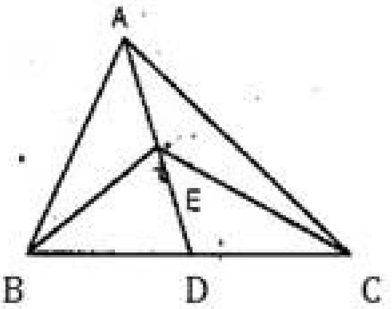
CBSE Class 9 Mathematics

NCERT Solutions

CHAPTER 9

Areas of Parallelograms and Triangles(Ex. 9.3)

1. In figure, E is any point on median AD of a $\triangle$ABC. Show that ar ($\triangle$ABE) = ar ($\triangle$ACE).



Ans. Given :- AD is the median of $\triangle$ABC

To Prove :- ar($\triangle$ABE) = ar($\triangle$ACE)

Proof :- In $\triangle$ABC, AD is a median.

ar ($\triangle$ABD) = ar ($\triangle$ACD) ....(i)

[$\because$ Median divides a $\triangle$ into two $\triangle$s of equal area]

Again in $\triangle$EBC, ED is a median

ar ($\triangle$EBD) = ar ($\triangle$ECD) ....(ii)

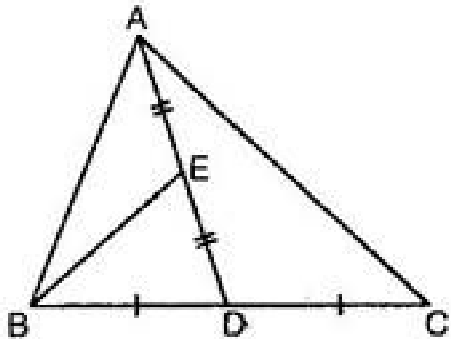
Subtracting eq. (ii) from (i),

ar ($\triangle$ABD) – ar ($\triangle$EBD) = ar ($\triangle$ACD) – ar ($\triangle$ECD)

$\Rightarrow$ ar ($\triangle$ABE) = ar ($\triangle$ACE)

2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).

Ans. Given: A $\triangle$ABC, AD is the median and E is the mid-point of median AD.



To prove: ar ($\triangle$BED) = $\frac{1}{4}$ ar ($\triangle$ABC)

Proof: In $\triangle$ABC, AD is the median.

$\therefore$ ar ($\triangle$ABD) = ar ($\triangle$ADC)

[$\because$ Median divides a $\triangle$ into two $\triangle$s of equal area]

$\Rightarrow$ ar ($\triangle$ABD) = $\frac{1}{2}$ ar ($\triangle$ ABC)..........(i)

In $\triangle$ABD,

E is a mid point of AD

AE = AD

=> BE is the median.

$\therefore$ ar ($\triangle$BED) = ar ($\triangle$BAE)

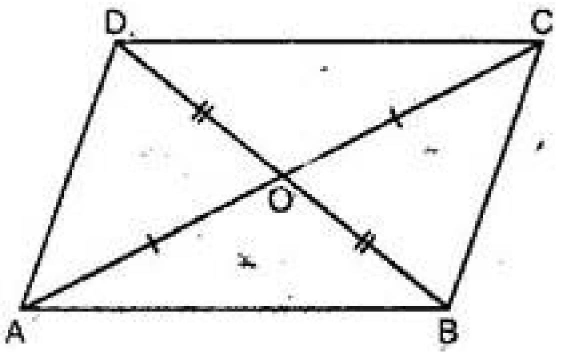
[$\because$ Median divides a $\triangle$ into two $\triangle$s of equal area]

$\Rightarrow$ ar ($\triangle$BED) = $\frac{1}{2}$ ar ($\triangle$ABD) .... (2)

from eq(1) and eq(2)

$\Rightarrow$ ar ($\triangle$BED) = $\frac{1}{2}\times\frac{1}{2}$ ar ($\triangle$ABC) = $\frac{1}{4}$ ar ($\triangle$ABC)

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Given. Parallelogram ABCD BD and AC are diagonal intersect at O

To Prove :- Diagonals of parallelogram divide it into four triangles of equal area.

