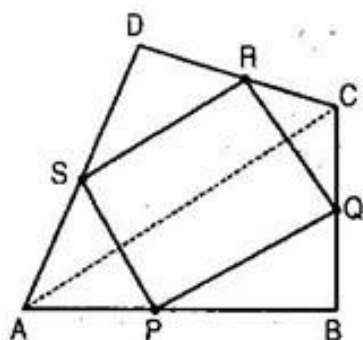


CBSE Class 9 Mathematics
NCERT Solutions
CHAPTER 8
Quadrilaterals(Ex. 8.2)

1. ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:



(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

Ans. In $\triangle ABC$, P is the mid-point of AB and Q is the mid-point of BC.

Then $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

(i) In $\triangle ACD$, R is the mid-point of CD and S is the mid-point of AD.

Then $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) Since $PQ = \frac{1}{2} AC$ and $SR = \frac{1}{2} AC$

Therefore, $PQ = SR$

(iii) Since $PQ \parallel AC$ and $SR \parallel AC$

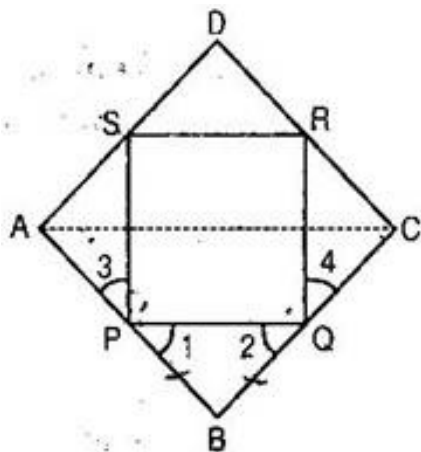
Therefore, $PQ \parallel SR$ [two lines parallel to given line are parallel to each other]

Now $PQ = SR$ and $PQ \parallel SR$

Therefore, PQRS is a parallelogram.

2. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.

Ans. Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.



To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In $\triangle ABC$, P is the mid-point of AB and Q is the mid-point of BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots\dots\dots(i)$$

In $\triangle ADC$, R is the mid-point of CD and S is the mid-point of AD.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \dots\dots\dots(ii)$$

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR$

\therefore PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

$$\therefore AB = BC$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC$$

$$\Rightarrow PB = BQ$$

$\therefore \angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in $\triangle CQR$ and $\triangle APS$, we have,

$AP = CQ$ [P and Q are the mid-points of AB and BC and $AB = BC$]

Similarly, $AS = CR$

and $PS = QR$ [Opposite sides of a parallelogram]

$\therefore \triangle APS \cong \triangle CQR$ [By SSS congruency]

$\Rightarrow \angle 3 = \angle 4$ [By C.P.C.T.]

Now we have

$$\angle 1 + \angle SPQ + \angle 3 = 180^\circ$$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$ [Linear pairs]

$$\therefore \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$$\therefore \angle SPQ = \angle PQR \dots\dots\dots\text{(iii)}$$

Now PQRS is a parallelogram [Proved above]

$$\therefore \angle SPQ + \angle PQR = 180^\circ \text{ [Interior angles] } \dots\dots\dots\text{(iv)}$$

Using eq. (iii) and (iv),

$$\angle SPQ + \angle SPQ = 180^\circ$$

$$\Rightarrow 2 \angle SPQ = 180^\circ$$

$$\Rightarrow \angle SPQ = 90^\circ$$

So, $\angle PQR = 90^\circ$ [Because, $\angle SPQ = \angle PQR$]

And, $\angle SRQ = \angle SPQ = 90^\circ$ [Because, opposite angles of a parallelogram are equal]

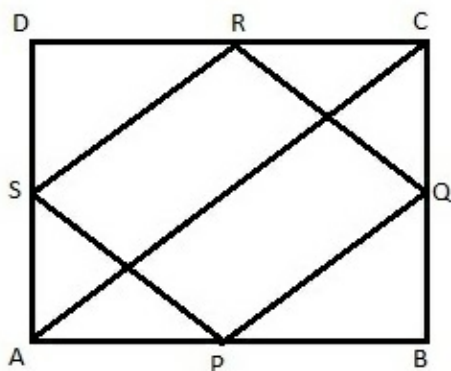
Again $\angle PSR = \angle PQR = 90^\circ$ [Because, opposite angles of a parallelogram are equal]

Thus, all the angles of parallelogram PQRS are 90° .

Hence PQRS is a rectangle. Hence Proved.

3. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans. Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In $\triangle ABC$, P and Q are the mid-points of sides AB, BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots\dots\dots(i)$$

In $\triangle ADC$, R and S are the mid-points of sides CD, AD respectively.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \dots\dots\dots(ii)$$

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR \dots\dots\dots(iii)$

\therefore PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$$\therefore AD = BC$$

$$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$$

$$\Rightarrow AS = BQ \dots\dots\dots(iv)$$

In $\triangle APS$ and $\triangle BPQ$,

$$AP = BP \text{ [P is the mid-point of AB]}$$

$$\angle PAS = \angle PBQ \text{ [Each } 90^\circ \text{]}$$

$$\text{And } AS = BQ \text{ [From eq. (iv)]}$$

$$\therefore \triangle APS \cong \triangle BPQ \text{ [By SAS congruency]}$$

$$\Rightarrow PS = PQ \text{ [By C.P.C.T.] } \dots\dots\dots(v)$$

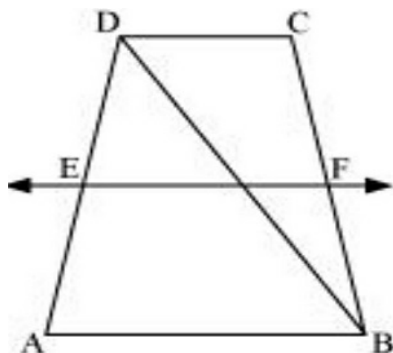
$$\Rightarrow \text{But } PQ = SR \text{ [proved in (ii)]}$$

$$\text{Also, } RQ = PS \text{ [Opposite sides of a parallelogram]}$$

Thus, all the sides of parallelogram PQRS are equal.

