE) = ar ($\triangle$BCF)

[$\because$ Area of two congruent figures is always equal]

4. In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).



Ans. Given :- ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ

AQ intersects DC at P

To Prove :- ar (BPC) = ar (DPQ).

Construction:- Join A and C.

$\triangle$APC and $\triangle$BPC are on the same base PC and between the same parallels PC and AB.

$\therefore$ ar ($\triangle$APC) = ar ($\triangle$BPC) ....(i)

Now ACBD is a parallelogram.

AD = BC [opposite sides of a parallelogram are always equal]

Also BC = CQ [given]

$\therefore$ AD = CQ

Now AD $\parallel$ CQ [Since CQ is the extension of BC]

And AD = CQ

$\therefore$ ADQC is a parallelogram.

[$\because$ If one pair of opposite sides of a quadrilateral is equal and parallel then it is a parallelogram]

Since diagonals of a parallelogram bisect each other.

$\therefore$ AP = PQ and CP = DP

Now in $\triangle$APC and $\triangle$DPQ,

AP = PQ [Proved above]

$\angle$APC = $\angle$DPQ [Vertically opposite angles]

PC = PD [Prove above]

$\therefore \triangle APC \cong \triangle DPQ$ ....(ii)

$\Rightarrow$ ar ($\triangle$APC) = ar ($\triangle$DPQ) [area of congruent figures is always equal]

From eq. (i) and (ii),

ar ($\triangle$BPC) = ar ($\triangle$DPQ)

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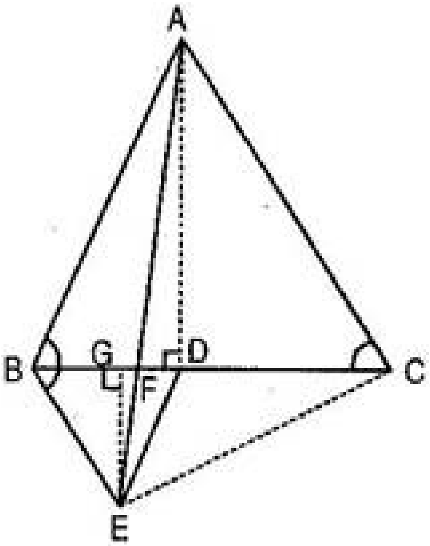
5. In figure, ABC and BDF are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that:



(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)

Ans. Join EC and AD.

Since ΔABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60^\circ$

Also ΔBDE is an equilateral triangle.

$\therefore \angle B = \angle D = \angle E = 60^\circ$

If we take two lines, AC and BE and BC as a transversal.

Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE $\parallel$ AC

Similarly, for lines AB and DE and BF as transversal.

Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE $\parallel$ AC

(i) Area of equilateral triangle BDE = $\frac{\sqrt{3}}{4}$ (BD)^2 ....(i)

Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4}$ (BC)^2 ....(ii)

Dividing eq. (i) by (ii),

$$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(BC)^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(2BD)^2} \quad [\because \text{ BD = DC}]$$

$\Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{(BD)^2}{(2BD)^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{1}{4}$

$\Rightarrow$ ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

(ii) In ΔBEC, ED is the median.

$\therefore$ ar (ΔBEC) = ar (ΔBAE) ....(i)

[Median divides the triangle in two triangles having equal area]

Now BE $\parallel$ AC

And ΔBEC and ΔBAE are on the same base BE and between the same parallels BE and AC.

$\therefore$ ar (ΔBEC) = ar (ΔBAE) ....(ii)

Using eq. (i) and (ii), we get

Ar (ΔBDE) = $\frac{1}{2}$ ar (ΔBAE)

(iii) We have ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC) [Proved in part (i)] ....(iii)

ar (ΔBDE) = $\frac{1}{4}$ ar (ΔBAE) [Proved in part (ii)]

ar (ΔBDE) = $\frac{1}{4}$ ar (ΔBEC) [Using eq. (iii)] ....(iv)

From eq. (iii) and (iv), we het

$\frac{1}{4}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔBEC)

$\Rightarrow$ ar (ΔABC) = 2 ar (ΔBEC)

(iv) ΔBDE and ΔAED are on the same base DE and between same parallels AB and DE.

