CBSE Class 9 Mathematics

NCERT Solutions

CHAPTER 2

Polynomials(Ex. 2.4)

1. Determine which of the following polynomials has $(x+1)$ a factor:

(i) x^3+x^2+x+1
(ii) $x^4+x^3+x^2+x+1$
(iii) $x^4+3x^3+3x^2+x+1$
(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Ans. (i) x^3+x^2+x+1

While applying the factor theorem, we get

$$p(x)=x^3+x^2+x+1$$
$$p(-1)=(-1)^3+(-1)^2+(-1)+1$$
=-1+1-1+1 =0

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as 0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$$p(x)=x^4+x^3+x^2+x+1$$
$$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$$
=1-1+1-1+1 =1

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+3x^3+3x^2+x+1$
$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$
=1-3+3-1+1 =1
We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.
Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$
While applying the factor theorem, we get
$p(x)=x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$
$p(-1)=(-1)^3-(-1)^2-\left(2+\sqrt{2}\right)(-1)+\sqrt{2}$
$=-1-1+2+\sqrt{2}+\sqrt{2}$
$=2\sqrt{2}$.
We conclude that on dividing the polynomial $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.
Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$.

2. Use the Factor Theorem to determine whether *g*(*x*) is a factor of *p*(*x*) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, $g(x)=x+1$
(ii) $p(x)=x^3+3x^2+3x+1$, $g(x)=x+2$
(iii) $p(x)=x^3-4x^2+x+6$, $g(x)=x-3$
Ans. (i) $p(x)=2x^3+x^2-2x-1$, $g(x)=x+1$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-1)*=0.

$$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$$

= 2+1-1-2 = 0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x)=x+2$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-2)*=0.

$$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$$

= -8+12-6+1 = -1

Therefore, we conclude that the *g(x)* is not a factor of *p(x)*.

(iii) $p(x)=x^3-4x^2+x+6$, $g(x)=x-3$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(3)*=0.

$$p(3)=(3)^3-4(3)^2+(3)+6$$

= 27-36+3+6 = 0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

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3. Find the value of *k*, if *x* – 1 is a factor of *p*(*x*) in each of the following cases:

(i) $p(x)=x^2+x+k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Ans. (i) $p(x) = x^2 + x + k$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = x^2 + x + k$, then $p(1) = 0$.

$p(1) = (1)^2 + (1) + k = 0$,

or

$k + 2 = 0$

$k = -2$

Therefore, we can conclude that the value of *k* is -2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then $p(1) = 0$.

$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0$,

or

$2 + k + \sqrt{2} = 0$

$k = -\left(2 + \sqrt{2}\right)$.

Therefore, we can conclude that the value of *k* is $-\left(2 + \sqrt{2}\right)$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, then $p(1) = 0$.

$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0,$

or

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1.$

Therefore, we can conclude that the value of *k* is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of p(x)

We conclude that if $(x - 1)$ is a factor of $p(x) = kx^2 - 3x + k$, then $p(1) = 0$.

$p(1) = k(1)^2 - 3(1) + k$

or $2k - 3 = 0 \quad \Rightarrow k = \frac{3}{2}$

Therefore, we can conclude that the value of *k* is $\frac{3}{2}$.

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4. Factorize:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Ans. (i) $12x^2 - 7x + 1$

$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$= 3x(4x - 1) - 1(4x - 1)$

$= (3x - 1)(4x - 1).$

Therefore, we conclude that on factorizing the polynomial $12x^2 - 7x + 1$, we get $(3x - 1)(4x - 1)$.

(ii) $2x^2 + 7x + 3$

$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x+3)+1(x+3)$

$= (2x+1)(x+3).$

Therefore, we conclude that on factorizing the polynomial $2x^2+7x+3$, we get $(2x+1)(x+3)$.

(iii) $6x^2+5x-6$

$6x^2+5x-6 = 6x^2+9x-4x-6$

$= 3x(2x+3)-2(2x+3)$

$= (3x-2)(2x+3).$

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.

(iv) $3x^2-x-4$

$3x^2-x-4 = 3x^2+3x-4x-4$

$= 3x(x+1)-4(x+1)$

$= (3x-4)(x+1).$

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

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5. Factorize:

(i) x^3-2x^2-x+2

(ii) x^3-3x^2-9x-5

(iii) $x^3+13x^2+32x+20$

(iv) $2y^3+y^2-2y-1$

Ans. (i) x^3-2x^2-x+2

We need to consider the factors of 2, which are $\pm1, \pm2$.