Hence, PQRS is a rhombus.

4. ABCD is a trapezium, in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans. Let diagonal BD intersect line EF at point G.

In $\triangle DAB$,

E is the mid-point of AD and $EG \parallel AB$ [$\because EF \parallel AB$ (given) P is the part of EF]

\therefore G is the mid-point of BD.

[Mid point theorem : A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

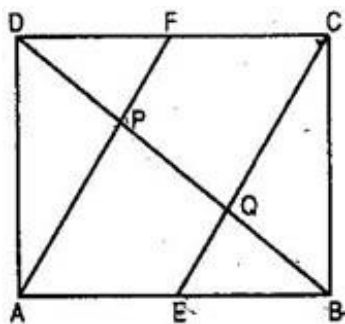
Now in $\triangle BCD$,

G is the mid-point of BD and $GF \parallel DC$ [$\because EF \parallel AB$ (given) and $AB \parallel DC$ (given)]

$\therefore EF \parallel DC$ and PF is a part of EF.

\therefore F is the mid-point of other side, BC of $\triangle BCD$. [Converse of mid-point of theorem]

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans. Since E and F are the mid-points of AB and CD respectively.

$$\therefore AE = \frac{1}{2} AB \text{ and } CF = \frac{1}{2} CD \dots\dots\dots(i)$$

But ABCD is a parallelogram.

$$\therefore AB = CD \text{ and } AB \parallel DC$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD \text{ and } AB \parallel DC$$

$$\Rightarrow AE = FC \text{ and } AE \parallel FC \text{ [From eq. (i)]}$$

\therefore AECF is a parallelogram.

$$\Rightarrow FA \parallel CE \Rightarrow FP \parallel CQ \text{ [FP is a part of FA and CQ is a part of CE](ii)}$$

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle DCQ$, F is the mid-point of CD and $\Rightarrow FP \parallel CQ$

\therefore P is the mid-point of DQ.

$$\Rightarrow DP = PQ \text{(iii)}$$

Similarly, In $\triangle ABP$, E is the mid-point of AB and $\Rightarrow EQ \parallel AP$

\therefore Q is the mid-point of BP.

$$\Rightarrow BQ = PQ \text{(iv)}$$

From eq. (iii) and (iv),

$$DP = PQ = BQ \text{(v)}$$

$$\text{Now } BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$$

$$\Rightarrow BQ = \frac{1}{3} BD \text{(vi)}$$

From eq. (v) and (vi),

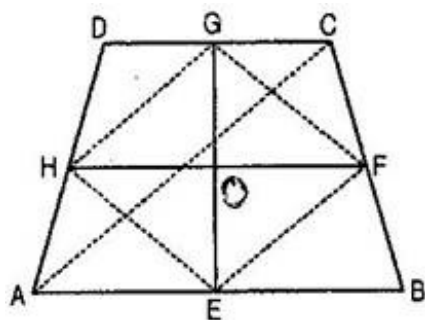
$$DP = PQ = BQ = \frac{1}{3} BD$$

$$\Rightarrow \text{Points P and Q trisect BD.}$$

So AF and CE trisect BD.

6. Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans. Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In $\triangle ABC$, E and F are the mid-points of respective sides AB and BC.

$$\therefore EF \parallel AC \text{ and } EF = \frac{1}{2} AC \dots\dots\dots(i)$$

Similarly, in $\triangle ADC$,

G and H are the mid-points of respective sides CD and AD.

$$\therefore HG \parallel AC \text{ and } HG = \frac{1}{2} AC \dots\dots\dots(ii)$$

From eq. (i) and (ii),

$$EF \parallel HG \text{ and } EF = HG$$

\therefore EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e., diagonals) EG and FH (of parallelogram EFGH) bisect each other.

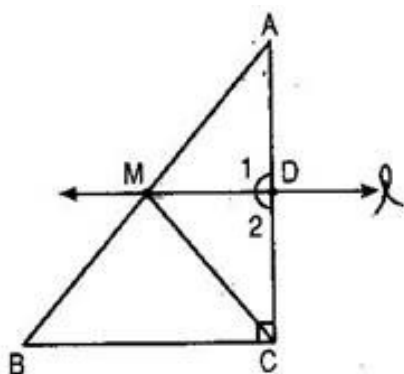
7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that:

(i) D is the mid point of AC

(ii) MD is perpendicular to AC, i.e. $MD \perp AC$

(iii) $CM = MA = \frac{1}{2} AB$

Ans. (i) In $\triangle ABC$, M is the mid-point of AB [Given]



$MD \parallel BC$

$\therefore AD = DC$ [Converse of mid-point theorem]

Thus D is the mid-point of AC.

(ii) $l \parallel BC$ (given) consider AC as a transversal.

$\therefore \angle 1 = \angle C$ [Corresponding angles]

$\Rightarrow \angle 1 = 90^\circ$ [$\angle C = 90^\circ$]

Thus $MD \perp AC$.

(iii) In $\triangle AMD$ and $\triangle CMD$,

$AD = DC$ [proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

$MD = MD$ [common]

$\therefore \triangle AMD \cong \triangle CMD$ [By SAS congruency]

$\Rightarrow AM = CM$ [By C.P.C.T.](i)

Given that M is the mid-point of AB.

$\therefore AM = \frac{1}{2} AB$ (ii)

From eq. (i) and (ii),

$CM = AM = \frac{1}{2} AB$