CBSE Class 9 Mathematics

NCERT Solutions

CHAPTER 8

Quadrilaterals(Ex. 8.1)

1. The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all angles of the quadrilateral.

Ans. Let in quadrilateral ABCD, $\angle A = 3x, \angle B = 5x, \angle C = 9x, \angle D = 13x$

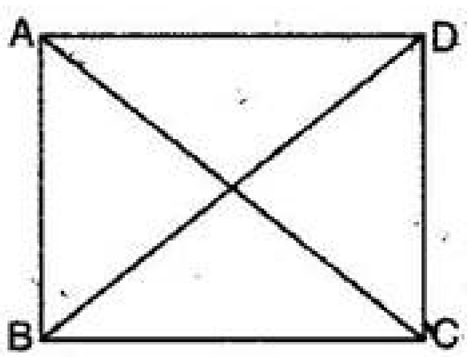
Since, sum of all the angles of a quadrilateral = 360^o

$\therefore\ \angle A + \angle B + \angle C + \angle D = 360^o$
$\Rightarrow 3x + 5x + 9x + 13x = 360^\circ$
$\Rightarrow\ 30x = 360^\circ$
$\Rightarrow\ x = 12^\circ$

Now $\angle A = 3x = 3 \times 12^o = 36^o$
$\angle B = 5x = 5 \times 12^o = 60^o$
$\angle C = 9x = 9 \times 12^o = 108^o$
And $\angle D = 13x = 13 \times 12^o = 156^o$
Hence angles of given quadrilateral are 36^o, 60^o, 108^o and 156^o

2. If the diagonals of a parallelogram are equal, show that it is a rectangle.

Ans. Given: ABCD is a parallelogram with diagonal AC = diagonal BD
To prove: ABCD is a rectangle.



Proof: In triangles ABC and ABD,

AB = AB [Common]

AC = BD [Given]

AD = BC [Opposite sides of a $||gm$]

$\therefore\ \Delta ABC \cong \Delta BAD$ [By SSS congruency]

$\Rightarrow \angle DAB = \angle CBA$ [By C.P.C.T.] ....(i)

But $\angle DAB + \angle CBA = 180^o$

[$\because$ The sum of consecutve angles of a parallelogram is$= 180^o$] ....(ii)

From eq. (i) and (ii),

$\angle DAB = \angle CBA = 90^o$

Simillarly, the other two angles are of 90^o each.

Hence ABCD is a rectangle.

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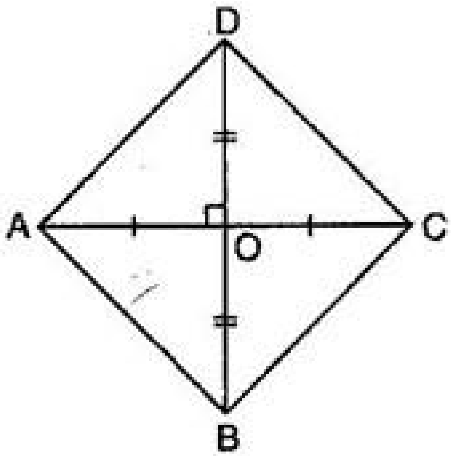
3. Show that if diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans. Let ABCD is a quadrilateral.



Given: The diagonals AC and BD bisect each other at right angle at point O.

$\therefore$ OA = OC, OB = OD

And $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^o$

To prove: ABCD is a rhombus, i.e.we have to prove that AB = BC = CD = DA

Proof: In ΔDOA and ΔBOC,

OA = OC [Given]

$\angle AOD = \angle BOC$ [Given]

OB = OD [Given]

$\therefore \ \Delta AOD \cong \Delta COB$ [By SAS congruency]

$\Rightarrow$ **AD** = **BC** [By C.P.C.T.] ....(i)

Again, In ΔAOB and ΔCOD,

OA = OC [Given]

$\angle AOB = \angle COD$ [Given]

OB = OD [Given]

$\therefore \Delta AOB \cong \Delta COD$ [By SAS congruency]

$\Rightarrow$ **AB** = **CD** [By C.P.C.T.] ....(ii)

Now In ΔAOB and ΔBOC,

OA = OC [Given]

$\angle AOB = \angle BOC$ [Given]

OB = OB [Common]

$\therefore$ ΔAOB $\cong$ Δ COB [By SAS congruency]

$\Rightarrow$ **AB** = **BC** [By C.P.C.T.] ....(iii)