Proof :- We know that diagnol of parallelogram bisect each other

Therefore O is a median of AC and BD

In $\triangle$ABC

AO = OC so we can say OB is median

therefore ar($\triangle$AOB) = ar($\triangle$BOC) .... (1)

Now in $\triangle$BCD

BO = OD so we can say OC is median

therefore ar($\triangle$BOC) = ar($\triangle$COD) .... (2)

Now in $\triangle$ACD

CO = OA so we can say OD is median

therefore ar($\triangle$COD) = ar($\triangle$AOD) .... (3)

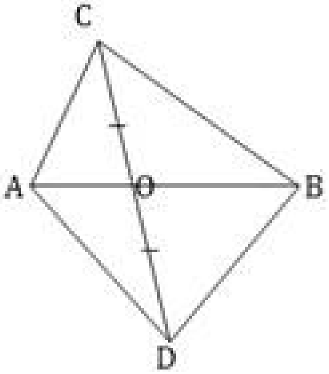
From eq(1) , eq(2) and eq(3)

we have

($\triangle$AOB) = ar($\triangle$BOC) = ar($\triangle$CO
D) = ar($\triangle$AOD)

So we can say Diagonals of parallelogram divide it into four triangles of equal area.

4. In figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Given:- $\triangle$ABC and $\triangle$DBC are on the same base AB . Line segment CD is bisected by AB at O

To Prove :- ar($\triangle$ABC) = ar($\triangle$ABD)

Proof:- Line segment CD is bisected by AB at O

In $\triangle$ACD

OC = OD

there fore AO is the median of $\triangle$ACD

ar($\triangle$ACO) = ar($\triangle$ADO) ....(1)

In $\triangle$BCD

OC = OD

there fore BO is the median of $\triangle$BCD

ar($\triangle$BCO) = ar($\triangle$BDO) ....(2)

Add (1) and (2)

ar($\triangle$ACO) + ar($\triangle$BCO) = ar($\triangle$ADO) + ar($\triangle$BDO)

ar($\triangle$ABC) = ar($\triangle$ABD)

5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a $\triangle$ABC.

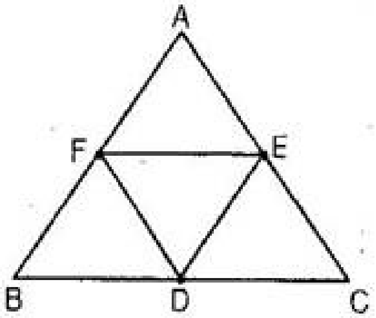
Show that:

(i) BDEF is a parallelogram.

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

Ans. (i) F is the mid-point of AB and E is the mid-point of AC.



$\therefore$ FE $\parallel$ BC and FE = $\frac{1}{2}$ BD

[$\because$ Line joining the mid-points of two sides of a triangle is parallel to the third and half of it]

$\Rightarrow$ FE $\parallel$ BD [BD is the part of BC]

And FE = BD

Also, D is the mid-point of BC.

$\therefore$ BD = $\frac{1}{2}$ BC

And FE $\parallel$ BC and FE = BD

Again E is the mid-point of AC and D is the mid-point of BC.

$\therefore$ DE $\parallel$ AB and DE = $\frac{1}{2}$ AB

$\Rightarrow$ DE $\parallel$ AB [BF is the part of AB]

And DE = BF

Again F is the mid-point of AB.

$\therefore$ BF = $\frac{1}{2}$ AB

But DE = $\frac{1}{2}$ AB

$\therefore$ DE = BF

Now we have FE $\|$ BD and DE $\|$ BF

And FE = BD and DE = BF

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

$\therefore$ ar ($\triangle$BDF) = ar ($\triangle$DEF) ....(i)

[diagonals of parallelogram divides it in two triangles of equal area]

DCEF is also parallelogram.