$\therefore$ ar (ΔBDE) = ar (ΔAED)

Subtracting ΔFED from both the sides,

ar (ΔBDE) – ar (ΔFED) = ar (ΔAED) – ar (ΔFED)

$\Rightarrow$ ar (ΔBFE) = ar (ΔAFD) ....(v)

(v) An in equilateral triangle, median drawn is also perpendicular to the side,

$\therefore$ AD $\perp$ BC

Now ar (ΔAFD) = $\frac{1}{2} \times FD \times AD$ ....(vi)

Draw EG $\perp$ BC

$\therefore$ ar (ΔFED) = $\frac{1}{2} \times FD \times EG$ ....(vii)

Dividing eq. (vi) by (vii), we get

$$\frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)}\frac{\frac{1}{2}\times FD\times AD}{\frac{1}{2}\times FD\times EG} \Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)}=\frac{AD}{EG}$$

$\Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)}=\frac{\frac{\sqrt{3}}{4}BC}{\frac{\sqrt{3}}{4}BD}$ [Altitude of equilateral triangle = $\frac{\sqrt{3}}{4}$ side]

$\Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)}=\frac{2BD}{BD}$ [D is the mid-point of BC]

$\Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)}=2$ $\Rightarrow$ ar (ΔAFD) = 2 ar (ΔFED) ......(viii)

Using the value of eq. (viii) in eq. (v),

Ar (ΔBFE) = 2 ar (ΔFED)

(vi) ar ($\triangle$AFC) = ar ($\triangle$AFD) + ar ($\triangle$ADC) = 2 ar ($\triangle$FED) + $\frac{1}{2}$ ar ($\triangle$ABC) [using (v)

= 2 ar ($\triangle$FED) + $\frac{1}{2}$ [4 $\times$ ar ($\triangle$BDE)] [Using result of part (i)]

= 2 ar ($\triangle$FED) + 2 ar ($\triangle$BDE) = 2 ar ($\triangle$FED) + 2 ar ($\triangle$AED)

[$\triangle$BDE and $\triangle$AED are on the same base and between same parallels]

= 2 ar ($\triangle$FED) + 2 [ar ($\triangle$AFD) + ar ($\triangle$FED)]

= 2 ar ($\triangle$FED) + 2 ar ($\triangle$AFD) + 2 ar ($\triangle$FED) [Using (viii)]

= 4 ar ($\triangle$FED) + 4 ar ($\triangle$FED)

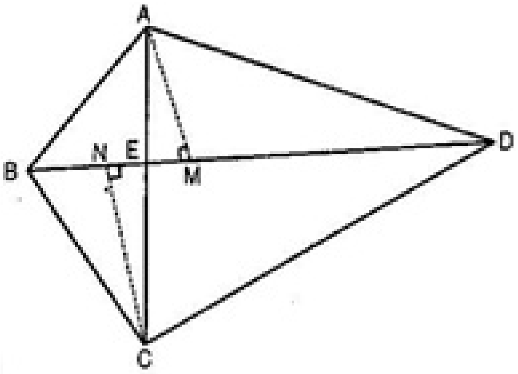
$\Rightarrow$ ar ($\triangle$AFC) = 8 ar ($\triangle$FED)

$\Rightarrow$ ar ($\triangle$FED) = $\frac{1}{8}$ ar ($\triangle$AFC)

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

ar (APB) $\times$ ar (CPD) = ar (APD) $\times$ ar (BPC)

Ans. Given: A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.



To Prove: ar ($\triangle$AED) $\times$ ar ($\triangle$BEC)

= ar ($\triangle$ABE) $\times$ ar ($\triangle$CDE)

Construction: From A, draw AM $\perp$ BD and from C, draw CN $\perp$ BD.

Proof: ar (ΔABE) = $\frac{1}{2} \times BE \times AM$ ....(i)

And ar (ΔAED) = $\frac{1}{2} \times DE \times AM$ ....(ii)

Dividing eq. (ii) by (i), we get,

$$\frac{\text{ar}(\Delta AED)}{\text{ar}(\Delta ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM} \Rightarrow \frac{\text{ar}(\Delta AED)}{\text{ar}(\Delta ABE)} = \frac{DE}{BE} \quad \text{..........(iii)}$$

Similarly $\frac{\text{ar}(\Delta CDE)}{\text{ar}(\Delta BEC)} = \frac{DE}{BE}$ ....(iv)

From eq. (iii) and (iv), we get

$$\frac{\text{ar}(\Delta AED)}{\text{ar}(\Delta ABE)} = \frac{\text{ar}(\Delta CDE)}{\text{ar}(\Delta BEC)}$$

$\Rightarrow$ ar (ΔAED) $\times$ ar (ΔBEC) = ar (ΔABE) $\times$ ar (ΔCDE)

Hence proved.

7. P and Q are respectively the mid-points of sides AB and BC or a triangle ABC and R is the mid-point of AP, show that:

(i) ar (PRQ) = $\frac{1}{2}$ **ar (ARC)**

(ii) ar (RQC) = $\frac{3}{8}$ **ar (ABC)**

(iii) ar (PBQ) = ar (ARC)

Ans. (i) PC is the median of ΔABC.