From eq. (i), (ii) and (iii),

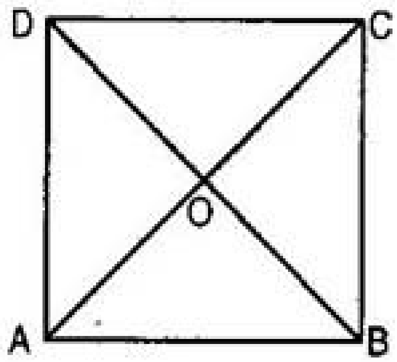
AD = BC = CD = AB

And the diagonals of quadrilateral ABCD bisect each other at right angle.

Therefore, ABCD is a rhombus.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans. Given: ABCD is a square. AC and BD are its diagonals intersect each other at point O.



To prove:(I) AC = BD

(ii) OA = OC and OB = OD

(iii) AC $\perp$ BD at point O.

Proof: In triangles ABC and BAD,

AB = AB [Common]

$\angle$ ABC = $\angle$ BAD = 90° [Because ABCD is a square]

BC = AD [Sides of a square]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$ BAD [By SAS congruency]

$\Rightarrow$ **AC** = **BD** [By C.P.C.T.]

Thus, the diagonals of a square are equal. **Hence proved (i)**.

Again in $\triangle$AOB and $\triangle$COD,

$\angle ABO = \angle CDO$ [Alternate angles]

$\angle AOB = \angle COD$ [Vertically opposite angles]

AB = CD [Sides of a square]

$\therefore\ \Delta AOB \cong \Delta COD$ [By AAS congruency]

$\Rightarrow$ OA = OC AND OB = OD [By C.P.C.T.]

Thus, diagonals of a square bisect each other. **Hence proved (ii).**

Now in triangles AOB and AOD,

AO = AO [Common]

AB = AD [Sides of a square]

OB = OD [Diagonals of a square bisect each other]

$\therefore\ \Delta$AOB $\cong$ Δ AOD [By SSS congruency]

$\angle$ AOB = $\angle$ AOD [By C.P.C.T.]

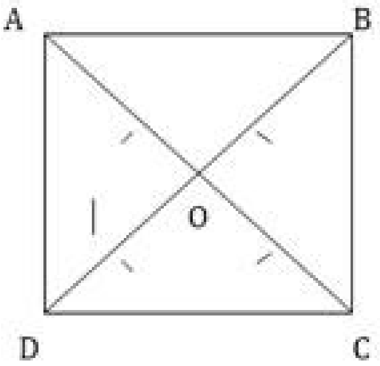
But $\angle$ AOB + $\angle$ AOD = 180° [Linear pair]

$\therefore$ $\angle$ AOB = $\angle$ AOD = 90°

$\Rightarrow$ OA $\perp$ BD or AC $\perp$ BD. Hence proved (iii).

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Ans. Let ABCD be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.



Given: AC = BD ....(i)

OA = OC ....(ii)

And OB = OD ....(iii)

To Prove: ABCD is a square. i.e all the sides are equal and all the angles are of 90^o each.

Proof: Now OA + OC = OB + OD

$\Rightarrow$ OC + OC = OB + OB [Using (i), (ii) & (iii)]

$\Rightarrow$ 2OC = 2OB

$\Rightarrow$ OC = OB ....(iv)

From eq. (i), (ii), (iii) and (iv), we get, OA = OB = OC = OD ....(v)

Now in $\triangle$AOB and $\triangle$COD,

OA = OD [proved]

$\angle$AOB = $\angle$COD [Vertically opposite angles]

OB = OC [proved]

$\therefore$ $\triangle$AOB $\cong$ $\triangle$COD [By SAS congruency]

$\Rightarrow$ **AB** = **CD** [By C.P.C.T.] ....(vi)

Similarly, $\triangle$BOC $\cong$ $\triangle$AOD [By SAS congruency]

$\Rightarrow$ **BC** = **AD** [By C.P.C.T.] ....(vii)

From eq. (vi) and (vii), it is concluded that **ABCD is a parallelogram because opposite sides of a quadrilateral are equal.**

Now in $\triangle$ABC and $\triangle$BAD,

AB = BA [Common]

BC = AD [proved above]

AC = BD [Given]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$BAD [By SSS congruency]

$\Rightarrow$ $\angle$ABC = $\angle$BAD [By C.P.C.T.] ....(viii)

But $\angle$ABC + $\angle$BAD = 180° [ABCD is a parallelogram] ....(ix)

$\Rightarrow$ $\angle$ABC + $\angle$ABC = 180° [Using eq. (viii) and (ix)]

$\Rightarrow$2$\angle$ABC = 180° $\Rightarrow$ $\angle$ABC = 90°

$\therefore \angle ABC = \angle BAD = 90^o$ ....(x)

But $\angle$ABC = $\angle$ADC [Opposite angles of a parallelogram are equal.]