$\therefore$ ar ($\triangle$DEF) = ar ($\triangle$DEC) ....(ii)

Also, AEDF is also parallelogram.

$\therefore$ ar ($\triangle$AFE) = ar ($\triangle$DEF) ....(iii)

From eq. (i), (ii) and (iii),

ar ($\triangle$DEF) = ar ($\triangle$BDF) = ar ($\triangle$DEC) = ar ($\triangle$AFE) ....(iv)

Now, ar ($\triangle$ABC) = ar ($\triangle$DEF) + ar ($\triangle$BDF) + ar ($\triangle$DEC) + ar ($\triangle$AFE) ....(v)

$\Rightarrow$ ar ($\triangle$ABC) = ar ($\triangle$DEF) + ar ($\triangle$DEF) + ar ($\triangle$DEF) + ar ($\triangle$DEF)

[Using (iv) & (v)]

$\Rightarrow$ ar ($\triangle$ABC) = 4 $\times$ ar ($\triangle$DEF)

$\Rightarrow$ ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC)

(iii) ar ($\|$ gm BDEF) = ar (ΔBDF) + ar (ΔDEF) = ar (ΔDEF) + ar (ΔDEF) [Using (iv)]

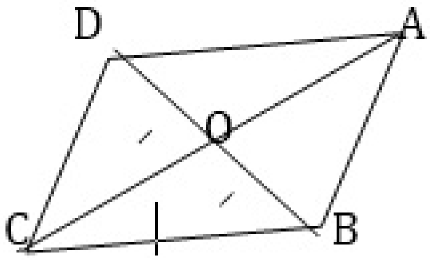
$\Rightarrow$ ar ($\|$ gm BDEF) = 2 ar (ΔDEF)

$\Rightarrow$ ar ($\|$ gm BDEF) = $2 \times \frac{1}{4}$ ar (ΔABC)

$\Rightarrow$ ar ($\|$ gm BDEF) = $\frac{1}{2}$ ar (ΔABC)

---

6. In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:



(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA $\|$ CB or ABCD is a parallelogram.

Ans. Given:- Diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD

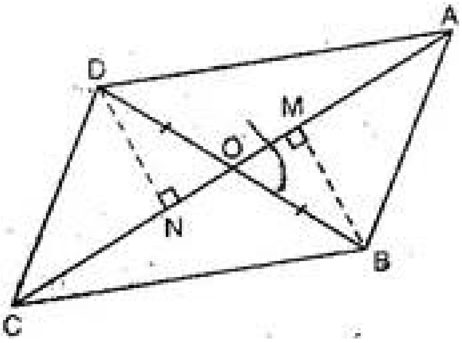
To Prove:- (i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA II CB or ABCD is a parallelogram.

Construction:- Draw BM $\perp$ AC and DN $\perp$ AC.

Proof:-



In $\triangle$DON and $\triangle$BOM,

OD = OB [Given]

$\angle$DNO = $\angle$BMO = 90° [By construction]

$\angle$DON = $\angle$BOM [Vertically opposite]

$\therefore$ $\triangle$DON $\cong$ $\triangle$BOM [By RHS congruency]

$\Rightarrow$ DN = BM [By CPCT]

Also ar ($\triangle$DON) = ar ($\triangle$BOM) ....(i)

Again, In $\triangle$DCN and $\triangle$ABM,

CD = AB [Given]

$\angle$DNC = $\angle$BMA = 90° [By construction]

DN = BM [Prove above]

$\therefore$ $\triangle$DCN $\cong$ $\triangle$BAM [By RHS congruency]

$\therefore$ ar ($\triangle$DCN) = ar ($\triangle$BAM) ....(ii)