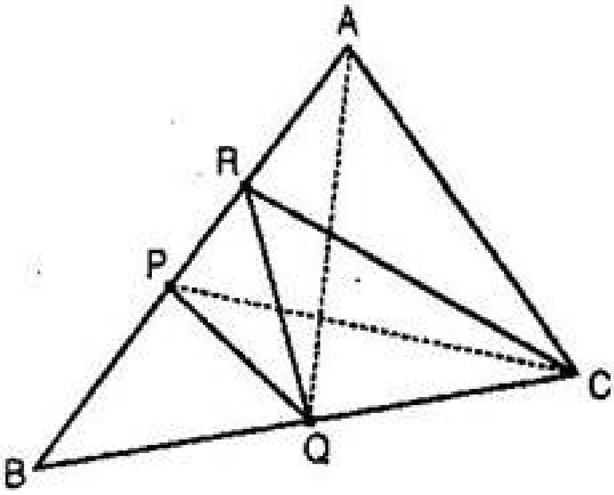
$\therefore$ ar ($\triangle$BPC) = ar ($\triangle$APC) ....(i)

RC is the median of $\triangle$APC.

$\therefore$ ar ($\triangle$ARC) = $\frac{1}{2}$ ar ($\triangle$APC) ....(ii)

[Median divides the triangle into two triangles of equal area]

PQ is the median of $\triangle$BPC.



$\therefore$ ar ($\triangle$PQC) = $\frac{1}{2}$ ar ($\triangle$BPC) ....(iii)

From eq. (i) and (iii), we get,

ar ($\triangle$PQC) = $\frac{1}{2}$ ar ($\triangle$APC) ....(iv)

From eq. (ii) and (iv), we get,

ar ($\triangle$PQC) = ar ($\triangle$ARC) ....(v)

We are given that P and Q are the mid-points of AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PA = $\frac{1}{2}$ AC

$\Rightarrow$ ar ($\triangle$APQ) = ar ($\triangle$PQC) ....(vi) [triangles between same parallel are equal in area]

From eq. (v) and (vi), we get

ar (ΔAPQ) = ar (ΔARC) ....(vii)

R is the mid-point of AP. Therefore RQ is the median of ΔAPQ.

$\therefore$ ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) ....(viii)

From (vii) and (viii), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC)

(ii) PQ is the median of ΔBPC

$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) = $\frac{1}{2}\times\frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ....(ix)

Also ar (ΔPRC) = $\frac{1}{2}$ ar (ΔAPC) [Using (iv)]

$\Rightarrow$ ar (ΔPRC) = $\frac{1}{2}\times\frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ....(x)

Adding eq. (ix) and (x), we get,

ar (ΔPQC) + ar (ΔPRC) = $\left(\frac{1}{4}+\frac{1}{4}\right)$ ar (ΔABC)

$\Rightarrow$ ar (quad. PQCR) = $\frac{1}{2}$ ar (ΔABC) ....(xi)

Subtracting ar (ΔPRQ) from the both sides,

ar (quad. PQCR) – ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔABC) – ar (ΔPRQ)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2}$ ar (ΔARC) [Using result (i)]

$\Rightarrow$ ar (ΔARC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2} \times \frac{1}{2}$ ar (ΔAPC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4}$ ar (ΔAPC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4} \times \frac{1}{2}$ ar (ΔABC) [PC is median of ΔABC]

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{8}$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\left(\frac{1}{2} - \frac{1}{8}\right) \times$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\frac{3}{8}$ ar (ΔABC)

(iii) ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC) [Using result (i)] $\Rightarrow$ 2 ar (ΔPRQ) = ar (ΔARC) ..(xii)

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) [RQ is the median of ΔAPQ] ....(xiii)

But ar (ΔAPQ) = ar (ΔPQC) [Using reason of eq. (vi)] ....(xiv)

From eq. (xiii) and (xiv), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔPQC) ....(xv)

But ar (ΔBPQ) = ar (ΔPQC) [PQ is the median of ΔBPC] ....(xvi)

From eq. (xv) and (xvi), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔBPQ) ....(xvii)

Now from (xii) and (xvii), we get,

$2\left(\frac{1}{2}\text{ar}(\Delta BPQ)\right)$ = ar (ΔARC) $\Rightarrow$ ar (ΔBPQ) = ar (ΔARC)

8. In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that:

(i) ΔMBC $\cong$ ΔABD

(ii) ar (BYXD) = 2 ar (MBC)