$\therefore$ $\angle$ABC = $\angle$ADC = ...(xi)

$\therefore$ $\angle$BAD = $\angle$BDC = 90° ....(xii)

From eq. (xi) and (xii), we get

$\angle$ABC = $\angle$ADC = $\angle$BAD = $\angle$BDC = 90° ....(xiii)

Now in $\triangle$AOB and $\triangle$BOC,

OA = OC [Given]

$\angle$AOB = $\angle$BOC = 90° [Given]

OB = OB [Common]

$\therefore$ $\triangle$AOB $\cong$ $\triangle$COB [By SAS congruency]

$\Rightarrow$ **AB** = **BC** ....(xiv)

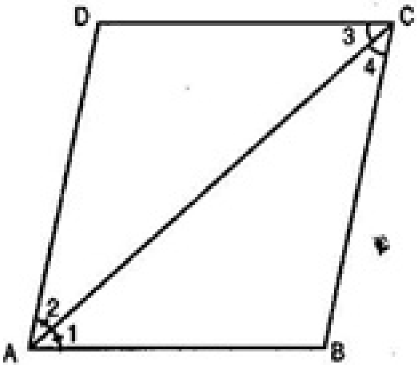
From eq. (vi), (vii) and (xiv), we get,

AB = **BC** = **CD** = **AD** ....(xv)

Now, from eq. (xiii) and (xv), we have a quadrilateral whose sides are equal make an angle of 90° with each other.

$\therefore$ ABCD is a square. **Hence Proved**

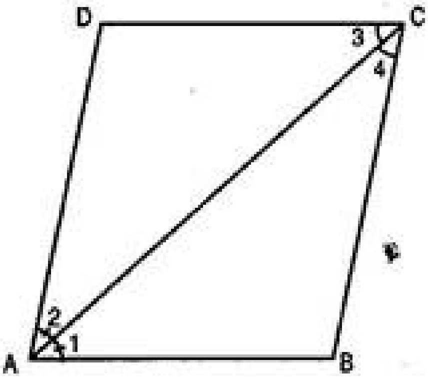
6. Diagonal AC of a parallelogram ABCD bisects $\angle$ A (See figure). Show that:



(i) It bisects $\angle$ C also.

(ii) ABCD is a rhombus.

Ans. Diagonal AC bisects $\angle$ A of the parallelogram ABCD.



(i) Since AB $\parallel$ DC and AC intersects them.

$\therefore$ $\angle 1 = \angle 3$ [Alternate angles] ....(i)

Similarly $\angle 2 = \angle 4$ ....(ii)

But $\angle 1 = \angle 2$ [Given] ....(iii)

$\therefore$ $\angle$ **3** = $\angle$ **4** [Using eq. (i), (ii) and (iii)]

Thus AC bisects $\angle$ C.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$ [Proved above]

Thus, $\angle 2 = \angle 3$

$\Rightarrow$ AD = CD [Sides opposite to equal angles] ....(iv)

But as ABCD is a parallelogram, therfore AB = CD and BC = AD

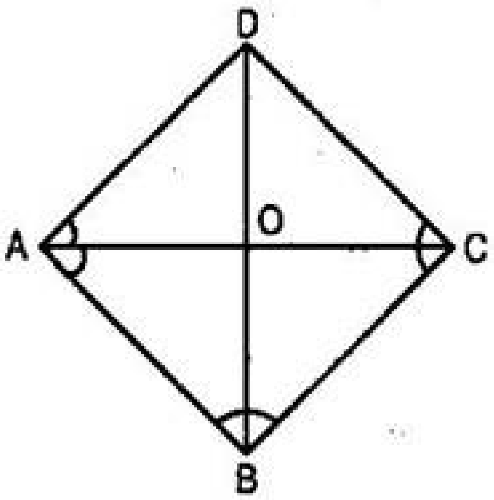
So, we get AB = CD = AD = BC [Using (iv)]

Hence ABCD is a rhombus.