Adding eq. (i) and (ii),

ar ($\triangle$DON) + ar ($\triangle$DCN) = ar ($\triangle$BOM) + ar ($\triangle$BAM)

$\Rightarrow$ ar ($\triangle$DOC) = ar ($\triangle$AOB)

(ii) Since ar ($\triangle$DOC) = ar ($\triangle$AOB)

Adding ar $\triangle$BOC both sides,

ar ($\triangle$DOC) + ar $\triangle$BOC = ar ($\triangle$AOB) + ar $\triangle$BOC

$\Rightarrow$ ar ($\triangle$DCB) = ar ($\triangle$ACB)

(iii) Since ar ($\triangle$DCB) = ar ($\triangle$ACB)

Therefore, these two triangles in addition to be on the same base CB lie between two same parallels CB and DA.

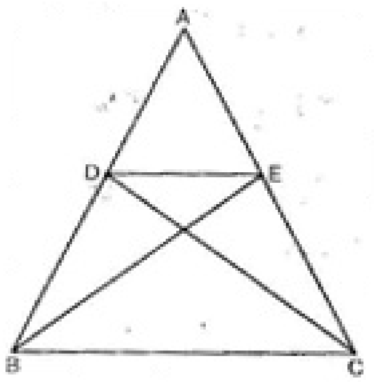
$\therefore$ DA $\|$ CB

Now AB = CD and DA $\|$ CB

Therefore, ABCD is a parallelogram.

7. D and E are points on sides AB and AC respectively of $\triangle$ABC such that ar (DBC) = ar (EBC). Prove that DE $\|$ BC.

Ans. Given: ar (DBC) = ar (EBC)



To Prove:- DE II BC.

Proof:- Since two triangles of equal area have common base BC.

Therefore, DE II BC [$\because$ Two triangles having same base (or equal bases) and equal areas lie between the same parallel]

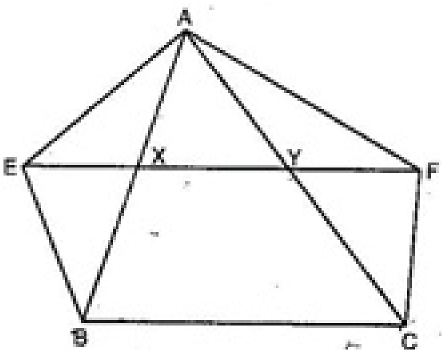
8. XY is a line parallel to side BC of triangle ABC. If BE $\|$ AC and CF $\|$ AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF).

Ans. Given: XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively.

To Prove:- ar(ABE) = ar(ACF)

Proof: $Area(= Area(\Delta ACF)$

$\triangle$ABE and parallelogram BCYE lie on the same base BE and between the same parallels BE and AC.



$\therefore$ ar ($\triangle$ABE) = $\frac{1}{2}$ ar ($\|$ gm BCYE) ....(i)

Also $\triangle$ACF and $\|$gm BCFX lie on the same base CF and between same parallel BX and CF.

$\therefore$ ar ($\triangle$ACF) = $\frac{1}{2}$ ar ($\|$ gm BCFX) ....(ii)

But $\|$gm BCYE and $\|$gm BCFX lie on the same base BC and between the same parallels BC and EF.

$\therefore$ ar ($\|$gm BCYE) = ar ($\|$gm BCFX) ....(iii)

From eq. (i), (ii) and (iii), we get,

ar ($\triangle$ABE) = ar ($\triangle$ACF)

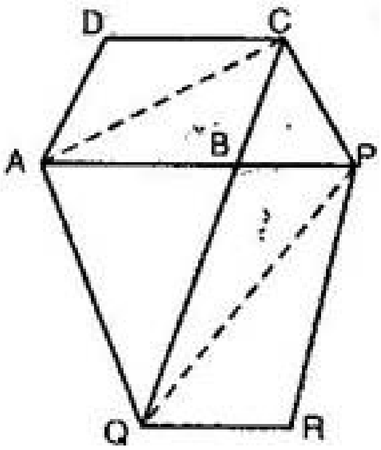
9. The side AB of parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar (ABCD) = ar (PBQR).