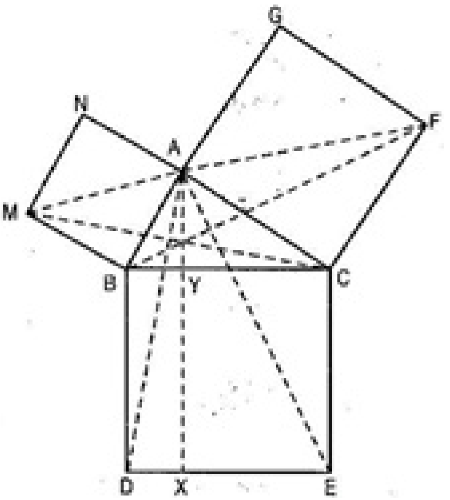
(iii) ar (BYXD) = ar (ABMN)

(iv) ΔFCB $\cong$ ΔACE

(v) ar (CYXE) = 2 ar (FCB)

(vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)



Ans. (i) $\angle$ABM = $\angle$CBD = 90°

Adding $\angle$ABC both sides, we get,

$\angle$ABM + $\angle$ABC = $\angle$CBD + $\angle$ABC $\Rightarrow$ $\angle$MBC = ABD ....(i)

Now in ΔMBC and ΔABD,

MB = AB [equal sides of square ABMN]

BC = BD [sides of square BCED]

$\angle$ MBC = $\angle$ ABD [proved above]

$\therefore$ $\triangle$ MBC $\cong$ $\triangle$ ABD [By SAS congruency]

(ii) From above, $\triangle$MBC $\cong$ $\triangle$ ABD

$\Rightarrow$ ar ($\triangle$MBC) = ar ($\triangle$ABD) $\Rightarrow$ ar ($\triangle$MBC) = ar (trap. ABDX) – ar ($\triangle$ADX)

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ (BD + AX) BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX.BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX (BY – DX)

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX. 0 [BY = DX]

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY

$\Rightarrow$ 2 ar ($\triangle$MBC) = BD.BY $\Rightarrow$ 2 ar ($\triangle$MBC) = ar (rect. BYXD)

Hence ar (BYXD) = 2 ar ($\triangle$MBC)

(iii) Join AM. ABMN is a square.

Therefore, NA $\parallel$ MB $\Rightarrow$ AC $\parallel$ MB

Now $\triangle$AMB and $\triangle$MBC are on the same base and between the same parallels MB and AC.

$\therefore$ ar ($\triangle$AMB) = ar ($\triangle$MBC) ....(ii)

From result (ii), we have ar (BYXD) = 2 ar ($\triangle$MBC) ....(iii)

Using eq. (ii) and (iii), we get, ar (BYXD) = 2 ar ($\triangle$AMB)

$\Rightarrow$ ar (BYXD) = ar (square ABMN)

[Diagonal AM of square ABMN divides it in two triangles of equal area]

(iv) In ΔFCB and ΔACE,

FC = AC [sides of square ACFG]

BC = CE [sides of square BCED]

$\angle$BCF = $\angle$ACE [$\because$ $\angle$ACF = $\angle$BCE = 90°]

Adding $\angle$ACB both sides,

$\angle$BCF + $\angle$ACB = $\angle$ACE + $\angle$ACB $\Rightarrow$ $\angle$BCF = $\angle$ACE

$\therefore$ ΔFCB $\cong$ ΔACE [By SAS congruency]

(v) From (iv), we have, ΔFCB $\cong$ ΔACE

$\Rightarrow$ ar (ΔFCB) = ar (ΔACE) $\Rightarrow$ ar (ΔFCB) = ar (trap. ACEX) – ar (ΔAEX)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ (CE + AX) CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX.CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX (CY – XE)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX. 0 [CY = XE]

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY

$\Rightarrow$ 2 ar (ΔFCB) = CE.CY $\Rightarrow$ 2 ar (ΔFCB) = ar (rect. CYXE)

Hence ar (BYXD) = 2 ar (ΔFCB)

(vi) Join AF. ACFG is a square.

$\therefore$ FC $\parallel$ AG $\Rightarrow$ FC $\parallel$ AB

Now ΔACF and ΔFCB are on the same base FC and between the same parallels FC and AB.

$\therefore$ ar (ΔACF) = ar (ΔFCB) ....(v)

From result (v), we get, ar (CYXE) = 2 ar (ΔFCB) ....(vi)

Using eq. (v) in (vi), we get, ar (CYXE) = 2 ar (ΔACF)

Diagonal AF of square ACFG divides it in two triangles of equal area.

$\therefore$ ar (CYXE) = ar (sq. ACFG) ....(vii)

(vii) Adding eq. (iv) and (vii), we get,

ar (BYXD) + ar (CYXE) = ar (ABMN) + ar (ACFG)

$\Rightarrow$ ar (BCED) = ar (ABMN) + ar (ACFG)