7. ABCD is a rhombus. Show that the diagonal AC bisects $\angle$ A as well as $\angle$ C and diagonal BD bisects $\angle$ B as well as $\angle$ D.

Ans. ABCD is a rhombus. Therefore, AB = BC = CD = AD

Let O be the point of intersection of diagonals.



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$\angle 1 = \angle 3$ [Angles opposite to equal sides are equal]

$\angle 1 = \angle 4$ [Alternate interior angles]

$\Rightarrow \angle 3 = \angle 4$

Thus, diagonal AC bisects $\angle C$

Also, $\angle 3 = \angle 2$ [Alternate interior angles]

So, we get ,

$\angle 1 = \angle 2$

Thus AC bisects $\angle A$ also.

Similarly,

$\angle 6 = \angle 8$ [Angles opposite to equal sides are equal]

$\angle 6 = \angle 7$ [Alternate interior angles]

$\Rightarrow \angle 7 = \angle 8$

Thus, diagonal BD bisects $\angle B$.

Again, $\angle 8 = \angle 5$ [Alternate interior angles]

So, we get ,

$\angle 6 = \angle 5$

Thus, BD bisects $\angle D$ also.

Hence Proved.

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8. ABCD is a rectangle in which diagonal AC bisects $\angle$ A as well as $\angle$ C. Show that:

(i) ABCD is a square.

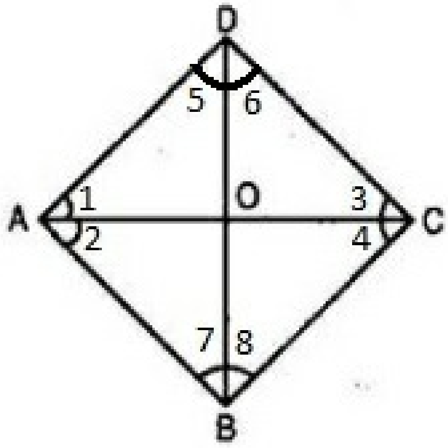
(ii) Diagonal BD bisects both $\angle$ B as well as $\angle$ D.

Ans. ABCD is a rectangle.

Therefore AB = DC ....(i)

And BC = AD

Also $\angle A = \angle B = \angle C = \angle D = 90^\circ$



(i) In $\triangle ABC$ and $\triangle ADC$

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

[AC bisects $\angle A$ and $\angle C$ (given)]

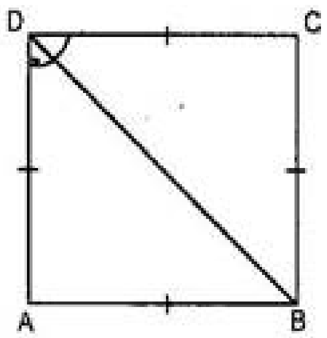
AC = AC [Common]

$\therefore$ $\triangle ABC \cong \triangle ADC$ [By ASA congruency]

$\Rightarrow$ AB = AD ....(ii)

From eq. (i) and (ii), AB = BC = CD = AD

Hence ABCD is a square.



(ii) In $\triangle ABD$ and $\triangle CBD$

AB = BC [Since ABCD is a square]

AD = DC [Since ABCD is a square]

BD = BD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$CBD [By SSS congruency]

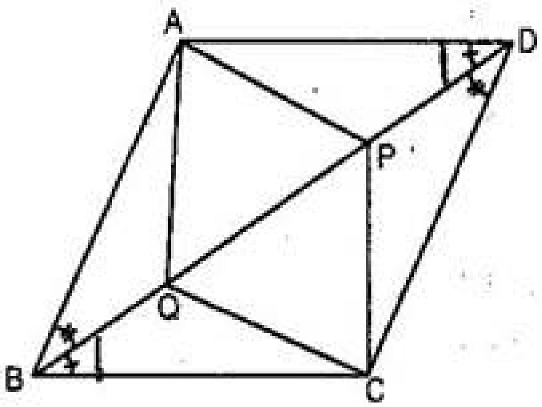
$\Rightarrow$ $\angle$ABD = $\angle$CBD [By C.P.C.T.] ....(iii)

And $\angle$ADB = $\angle$CDB [By C.P.C.T.] ....(iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle$B and $\angle$D.

Hence, Proved.