Ans. Given: ABCD is a parallelogram, CP$\parallel$AQ and PBQR is a parallelogram.

To prove: ar (ABCD) = ar (PBQR)

Construction: Join AC and QP.

Proof: Since AQ$\parallel$CP



$\therefore$ ar ($\triangle$AQC) = ar ($\triangle$AQP)

[Triangles on the same base and between the same parallels are equal in area]

Subtracting ar ($\triangle$ABQ) from both sides, we get

ar ($\triangle$AQC – ar ($\triangle$ABQ) = ar ($\triangle$AQP) – ar ($\triangle$ABQ)

$\Rightarrow$ ar ($\triangle$ABC) = ar ($\triangle$QBP) ....(i)

Now ar ($\triangle$ABC) = $\frac{1}{2}$ ar ($\parallel$gm ABCD)

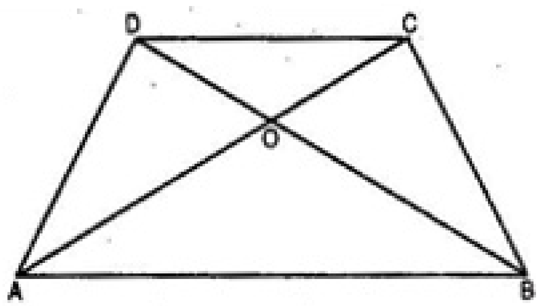
[Diagonal divides a parallelogram in two triangles of equal area]

And ar ($\triangle$PQB) = $\frac{1}{2}$ ar ($\parallel$gm PBQR)

From eq. (i), (ii) and (iii), we get

ar ($\parallel$gm ABCD) = ar ($\parallel$gm PBQR)

10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(AOD) = ar (BOC).



Given:- Trapezium ABCD with ABIIDC Diagonals AC and BD intersect each other at O

To Prove :- ar(AOD) = ar (BOC).

Proof :-

$\triangle$ABD and $\triangle$ABC lie on the same base AB and between the same parallels AB and DC.

$\therefore$ ar ($\triangle$ABD) = ar ($\triangle$ABC)

Subtracting ar ($\triangle$AOB) from both sides,

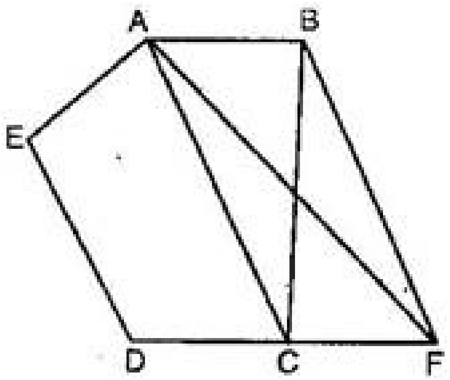
ar ($\triangle$ABD) – ar ($\triangle$AOB) = ar ($\triangle$ABC) – ar ($\triangle$AOB)

$\Rightarrow$ ar ($\triangle$AOD) = ar ($\triangle$BOC)

11. In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that:

(i) ar (ACB = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Ans. (i) Given that BF II AC

To Prove :-

(i) ar (ACB = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Proof:-

$\triangle$ACB and $\triangle$ACF lie on the same base AC and between the same parallels AC and BF.