Let us substitute x=1 in the polynomial x^3-2x^2-x+2, to get

$(1)^3-2(1)^2-(1)+2$ =1-1-2+2=0,so we can say that P(1)=0

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial

x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$
\begin{array}{r|l}
 & x^2-x-2 \\
\hline
x-1 & x^3-2x^2-x+2 \\
 & x^3-x^2 \\
 & \underline{-\quad +} \\
 & -x^2-x \\
 & -x^2+x \\
 & \underline{+\quad -} \\
 & -2x+2 \\
 & -2x+2 \\
 & \underline{+\quad -} \\
 & 0
\end{array}
$$

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$=(x-1)(x^2+x-2x-2)$

$=(x-1)[x(x+1)-2(x+1)]$

$=(x-1)(x-2)(x+1)$.

Therefore, we can conclude that on factorizing the polynomial x^3-2x^2-x+2, we get $(x-1)(x-2)(x+1)$.

(ii) x^3-3x^2-9x-5

We need to consider the factors of -5, which are $\pm1, \pm5$.

Let us substitute x= -1 in the polynomial x^3-3x^2-9x-5, to get

$(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$,so p(-1)=0

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial x^3-3x^2-9x-5.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$\begin{array}{r|l} x+1 & \overline{x^3-3x^2-9x-5} \quad (x^2-4x-5 \\ & \underline{\overset{-}{x^3}+\overset{-}{x^2}} \\ & -4x^2-9x \\ & \underline{\overset{+}{-4x^2}\ \overset{+}{-4x}} \\ & -5x-5 \\ & \underline{\overset{+}{-5x}\ \overset{+}{-5}} \\ & \underline{\quad 0} \end{array}$$

$x^3-3x^2-9x-5=(x+1)(x^2-4x-5)$

$=(x+1)(x^2+x-5x-5)$

$=(x+1)[x(x+1)-5(x+1)]$

$=(x+1)(x-5)(x+1).$

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$.

(iii) $x^3+13x^2+32x+20$

We need to consider the factors of 20, which are $\pm5, \pm4, \pm2, \pm1$.

Let us substitute x= -1 in the polynomial $x^3+13x^2+32x+20$, to get

$(-1)^3+13(-1)^2+32(-1)+20$ =-1+13-32+20=-20+20=0

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3+13x^2+32x+20$.

Let us divide the polynomial $x^3+13x^2+32x+20$ by $(x+1)$, to get

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\cline{2-2}
x+1 & x^3+13x^2+32x+20 \\
 & \underline{\underset{-}{x^3}+\underset{-}{x^2}} \\
 & 12x^2+32x \\
 & \underline{\underset{-}{12x^2}+\underset{-}{12x}} \\
 & 20x+20 \\
 & \underline{\underset{-}{20x}+\underset{-}{20}} \\
 & \underline{\quad 0}
\end{array}
$$

$$x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$$
$$=(x+1)(x^2+2x+10x+20)$$
$$=(x+1)[x(x+2)+10(x+2)]$$
$$=(x+1)(x+10)(x+2).$$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$.

(iv) $2y^3+y^2-2y-1$

We need to consider the factors of -1, which are ± 1.

Let us substitute y=1 in the polynomial $2y^3+y^2-2y-1$, to get

$2(1)^3+(1)^2-2(1)-1$ =2+1-2-1=3-3=0,so y(1)=0

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$
\begin{array}{r|l}
 & 2y^2+3y+1 \\
\hline
y-1 & 2y^3+y^2-2y-1 \\
 & \underline{\overset{-}{2y^3}\overset{+}{-2y^2}} \\
 & 3y^2-2y \\
 & \underline{\overset{-}{3y^2}\overset{+}{-3y}} \\
 & y-1 \\
 & \underline{\overset{-}{y}\overset{+}{-1}} \\
 & \underline{\quad 0}
\end{array}
$$

$2y^3+y^2-2y-1=(y-1)(2y^2+3y+1)$

$=(y-1)(2y^2+2y+y+1)$

$=(y-1)[2y(y+1)+1(y+1)]$

$=(y-1)(2y+1)(y+1).$

Therefore, we can conclude that on factorizing the polynomial $2y^3+y^2-2y-1$, we get $(y-1)(2y+1)(y+1)$.