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (See figure). Show that:



(i) $\triangle$APD $\cong$ $\triangle$CQB

(ii) AP = CQ

(iii) $\triangle$AQB $\cong$ $\triangle$CPD

(iv) AQ = CP

(v) APCQ is a parallelogram.

Ans. (i) In $\triangle$APD and $\triangle$CQB,

DP = BQ [Given]

$\angle$ADP = $\angle$QBC [Alternate angles (AD $\parallel$ BC and BD is transversal)]

AD = CB [Opposite sides of parallelogram]

$\therefore\ \triangle APD \cong \triangle CQB$ [By SAS congruency]

(ii) Since $\triangle APD \cong \triangle CQB$

$\Rightarrow$ AP = CQ [By C.P.C.T.]

(iii) In $\triangle AQB$ and $\triangle CPD$,

BQ = DP [Given]

$\angle ABQ = \angle PDC$ [Alternate angles (AB $\parallel$ CD and BD is transversal)]

AB = CD [Opposite sides of parallelogram]

$\therefore\ \triangle AQB \cong \triangle CPD$ [By SAS congruency]

(iv) Since $\triangle AQB \cong \triangle CPD$

$\Rightarrow$ AQ = CP [By C.P.C.T.]

(v) In quadrilateral APCQ,

AP = CQ [proved in part (i)]

AQ = CP [proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

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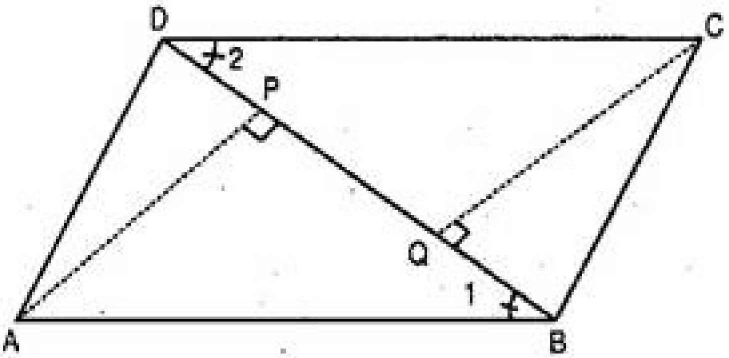
10. ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:



(i) $\triangle$ **APB** $\cong$ $\triangle$ **CQD**

(ii) AP = CQ

Ans. Given: ABCD is a parallelogram. AP $\perp$ BD and CQ $\perp$ BD

To prove: (i) $\triangle$APB $\cong$ $\triangle$CQD (ii) AP = CQ

Proof: (i) In $\triangle$APB and $\triangle$CQD,

$\angle 1 = \angle 2$ [Alternate interior angles]

AB = CD [Opposite sides of a parallelogram are equal]

$\angle$APB = $\angle$CQD = 90°

$\therefore$ $\triangle$APB $\cong$ $\triangle$CQD [By AAS Congruency]

(ii) Since $\triangle$APB $\cong$ $\triangle$CQD

$\therefore$ AP = CQ [By C.P.C.T.]

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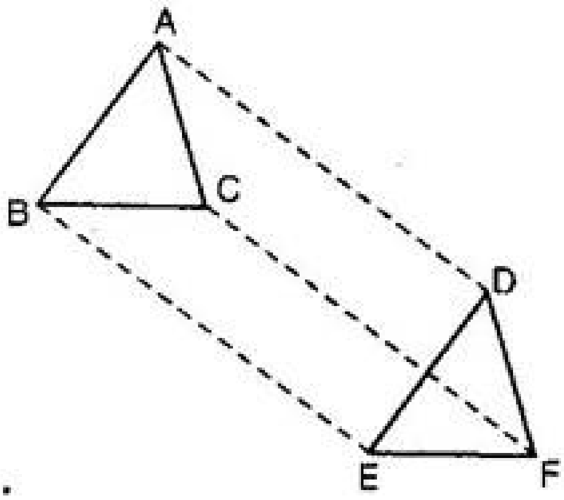
11. In $\triangle$ABC and $\triangle$DEF, AB = DE, AB $\parallel$ DE, BC = EF and BC $\parallel$ EF. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:



(i) Quadrilateral ABED is a parallelogram.

(ii) Quadrilateral BEFC is a parallelogram.