$\therefore$ ar ($\triangle$ACB) = ar ($\triangle$ACF) ....(i)

(ii) Now add ar (AEDC) on both side of equation (i)

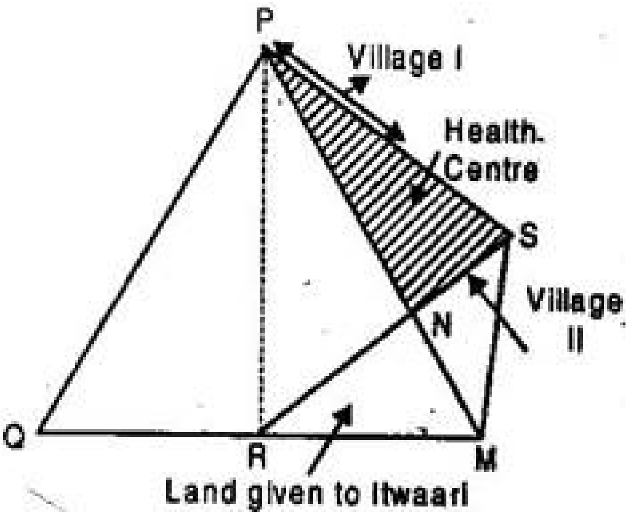
$\Rightarrow$ ar ($\triangle$ACB) + ar (AEDC) = ar ($\triangle$ACF) +ar (AEDC)

$\Rightarrow$ ar (ABCDE) = ar (AEDF)

12. A villager Itwaari has a plot of land of the shape of quadrilateral. The Gram Panchyat of two villages decided to take over some portion of his plot from one of the corners to construct a health centre. Itwaari agrees to the above personal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Ans. Let Itwari has land in shape of quadrilateral PQRS.

Draw a line through 5 parallel to PR, which meets QR produced at M.



Let diagonals PM and RS of new formed quadrilateral intersect each other at point N.

We have PR || SM [By construction]

$\therefore$ ar ($\triangle$PRS) = ar ($\triangle$PMR)

[Triangles on the same base and same parallel are equal in area]

Subtracting ar ($\triangle$PNR) from both sides,

ar ($\triangle$PRS) – ar ($\triangle$PNR) = ar ($\triangle$PMR) – ar ($\triangle$PNR)

$\Rightarrow$ ar ($\triangle$PSN) = ar ($\triangle$MNR)

It implies that Itwari will give corner triangular shaped plot PSN to the Grampanchayat for health centre and will take equal amount of land (denoted by $\triangle$MNR) adjoining his plot so as to form a triangular plot PQM.

---

13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY)

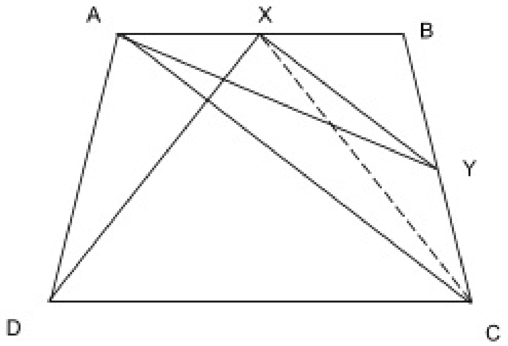
Given:- ABCD is a trapezium with AB II DC A line parallel to AC intersects AB at X and BC at Y

To Prove :- ar ($\triangle$ADX) = ar ($\triangle$ACY)

Constuction:- Join CX,

Proof:-

$\triangle$ADX and $\triangle$ACX lie on the same base XA and between the same parallels XA and DC.



$\therefore$ ar ($\triangle$ADX) = ar ($\triangle$ACX) ....(i)

Also $\triangle$ACX and $\triangle$ACY lie on the same base

AC and between same parallels CY and XA.

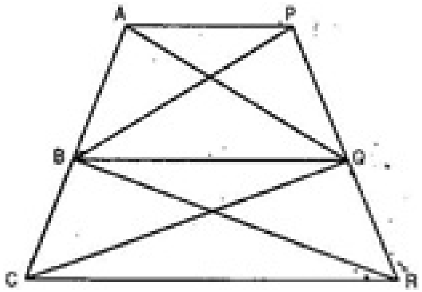
$\therefore$ ar ($\triangle$ACX) = ar ($\triangle$ACY) ....(ii)

From (i) and (ii),

ar ($\triangle$ADX) = ar ($\triangle$ACY)

---

14. In figure, AP $\parallel$ BQ $\parallel$ CR. Prove that ar (AQC) = ar (PBR).



Given :- AP II BQ II CR.

To Prove :- ar (AQC) = ar (PBR)

Proof:- $\triangle$ABQ and BPQ lie on the same base BQ and between same parallels AP and BQ.

$\therefore$ ar ($\triangle$ABQ) = ar ($\triangle$BPQ) ....(i)

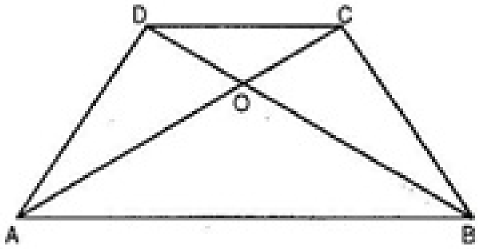
$\triangle$BQC and $\triangle$BQR lie on the same base BQ and between same parallels BQ and CR.

$\therefore$ ar ($\triangle$BQC) = ar ($\triangle$BQR) ....(ii)

Adding eq (i) and (ii), ar ($\triangle$ABQ) + ar ($\triangle$BQC) = ar ($\triangle$BPQ) + ar ($\triangle$BQR)

$\Longrightarrow$ ar ($\triangle$AQC) = ar ($\triangle$PBR)

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar(BOC). Prove that ABCD is a trapezium.



Given :- ar ($\triangle$AOD) = ar ($\triangle$BOC)

To prove :-ar (AOD) = ar(BOC)

Proof: ar ($\triangle$AOD) = ar ($\triangle$BOC)

Adding $\triangle$AOB both sides,

ar ($\triangle$AOD) + ar ($\triangle$AOB) = ar ($\triangle$BOC) + ar ($\triangle$AOB)

$\Longrightarrow$ ar ($\triangle$ABD) = ar ($\triangle$ABC)

Since if two triangles equal in area, lie on the same base then, they lie between same parallels. We have $\triangle$ABD and $\triangle$ABC lie on common base AB and are equal in area.

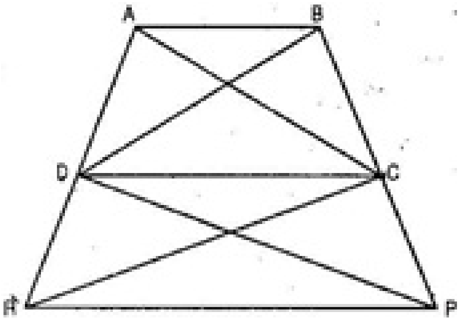
$\therefore$ They lie in same parallels AB and DC.

$\Longrightarrow$ AB$\parallel$DC

Now in quadrilateral ABCD, we have AB$\parallel$DC

Therefore, ABCD is trapezium. [$\because$ In trapezium one pair of opposite sides is parallel]

16. In figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Given :- ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC)

To Prove :- the quadrilaterals ABCD and DCPR are trapeziums.

Proof:- Given that $\triangle$DRC and $\triangle$DPC lie on the same base DC and ar ($\triangle$DPC) = ar ($\triangle$DRC) .....(i)

$\therefore$ DC II RP

[If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, DCPR is trapezium. [$\because$ In trapezium one pair of opposite sides is parallel]

Also ar ($\triangle$BDP) = ar ($\triangle$ARC) ....(ii)

Subtracting eq. (i) from (ii),

ar ($\triangle$BDP) – ar ($\triangle$DPC) = ar ($\triangle$ARC) – ar ($\triangle$DRC)

$\Rightarrow$ ar ($\triangle$BDC) = ar ($\triangle$ADC)

Therefore, AB $\parallel$ DC [If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, ABCD is trapezium.