(iii) AD $\parallel$ CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

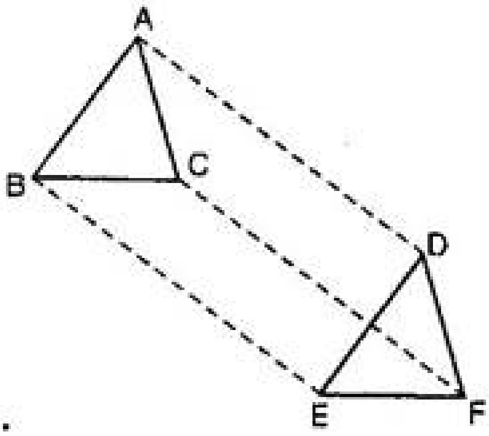
(v) AC = DF

(vi) $\triangle$ ABC $\cong$ $\triangle$ DEF

Ans. (i) AB = DE [Given]

And AB $\parallel$ DE [Given]

$\therefore$ ABED is a parallelogram.



(ii) BC = EF [Given]

And BC $\parallel$ EF [Given]

$\therefore$ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

$\therefore$ AD $\parallel$ BE and AD = BE ....(i)

Also BEFC is a parallelogram.

$\therefore$ CF $\parallel$ BE and CF = BE ....(ii)

From (i) and (ii), we get

$\therefore$ AD $\parallel$ CF and AD = CF

(iv) As AD $\parallel$ CF and AD = CF

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

$\therefore$ AC = DF

(vi) In $\triangle$ABC and $\triangle$DEF,

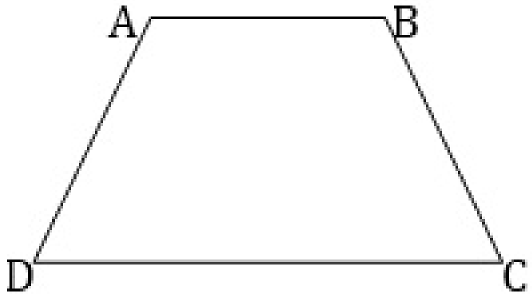
AB = DE [Given]

BC = EF [Given]

AC = DF [Proved in (iii)]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$DEF [By SSS congruency]

12. ABCD is a trapezium in which AB $\parallel$ CD and AD = BC (See figure). Show that:



(i) $\angle$A = $\angle$B

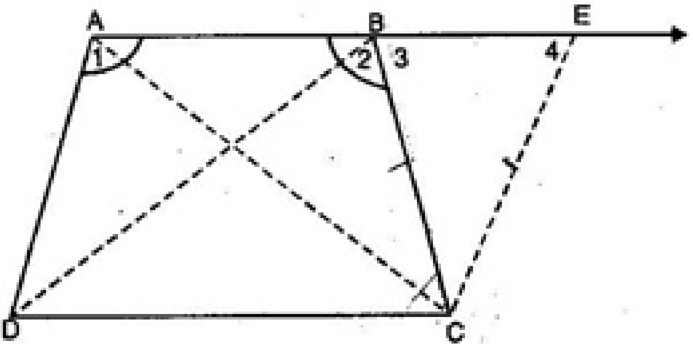
(ii) $\angle$C = $\angle$D

(iii) $\triangle$ABC $\cong$ $\triangle$BAD

(iv) Diagonal AC = Diagonal BD

Ans. Given: ABCD is a trapezium.

AB $\parallel$ CD and AD = BC



To prove: (i) $\angle$A = $\angle$B

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) Diagonal AC = Diagonal BD

Construction: Draw CE $\parallel$ AD and extend AB to intersect CE at E.

Proof: **(i)** As AECD is a parallelogram.

[By construction, CE is parallel to AD and AE is parallel to CD]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now $\angle 1 + \angle 4 = 180^\circ$ [Consecutive Interior angles of a parallelogram]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ [$\because$ $\angle 3 = \angle 4$, so gets cancelled with each other] .... (i)

$\Rightarrow \angle A = \angle B$

(ii) $\angle 3 = \angle C$ [Alternate interior angles]

And $\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [$\triangle$BCE is an isosceles triangle]

$\therefore$ $\angle C = \angle D$

(iii) In $\triangle$ABC and $\triangle$BAD,

AB = AB [Common]

$\angle 1 = \angle 2$ [Proved , see eqn (i)]

AD = BC [Given]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS congruency]

(iv) We had observed that,

$\therefore \Delta ABC \cong \Delta BAD$

$\Rightarrow$ AC = BD [By C.P.C.T.]